MATHEMATICS 1LS3 TEST 3

Day Class
Duration of Examination: 60 minutes
McMaster University, 26 November 2018

E. Clements, J. Hofscheier, M. Lovrić

First name (PLEASE PRINT): SOLUTIONS

Family name (PLEASE PRINT): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. USE A PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

Total number of points is 40. Marks are indicated next to the problem number. Calculator allowed: McMaster standard calculator Casio fx991MS or Casio fx991MS PLUS or lower Casio which has two lines of display and no graphing capabilities.

EXCEPT ON QUESTIONS 1 AND 2, you must show work to receive full credit.

Problem	Points	Mark
1	10	
2	6	
3	5	
4	5	
5	7	
6	7	
TOTAL	40	

Continued on next page

Name:_____
Student No.: _____

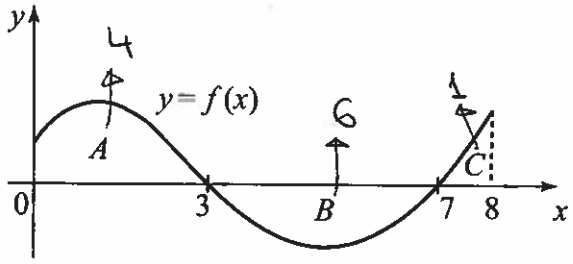
1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] $\int_1^2 \frac{1}{(x-3)^2}\,dx =$

(A) 0 (B) $-1/2$ (C) $-1/3$ (D) $1/2$
(E) $1/3$ (F) $3/2$ (G) $-3/2$ (H) $2/3$

$$= -\frac{1}{x-3}\Big|_1^2 = -(-1) - \left(\frac{1}{2}\right) = \frac{1}{2}$$

(b)[2] In the graph below, the area of A is 4, the area of B is 6 and the area of C is 1 (in some units squared). Identify all correct statements.



(I) $\int_0^8 f(x)\,dx = -1$ (II) $\int_3^8 2f(x)\,dx = -5$ (III) $\int_3^7 (f(x)+3)\,dx = 6$

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

$= 4 - 6 + 1 = -1$

$2 \cdot (-6+1)$
$= -10$

$= \int_3^7 f(x)\,dx \to -6$
$+ \int_3^7 3\,dx \to 12$

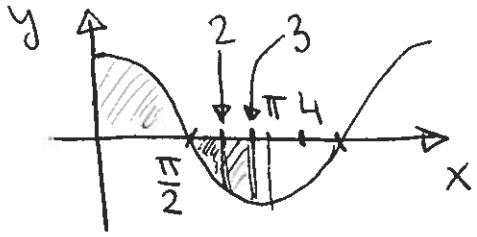
Continued on next page

Name:_____
Student No.: _____

(c)[2] Which of the following definite integral(s) is/are positive? (Hint: Think! No need to calculate the integrals.)

(I) $\int_0^2 \cos x\,dx$ (+) (II) $\int_0^3 \cos x\,dx$ (+) (III) $\int_0^4 \cos x\,dx$ (−)

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three



(d)[2] The average value of $f(x) = \sin x$ on $[0, \pi]$ is

(A) 0 (B) 1 (C) π (D) $\pi/2$
(E) $\pi/4$ (F) $2/\pi$ (G) $1/\pi$ (H) $\pi/8$

$$\text{avg} = \frac{1}{\pi}\int_0^{\pi} \sin x = \frac{2}{\pi}$$

(e)[2] Which of the following improper integrals are *convergent*?

(I) $\int_1^\infty x^{-1.8}\,dx$ ✓ (II) $\int_1^\infty x^{-1}\,dx$ ✗ (III) $\int_1^\infty x^{-0.11}\,dx$ ✗

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

Continued on next page

Name:_____

Student No.: _____

2. True/false questions: circle ONE answer. No justification is needed.

(a)[2] $P(t) = 9e^{0.1t}$ is a solution of the initial value problem $P'(t) = 0.9P(t)$, $P(0) = 10$.

TRUE (FALSE)

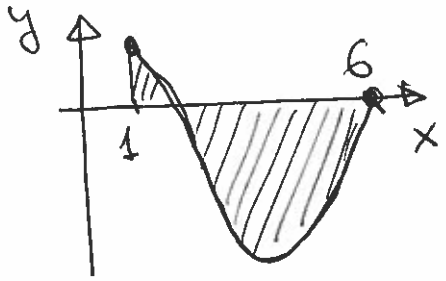
$P' = 0.9e^{0.1t}$

$0.9P = 8.1e^{0.1t}$

(b)[2] It is known that $\int_1^6 f(x)\,dx = -10$. Thus, $f(x) < 0$ for all x in $[1, 6]$.

