Assignment 1, Math 712
Due Jan. 30, in class

1. Prove the Tarski-Vaught Lemma: Suppose that $\mathcal{M} \subset \mathcal{N}$ are $\mathcal{L}$-structures. Moreover, assume that whenever $\varphi(y, x_1, \ldots, x_n)$ is an $\mathcal{L}$-formula such that $\mathcal{N} \models \exists y \varphi(y, a_1, \ldots, a_n)$ for $a_1, \ldots, a_n \in M$ then there is $b \in M$ such that $\mathcal{N} \models \varphi(b, a_1, \ldots, a_n)$. Then the conclusion is that $\mathcal{M} \prec \mathcal{N}$.

2. Prove the Łoś Theorem.

3. Prove that if $(\mathcal{M}_i : i < \alpha)$ is an elementary chain of $\mathcal{L}$-structures i.e. $M_i \prec M_j$ whenever $i < j < \alpha$, then the natural $\mathcal{L}$-structure formed on the union of the M_i's is an elementary extension of each $\mathcal{M}_i$.

4. Show that if $(r_i : i \in I)$ is a bounded I-indexed set of real numbers then $\lim_{\mathcal{U}} r_i$ is well-defined for any ultrafilter $\mathcal{U}$ on I.

5. Prove that if (X, d) is a compact metric space and $\mathcal{U}$ is any u.f. then $\prod_{\mathcal{U}}(X, d) \cong (X, d)$.