Math 5531   Lecture 10 - LP (Revisited)

In this talk, we will learn how to solve an LP problem using Excel Spreadsheet through an example.

Gaspar's Diet Problem:

Gaspar's diet requires that all the food he eat come from one of the four "basic food groups": chocolate cake, ice cream, coca cola, and cheesecake). The daily nutrition requirement for Gaspar is: 1800 calories, 20 oz of chocolate, 30 oz of sugar, and 25 oz of fat. The nutritional content per unit of each food is shown in Table 1.

	Calories	Chocolate (oz)	Sugar (oz)	Fat (oz)
1 piece of chocolate cake	400	3	2	2
1 scoop of ice cream	200	2	2	4
1 can of coca cola	150	0	4	1
1 piece of cheesecake	500	0	4	5

Table 1

The market prices of these food are:

	Price
1 piece of chocolate cake	$2.50
1 scoop of ice cream	$0.70
1 can of coca cola	$0.85
1 piece of cheesecake	$3.00

Formulate (and solve) a LP problem to give Gaspar a daily shopping list under his tight student's budget.

LP Formulation

We can set up the following LP problem by letting

$x_1 =$ number of pieces of chocolate cake

$x_2 =$ number of scoops of ice cream

$x_3 =$ number of cans of coca cola

$x_4 =$ number of pieces of cheesecake

$$\min z = 2.5x_1 + 0.7x_2 + 0.85x_3 + 3x_4$$
$$s.t. : \ 400x_1 + 200x_2 + 150x_3 + 500x_4 \geq 1800, \text{ (calories to keep him alive)}$$
$$: \ 3x_1 + 2x_2 \geq 20, \text{ (oz of chocolate to keep him happy)}$$
$$: \ 2x_1 + 2x_2 + 4x_3 + 4x_4 \geq 30, \text{ (oz of sugar to keep him smart)}$$
$$: \ 2x_1 + 4x_2 + x_3 + 5x_4 \geq 25, \text{ (oz of fat to keep him strong)}$$
$$: \ x_1, x_2, x_3, x_4 \geq 0.$$

Setup the LP spreadsheet

Step 1. Input all the parameters of the problem.

Gaspar's Diet Problem							
	Chocolate Cake	Ice Cream	Coca Cola	Cheesecake	Total		Required
Cost	2	0.7	0.85	3			
Calories	400	200	150	500			1800
Chocolate	3	2	0	0			20
Sugar	2	2	4	4			30
Fat	2	4	1	5			25

Step 2. Enter the formulas for the "total" columns after fill the decision variables with feasible values, say 4, 4, 2, 2. ($18.50 is the corresponding cost)

Gaspar's Diet Problem							
	Chocolate Cake	Ice Cream	Coca Cola	Cheesecake	Total		Required
	4.00	**4.00**	**2.00**	**2.00**			
Cost	2	0.7	0.85	3	**18.50**		
Calories	400	200	150	500	**3700.00**		1800
Chocolate	3	2	0	0	**20.00**		20
Sugar	2	2	4	4	**32.00**		30
Fat	2	4	1	5	**36.00**		25

Step 3. Open the "Solver Parameter" box under "tools" menu.

- Set the target cell: total cost. Choose "min";
- Define "changing cells", which are the decision variables;
- Define "constraints", don't forget the "nonnegative constraints for the decision variables.
- Now you are ready to hit "Solve"...

Gaspar's Diet Problem							
	Chocolate Cake	Ice Cream	Coca Cola	Cheesecake	Total		Required
	0.00	**10.00**	**2.50**	**0.00**			
Cost	2	0.7	0.85	3	**9.13**		
Calories	400	200	150	500	**2375.00**		1800
Chocolate	3	2	0	0	**20.00**		20
Sugar	2	2	4	4	**30.00**		30
Fat	2	4	1	5	**42.50**		25

The best diet plan for Gaspar is to have 10 scoops of ice cream and 2.5 cans of coca cola per day, nothing else. This will cost him $9.13 a day. Too much calorie and fat!!!

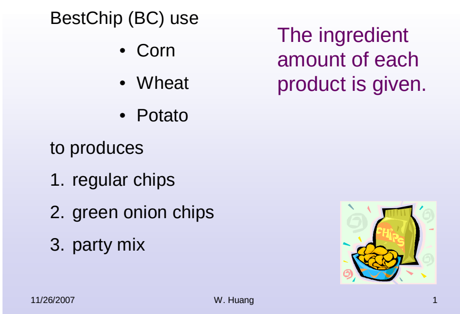
Explore the alternatives:

- First we note that Gaspar is in the risk of gaining weight. We need to restrict his daily intake of calories with 2000. We need to add another constraint. And hit the "solve", we have

