Lecture 1: A Snapshot of OR

OR From Wikipedia:

"Operations Research or Operational Research (OR) is an interdisciplinary branch of mathematics which uses methods like mathematical modeling, statistics, and algorithms to arrive at optimal or good decisions in complex problems which are concerned with optimizing the maxima (profit, faster assembly line, greater crop yield, higher bandwidth, etc.) or minima (cost loss, lowering of risk, etc.) of some objective function.

The eventual intention behind using Operations Research is to elicit a best possible solution to a problem mathematically, which improves or optimizes the performance of the system.

The terms operations research and management science (MS) are often used synonymously. When a distinction is drawn, MS generally implies a closer relationship to the problems of business management.

Operations research also closely relates to Industrial Engineering (IE). IE takes more of an engineering point of view, and industrial engineers typically consider OR techniques to be a major part of their toolset.

Some of the primary tools used by operations researchers are statistics, optimization, stochastic, queueing theory, game theory, graph theory, decision analysis, and simulation. Because of the computational nature of these fields, OR also has ties to computer science (CS), and operations researchers regularly use custom-written or off-the-shelf software.

Operations research is distinguished by its ability to look at and improve an entire system, rather than concentrating only on specific elements (though this is often done as well). An operations researcher faced with a new problem is expected to determine which techniques are most appropriate given the nature of the system, the goals for improvement, and constraints on time and computing power. For this and other reasons, the human element of OR is vital. Like any other tools, OR techniques cannot solve problems by themselves."

Mathematical Programming and more:

Decision variable(s): x
Objective function(s): $f(x)$
Decision preference: min, max, etc.
Constraints: a set Ω.
A mathematical programming problem takes the form of

$$\max_{x\in\Omega} f(x) \text{ or } \min_{x\in\Omega} f(x), \ldots$$

In a traditional/classical MP problem:

- $x \in R^n$;
- $f : R^n \to R$, and differentiable to the order we need;
- min or max;
- Ω is defined by m equations or inequalities:

$$g_j(x) = \left(\text{or } \geq, \leq, >, <\right) 0, \ \ j = 1, 2, \ldots, m,$$

where $g_j(\cdot) : R^n \to R$ and differentiable to the order we need.

More general versions:

- Linear vs. Nonlinear (LP, NLP, also QP, CP, etc.)
- Discrete vs. Continuous, and non-numerical (IP, CO)
- Finite vs. Infinite dimension (Funtionals, Calculus of Variations, ...)
- Smooth vs. Nonsmooth (Nonsmooth optimization, subderivatives,...)
- Deterministic vs. Stochastic (uncertainty in the parameters and constraints, etc., Stochastic Programming)
- Non Min/Max objective (Equilibrium strategy in Game Theory, Pareto Solution in Multi-Objective Programming,...)
- ......

Who is doing what?

Modeling: convert a real problem into a mathematical model (decision variables, objective function, constraints, what problem it is?)

Solution issues: Is the problem well defined? What is a solution anyway? (in SP, a map or a policy) Does the solution exist? or unique? What properties a solution has? (necessary and sufficient conditions etc.) Can a solution be found in reasonable time? (in CO, complexity issues.) Can the solution be approached? (in NLP, convergence and rate of convergence.) Is the solution what we want? (Local vs. global.) Is the solution stable? (Sensitivity analysis.) Asymptotic behavior,...

Algorithms: It is important to identify the type of problem we are facing, for each type requires different computational approach. The level of difficulty varies from one model to another.

- LP - Simplex methods and more;
- NLP - direct search methods, gradient methods, Newton-type methods;
- IP - cutting plane, branch-and-bound;
- Dealing with constraints - feasible direction, penalty function, Lagrangian method, duality;
- Infinite dimensional variables - discretization, reduced subspace using TBD parameters, duality;
- Discrete problems - total search (finite feasible set), smart search (branch-and-bound, DP, etc.);
- Complicate or hard problems - approximation methods (worst-case analysis, performance ratio, etc.), heuristic methods, simulation, ...
- Computational issues: ...

Challenges and Open Problems:

Many!