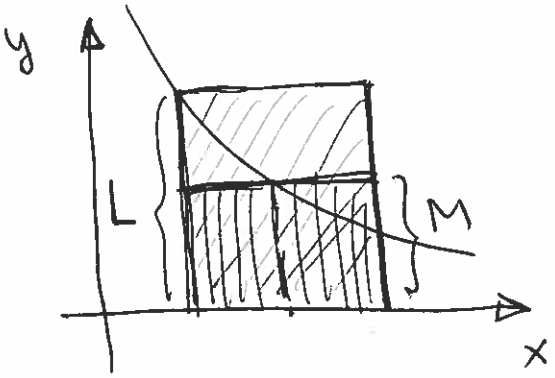
TRUE (FALSE)

Could be:



(c)[2] The left and the midpoint Riemann sums of $f(x) = x^{-1/3}$ on $[2, 12]$ satisfy $M_{15} < L_{15}$.

(TRUE) FALSE



Key: f(x) is a decreasing function

Continued on next page

Name:_____

Student No.: _____

Questions 3-6: You must show correct work to receive full credit.

3. In December 2016, there was a notable increase in influenza cases (caused by the H3N2-like virus) in Winnipeg. Some researchers suggested that the number of influenza cases in Winnipeg could be modelled by

$$I'(t) = 240e^{-0.5t} - 40e^{-0.8t}$$

where t is time in days, with $t = 0$ representing 12 December 2016. On 12 December 2016, there were 230 reported cases of influenza in Winnipeg.

(a) [2] Estimate the number of influenza cases in Winnipeg on 14 December 2016 using Euler's Method with a step size of $\Delta t = 2$. Round your answer to the nearest integer.

$t_0 = 0$, $\Delta t = 2$, $I_0 = 230$ ... 12 Dec.

14 Dec with $\Delta t = 2$ means we need one step:

$t_1 = t_0 + \Delta t = 2$

$I_1 = I_0 + I_0' \Delta t = 230 + 200 \cdot 2 = 630$

$I'(0) = 240 - 40 = 200$

(b) [3] Find a formula for $I(t)$ algebraically and use this formula to find the actual number of influenza cases in Winnipeg on 14 December 2016. Round your answer to the nearest integer.

$$I(t) = \int (240e^{-0.5t} - 40e^{-0.8t})\,dt$$

$$= -480e^{-0.5t} + 50e^{-0.8t} + C$$

$I(0) = 230 \Rightarrow 230 = -480 + 50 + C$

so $C = 660$

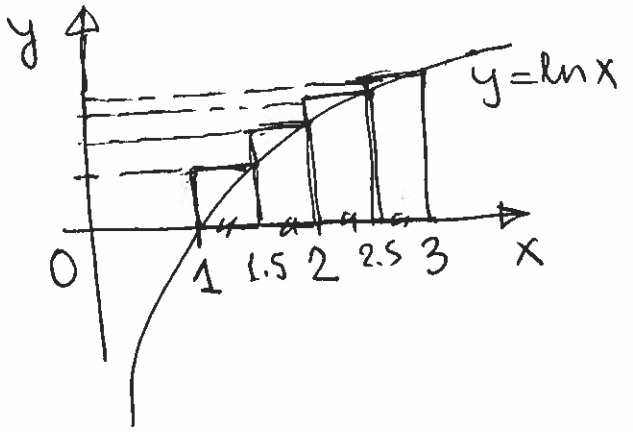
thus $I(t) = -480e^{-0.5t} + 50e^{-0.8t} + 660$

and $I(2) = -480e^{-1} + 50e^{-1.6} + 660$

≈ 493.5, ie. 494 (accept 493)

Continued on next page

4. (a) [2] Find an approximation of the area of the region below the graph of $y = \ln x$ and over the interval $[1, 3]$, using a Riemann sum with 4 rectangles and right endpoints. Round your answer to three decimal places. Sketch the function and the four rectangles involved.



$$R_4 = \frac{1}{2}(\ln 1{,}5 + \ln 2 + \ln 2{,}5 + \ln 3)$$

$$\approx 1{,}557$$

(b) [3] Find the exact area of the region in part (a) by evaluating $\int_1^3 \ln x \, dx$. Round your answer to three decimal places.

$$\int_1^3 \ln x \, dx = \left\{ \begin{array}{l} u = \ln x \longrightarrow u' = \frac{1}{x} \\ v' = 1 \longrightarrow v = x \end{array} \right\}$$