Gaspar's Diet Problem							
	Chocolate Cake	Ice Cream	Coca Cola	Cheesecake	Total		Required
	0.00	**7.00**	**4.00**	**0.00**			
Cost	2	0.7	0.85	3	**8.30**		
Calories	400	200	150	500	**2000.00**		1800
Calories	400	200	150	500	**2000.00**		2000
Chocolate	3	2	0	0	**14.00**		20
Sugar	2	2	4	4	**30.00**		30
Fat	2	4	1	5	**32.00**		25

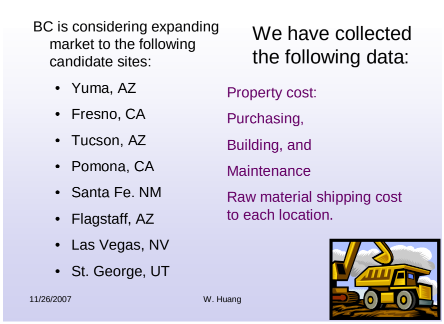
Wait a minute... this solution is not feasible! We need to start with a feasible solution.

Case Study: BestChip



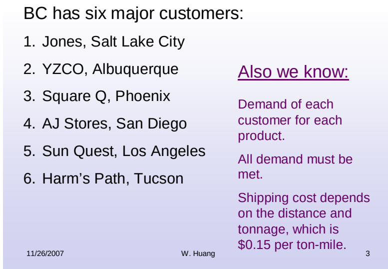
Product-Ingredient Mix

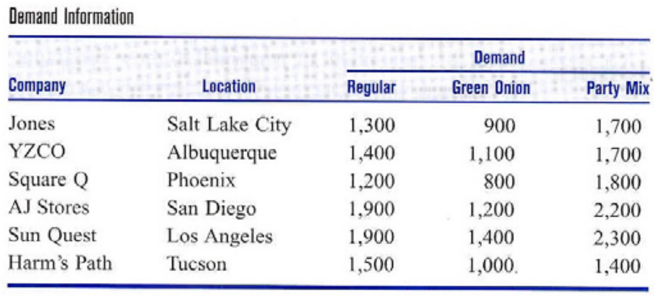
Product	Ingredient Corn	Wheat	Potato
Regular chips	70	20	10
Green onion chips	30	15	55
Party mix	20	50	30



Site Information and Material Shipping Cost

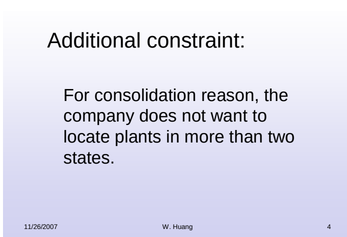
Site Location	Purchase Cost ($/Year)	Material Shipping Cost ($/Ton) Corn	Wheat	Potato
Yuma, AZ	125,000	10	5	16
Fresno, CA	130,000	12	8	11
Tucson, AZ	140,000	9	10	15
Pomona, CA	160,000	11	7	14
Santa Fe, NM	150,000	8	14	10
Flagstaff, AZ	170,000	10	12	11
Las Vegas, NV	155,000	13	12	9
St. George, UT	115,000	14	15	8

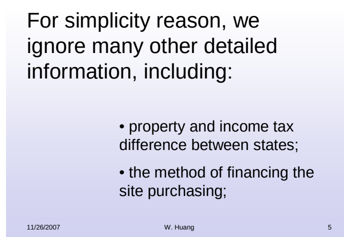


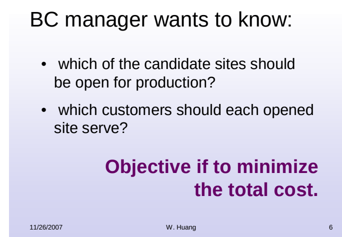


Demand Information

Company	Location	Demand		
		Regular	Green Onion	Party Mix
Jones	Salt Lake City	1,300	900	1,700
YZCO	Albuquerque	1,400	1,100	1,700
Square Q	Phoenix	1,200	800	1,800
AJ Stores	San Diego	1,900	1,200	2,200
Sun Quest	Los Angeles	1,900	1,400	2,300
Harm's Path	Tucson	1,500	1,000	1,400







Other constraints:

- each plant, when opened, can produce 20,000 tons of product (mixed) per year.

