MATHEMATICS 1LS3 TEST 1

Day Class
Duration of Examination: 60 minutes
McMaster University, 1 October 2018

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First name (PLEASE PRINT): SOLUTIONS

Family name (PLEASE PRINT): __________

Student No.: __________

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

Total number of points is 40. Marks are indicated next to the problem number. Calculator allowed: McMaster standard calculator Casio fx991MS or Casio fx991MS PLUS or lower Casio which has two lines of display and no graphing capabilities.

EXCEPT ON QUESTIONS 1 AND 2, you must show work to receive full credit.

Problem	Points	Mark
1	10	
2	6	
3	4	
4	7	
5	7	
6	6	
TOTAL	40	

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1. Multiple choice questions: circle ONE answer. No justification is needed.

(a)[2] The average half-life of acetaminophen (active ingredient in tylenol) is 2.5 hours. Assume that a patient is given a dose of 1000 mg of acetaminophen.

Identify all correct statements.

(I) After 5 hours, 250 mg of acetaminophen is left unabsorbed in patient's body. ✓

(II) After 2 hours, 450 mg of acetaminophen is left unabsorbed in patient's body. ✗

(III) After 10 hours, less than 100 mg of acetaminophen is left unabsorbed in patient's body. ✓

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

$$1000 \xrightarrow{2.5\text{hrs}} 500 \xrightarrow{2.5} 250 \xrightarrow{2.5} 125 \xrightarrow{2.5\text{hrs}} 62.5$$

(b)[2] The resistance R of the flow of blood through a curved blood vessel (assumed to have the shape of a cylindrical tube) is given by

$$R = \frac{Kl\gamma^2}{d^4}$$

where l is the length of the tube, d is its diameter and $\gamma \geq 1$ is the curvature. The positive constant K represents the viscosity of the blood (viscosity is a measure of the resistance of fluid to stress; water has low viscosity, honey has high viscosity).

Identify all correct statements.

(I) If the curvature γ doubles, then the resistance R doubles. ✗

(II) If the viscosity K doubles, then the resistance R doubles. ✓

(III) If the diameter d doubles, then the resistance R increases $2^4 = 16$-fold. ✗

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

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(c)[2] What is the domain of the function $f(x) = \ln(5-x) + \log_{10}(x+2)$?

(A) $x < 0$ (B) $x < -2$ (C) $x < 5$ (D) $x < -2$ and $x > 5$
(E) $-2 < x < 0$ (F) $-2 < x < 5$ (G) $0 < x < 5$ (H) $x < -2$ and $x > 0$

$\ln(5-x) \to 5-x>0 \to x<5$
$\log_{10}(x+2) \to x+2>0 \to x>-2$
BOTH needed

(d)[2] Identify all correct statements about the function $f(x) = \dfrac{x^2-1}{x^2+1}$.

(I) $f(x)$ is continuous at $x = 0$. ✓
(II) $x = -1$ is a vertical asymptote of the graph of $f(x)$. ✗
(III) $y = 1$ is a horizontal asymptote of the graph of $f(x)$. ✓

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(e)[2] The formula (adapted from M. Benton and D. Harper, *Basic Palaeontology*)

$$\text{Sk} = e^{0.08T}\,\text{Sp}^{0.84}$$

relates the skull length, Sk, of a larger dinosaur to its spine length, Sp, at the time of death T.

Identify all correct statements.

$\ln \text{Sk} = 0.08T + 0.84 \ln \text{Sp}$

(I) The semilog graph of Sk as a function of Sp is a line. ✗

(II) The semilog graph of Sk as a function of T is a line. ✓

(III) The double log graph of Sk as a function of Sp is a line. ✓

(Note: It does not matter whether ln or log is used.)

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

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2. **True/false questions: circle ONE answer. No justification is needed.**

(a)[2] The formula $H = Mx+4$, where M is a constant, represents a proportional relationship between H and x.

TRUE FALSE

cannot be proportional (if non-zero)

(b)[2] An oscillatory input (intensity function of a group of spiking neurons) is given by the formula $\hat{\lambda}_1(t) = v_0 + a\cos(2\pi f_m(t+d))$. The period of $\hat{\lambda}_1(t)$ is f_m.

TRUE FALSE

$$\text{period} = \frac{2\pi}{2\pi f_m} = \frac{1}{f_m}$$

(c)[2] The limit $\lim_{x\to 0} \frac{|x-1|}{x-1}$ is a real number.

TRUE FALSE

$$= \frac{|0-1|}{0-1} = -1$$

direct substitution

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MATH 1LS3 * Test 1 * 1 October 2018 Name:___________
Student No.: ___________

Questions 3-6: You must show correct work to receive full credit.

3. The visibility index tells us how clearly we can see an object submerged in water. In saltwater, the visibility index is given by the function

$$v(d) = \frac{1.75}{0.79 + \ln(8.3d + 2.61)}$$

where d is the depth in metres, $0 \leq d \leq 50$ (so $d = 0$ labels the surface, and $d = 3$ is 3 m below the surface).

(a)[1] State (in one sentence) what question related to the visibility index is answered by finding the inverse function.

if we know the visibility index, what is the corresponding depth ?

