ASSIGNMENT 10

1.(a) To define how close we wish the values of f(x) to be to L, we take an interval around L (for instance, (L-0.1, L+0.1), (L-0.005, L+0.005), etc.). Then $\lim_{x\to a} f(x) = L$ means that we can make the values of f fall within these intervals if we pick x close enough to a (but not x=a)

(b) Numeric calculation of limits is not reliable.

(c) The limit is 1

(d) The limit of a sum (difference, product, quotient) is the sum (diff., prod., quotient) of the limits, provided that all limits involved are real numbers

(e) For algebraic and some transcendental functions,

$$\lim_{x\to a} f(x) = f(a)$$

if a is in the domain of f

2.(a) The values of f(x) can be made larger than any number if x is close enough to a

(b) As $x\to 0$ from the right, the values of f(x) increase beyond any bounds (ie, $\lim_{x\to 0^+} f(x) = +\infty$); thus, the limit of $\frac{1}{x}$ as $x\to 0$ cannot be a real #. As $x\to 0^-$, the values of f(x) fall below any bounds (ie, $\lim_{x\to 0^-} f(x) = -\infty$).

Thus, $\lim_{x\to 0} f(x)$ does not exist.

(c) A vertical line $x=a$ is a V.A. of the graph of $y=f(x)$ if $\lim f(x)=+\infty$ or $-\infty$ as $x\to a$, or $x\to a^+$, or $x\to a^-$

(d) A horizontal line $y=L$ is a H.A. of the graph of $y=f(x)$ if $\lim f(x)=L$ as $x\to\infty$ or $x\to-\infty$

(e) $x^{3.7}$

(f) $x^{-3.7}$

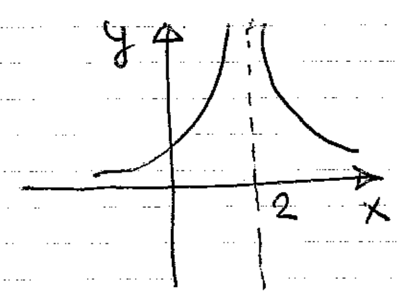
3. (a) $f(x)$ is continuous at every point in that interval

(b) The constant function, the identity function, $y=e^x$, $y=\ln x$, $y=|x|$ and $y=\sin x$, $y=\cos x$ are continuous at all points where they are defined

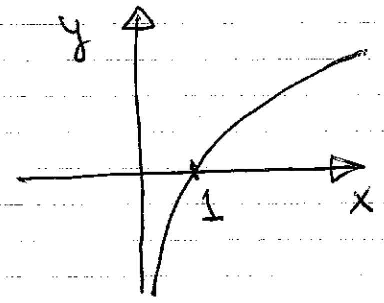
(c) The sum (product, difference, quotient) of cont. functions are continuous functions (in case of quotient, we need to assume that the denominator is not 0)

(d) $y=\frac{1}{x}$; $y=\ln 0$; $y=\begin{cases} 1 & x>0 \\ 2 & x\le 0\end{cases}$

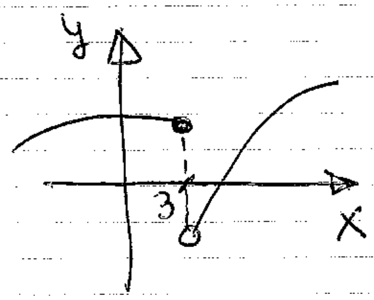
or graphs:



$\frac{1}{(x-2)^2}$

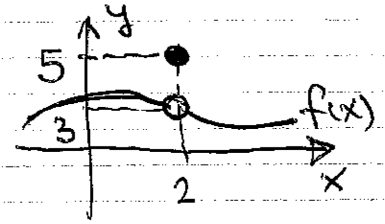


$\log_{10} x$



4.(a) $\lim_{x\to 2} f(x) = f(2)$

(b)



$\lim_{x\to 2} f(x) = 3 \neq f(2) = 5$

(c) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2-1) = 3$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (2.5 + \frac{1}{x}) = 3$

$\lim_{x\to 2} f(x) = 3$

Since $f(2) = 2.5 + \frac{1}{2} = 3$, it follows that f is cont. at 2

5. Need to check each piece, and all x values where the definition of f changes

$1 - \frac{1}{x}$ is continuous if $x \neq 0$ .... so $f(x)$ is not cont. at $x=0$

$\frac{1}{x-1}$ is continuous if $x \neq 1$ ... $f(x)$ is cont. at $x=1$ since $f(x) = 1 - \frac{1}{x}$ if $x=1$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (1 - \frac{1}{x}) = 1 - \frac{1}{2} = \frac{1}{2}$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{1}{x-1} = \frac{1}{2-1} = 1$

$\lim_{x\to 2} f(x)$ does not exist

$\Longrightarrow$ f is not cont. at $x=0$ and at $x=2$

6. ln $\Rightarrow$ $e^x - 1 > 0$, $e^x > 1$ so $x > \ln 1 = 0$

So, as long as $x > 0$, $\ln(e^x - 1)$ is cont. at x.