LP (or ILP) formulation:

Number the plants, customers, products, ingredients, and states:

$i =$	Plant in
1	Yuma, AZ
2	Fresno, CA
3	Tueson, AZ
4	Pomona, CA
5	Santa Fe, NM
6	Flagstaff, AZ
7	Las Vegas, NY
8	St. George, UT

$j =$	Customer & Location
1	Jones, Salt Lake City
2	YZCO, Albuquerque
3	Square Q, Phenix
4	AJ Stores, San Diego
5	Sun Quest, Los Angeles
6	Harm's Path, Tucson

$k =$	Product
1	Regular chips
2	Green onion chips
3	Party mix

$l =$	ingredients
1	Corn
2	Wheat
3	Potato

$s =$	1	2	3	4	5
States	AZ	CA	NM	NV	UT

We define the following decision variables:

x_{ijk} = tons of product k sent from plant i to customer j;

$y_i = 1$, if plant i is open for production, 0 otherwise;

$\delta_s = 1$, if states s is used, 0 otherwise.

We also define the following intermediate variables:

- f_i = fixed cost associated with opening plant i.

i	f_i
1	125,000
2	130,000
3	140,000
4	160,000
5	150,000
6	170,000
7	155,000
8	115,000

- c_{ij} = cost to ship 1 ton of final products from plant i to customer j.

Distances		SL City	Albuq.	Phoenix	S. Diego	LA	Tucson
		1	2	3	4	5	6
1	Yuma	712	646	186	172	282	242
2	Fresno	818	917	592	350	222	708
3	Tucson	779	497	116	412	488	-
4	Pomona	661	760	345	114	29	460
5	Santa Fe	626	63	526	877	848	558
6	Flagstaff	519	326	144	495	466	260
7	Las Vegas	420	574	288	338	271	404
8	St. George	302	604	434	455	388	550

Unit shipping cost from Plants i to Customers j ($0.15 times the mile distance)

Cij	j = 1	j = 2	j = 3	j = 4	j = 5	j = 6
i = 1	$106.83	$96.89	$27.89	$25.74	$42.33	$36.24
i = 2	$122.66	$137.51	$88.85	$52.46	$33.26	$106.19
i = 3	$116.84	$74.51	$17.39	$61.86	$73.20	$0.00
i = 4	$99.12	$113.97	$51.69	$17.15	$4.41	$69.03
i = 5	$93.95	$9.45	$78.92	$131.49	$127.20	$83.63
i = 6	$77.82	$48.92	$21.63	$74.19	$69.93	$38.96
i = 7	$62.96	$86.03	$43.14	$50.69	$40.62	$60.62
i = 8	$45.35	$90.65	$65.09	$68.19	$58.13	$82.43

- d_{kj} = demand in tons of product k from customer j.

d_{kj}	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
$k = 1$	1300	1400	1200	1900	1900	1500
$k = 2$	900	1100	800	1200	1400	1000
$k = 3$	1700	1700	1800	2200	2300	1400

- m_{il} = cost in dollars of sending 1 ton of raw material l to plant i.

m_{il}	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$
$l = 1$	10	12	9	11	8	10	13	14
$l = 2$	5	8	10	7	14	12	12	15
$l = 3$	16	11	15	14	10	11	9	8

- r_{lk} = percentage of product k that is raw material l.

r_{lk}	$l = 1$	$l = 2$	$l = 3$
$k = 1$	70	20	10
$k = 2$	30	15	55
$k = 3$	20	50	30

We now have the following mixed ILP model

$$\min z = \text{Location purchasing cost} + \text{Shipping for Products} + \text{Shipping for Raw Materials}$$

$$= \sum_{i=1}^{8} f_i y_i + \sum_{i=1}^{8}\sum_{j=1}^{6} c_{ij} \sum_{k=1}^{3} x_{ijk} + \sum_{i=1}^{8}\sum_{l=1}^{3} m_{il} \sum_{k=1}^{3}\sum_{j=1}^{6} r_{lk} x_{ijk},$$

$$s.t\ :\ \sum_{i=1}^{8} x_{ijk} \geq d_{kj},\ j = 1,2,\ldots,6,\ k = 1,2,3 \qquad \text{(Demands need to be met)}$$

$$:\ \sum_{j=1}^{6}\sum_{k=1}^{3} x_{ijk} \leq 20000 y_i,\ i = 1,2,\ldots,8, \qquad \text{(Plant capacity constraints)}$$

$$:\ y_1 + y_3 + y_6 \leq 10\delta_1, \qquad \text{(State AZ is counted)}$$

$$:\ y_2 + y_4 \leq 10\delta_1, \qquad \text{(State CA is counted)}$$

$$:\ y_5 \leq 10\delta_3, \qquad \text{(State NM is counted)}$$

$$:\ y_7 \leq 10\delta_5, \qquad \text{(State NV is counted)}$$

$$:\ y_8 \leq 10\delta_5, \qquad \text{(State UT is counted)}$$

$$:\ \sum_{s=1}^{5} \delta_s \leq 2. \qquad \text{(No more than 2 states are used)}$$

This model has $8 + 5 = 13$ 0-1 integer variables and $8 \times 6 \times 3 = 144$ nonnegative continuous variables.