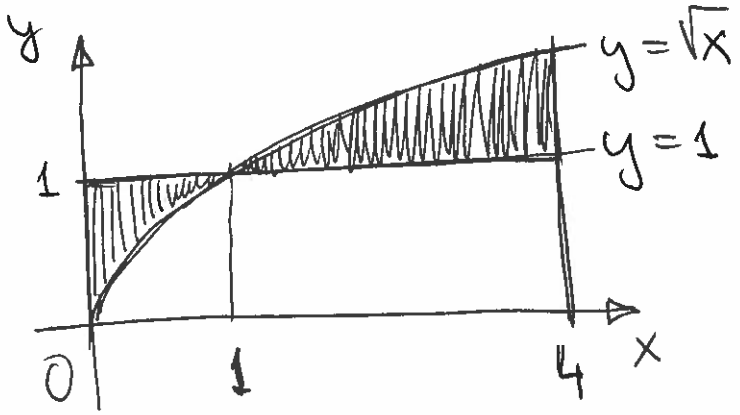
$$= uv - \int v u' \, dx = x \ln x - \int 1 \, dx = x \ln x - x + C$$

$$\int_1^3 \ln x \, dx = (x \ln x - x)\Big|_1^3 = (3\ln 3 - 3) - (\underbrace{\ln 1}_{0} - 1)$$

$$= 3\ln 3 - 2 \approx 1{,}296$$

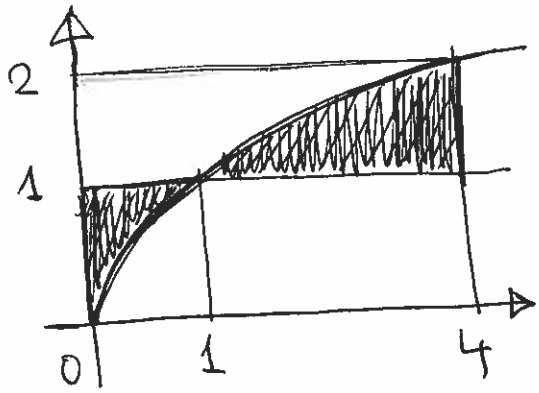
Continued on next page

5. (a)[3] Sketch (shade) the region bounded by the graphs of $y = \sqrt{x}$, $y = 1$, $x = 0$ and $x = 4$. Set up, but **do not evaluate**, the formula for its area. Your formula should not include absolute value.



$$A = \int_0^1 (1 - \sqrt{x})\,dx + \int_1^4 (\sqrt{x} - 1)\,dx$$

(b)[4] Set up a formula for the volume of the solid obtained by rotating the region in part (a) about the y-axis. Your formula should not include absolute value. **Do not evaluate the integral.**



$y = \sqrt{x} \to x = y^2$

$$V = \pi \int_0^1 (y^2)^2 - (0)^2\,dy + \pi \int_1^2 (4)^2 - (y^2)^2\,dy$$

$$= \pi \int_0^1 y^4\,dy + \pi \int_1^2 (16 - y^4)\,dy$$

Continued on next page

6. A blood concentration of acetaminophen (common pain reliever) higher than 200 mcg/mL (micrograms per millilitre), reached 4 hours after ingestion, is known to increase the risk of liver damage. Even without taking a medication, a small amount of acetaminophen can be found in the body; consequently, we assume that the initial concentration is 8 mcg/mL. (Source: University of Rochester Medical Centre.)

Suppose that the concentration of acetaminophen in the blood of a patient, when following a certain protocol (such as after a minor surgery), changes according to $c'(t) = 40.2te^{-0.1t^2}$, measured in mcg/mL per hour. The dosing protocol (ingestion) starts when $t = 0$.

(a) [3] Find the indefinite integral $\int 40.2te^{-0.1t^2}\,dt$.

$$= \left\{ \begin{array}{l} u = -0.1t^2 \\ du/dt = -0.2t \longrightarrow tdt = \dfrac{du}{-0.2} \end{array} \right\}$$

$$= 40.2 \int e^u \frac{du}{-0.2} = -201 e^{-0.1t^2} + C$$

(b) [2] Determine whether a patient, subjected to the dosing protocol described above, faces an increased risk of liver damage.

need to

Compute $C(4)$ $\quad c(t) = -201 e^{-0.1t^2} + C$

$$c(0) = 8 \longrightarrow c(t) = -201 e^{-0.1t^2} + 209$$

so $\quad c(4) = -201 e^{-0.1\cdot(4)^2} + 209 \approx 168.42 < 200$

NO

(c) [2] What does the integral $\int_0^\infty 40.2te^{-0.1t^2}\,dt$ represent, and what are its units?

rate of change

it's the total change in concentration

units: mcg/mL

THE END