(b)[3] Find the inverse function of $v(d)$. → solve for d

$$0,79v + v\ln(8,3d + 2,61) = 1,75$$

$$\ln(8,3d + 2,61) = \frac{1,75 - 0,79v}{v}$$

$$8,3d + 2,61 = e^{\frac{1,75-0,79v}{v}}$$

so

$$d = \underbrace{\frac{1}{8,3}}_{\approx 0,1205}\left(e^{\frac{1,75-0,79v}{v}} - 2,61\right)$$

can be written as $\frac{1,75}{v} - 0,79$

or: $d = 0,1205e^{\frac{1,75}{v} - 0,79} - 0,3145$

4. (a)[2] What is the domain of the function $f(x) = 9.71 - \arcsin(2x - 5.4)$?

$$-1 \leq 2x - 5.4 \leq 1$$
$$4.4 \leq 2x \leq 6.4$$
$$2.2 \leq x \leq 3.2 \quad \text{or} \quad [2.2, 3.2]$$

(b)[3] The body mass index is defined as BMI$=m/h^2$ where the mass m is in kilograms and the height h in metres. Find a formula for the body mass index in the case where the mass is in kilograms, but the height is measured in centimetres.

$$\text{BMI} = \frac{m}{h^2}\,\frac{[kg]}{[m^2]} = \frac{m\,[kg]}{h^2 \cdot (100)^2\,[cm^2]}$$
$$= \frac{1}{10^4} \cdot \frac{m\,[kg]}{h^2\,[cm^2]}$$

thus $\text{BMI} = 10^4 \cdot \dfrac{m\,[kg]}{h^2\,[cm^2]}$

(c)[2] Consider the ~~data in~~ the table. Which coordinate system (the usual xy-coordinate system, semilog or double-log) is the most suitable to represent this data set? Why?

x	y
0.05	4
0.1	18
1	4400
20	2.6
240	0.1
2500	0.0029

double-log

the ranges of both x and y are large in terms of the sizes of numbers

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5. (a)[3] Consider the formula for human population growth

$$P(t) = 4.43\left(\frac{\pi}{2} + \arctan\frac{t-2007}{42}\right)$$

where t is a calendar year and $P(t)$ is in billions. Find the range of $P(t)$. Based on it, state the maximum world population (two decimal places suffice) predicted by this model.

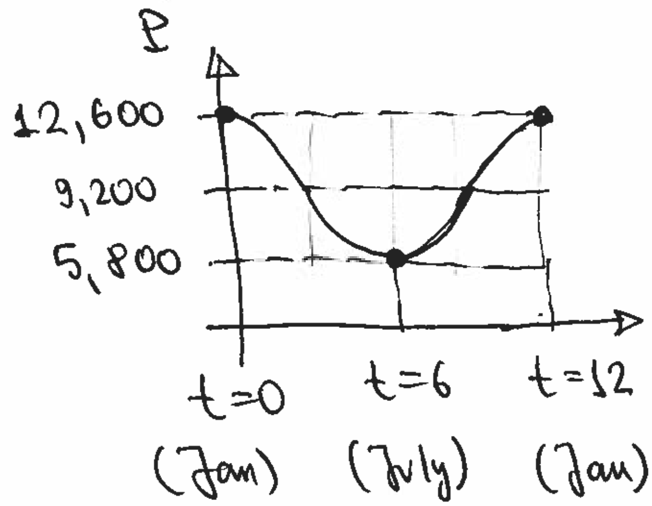
range of $\arctan\frac{t-2007}{42} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

range of $\frac{\pi}{2} + \arctan\frac{\cdot}{\cdot} \to (0, \pi)$

range of $P(t) \to (0, 4.43\pi)$

max world population $= 4.43\pi \approx 13.92$ billion

(b)[4] A population of river sharks (freshwater sharks) in New Zealand changes periodically with a period of 12 months, and is measured at the start of each month. In January, it reaches a maximum of 12,600, and in July it reaches a minimum of 5,800. By selecting an appropriate trigonometric function, find a formula which describes how the population of river sharks changes with time.



use cos t

period $= \frac{2\pi}{a} = 12 \to a = \frac{\pi}{6}$

so $\cos\left(\frac{\pi}{6}t\right)$

average $= \frac{12{,}600 + 5{,}800}{2}$

$= 9{,}200$

amplitude $= 12{,}600 - 9{,}200$

$= 3{,}400$

$$P(t) = 9{,}200 + 3{,}400 \cdot \cos\left(\frac{\pi}{6}t\right)$$

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Name:________________
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6. (a)[3] Find $\lim_{x\to\infty} \left(\ln(2x^3+4) - \ln(x^2+x^3+2)\right)$ or else say that it does not exist.

$$= \lim_{x\to\infty} \ln \frac{2x^3+4}{x^3+x^2+2} = \ln 2$$

because

$$\lim_{x\to\infty} \frac{2x^3+4}{x^3+x^2+2} = \lim_{x\to\infty} \frac{2x^3}{x^3} = 2$$

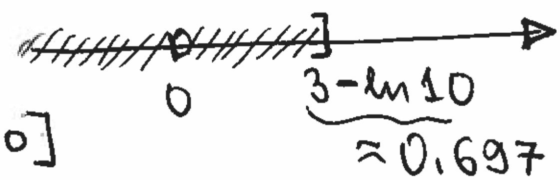
(b)[3] Identify all x for which the function $f(x) = \dfrac{\sqrt{e^{3-x}-10}}{x}$ is continuous.

$\sqrt{\dots}$ $\quad e^{3-x} - 10 \geq 0$

$e^{3-x} \geq 10$, $\quad 3-x \geq \ln 10$

and $x \leq 3 - \ln 10$

fraction .... $\quad x \neq 0$



answer

$(-\infty, 0)$ and $(0, 3-\ln 10]$

or: $(-\infty, 3-\ln 10]$ and not $x=0$

THE END