ASSIGNMENT 11

1. (a)
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{3(x+h)\cancel{-5}-(3x\cancel{-8})}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{3x}+3h-\cancel{3x}}{h}=\lim_{h\to 0}\frac{3\cancel{h}}{\cancel{h}}=3$$

(b)
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{1-(x+h)^2-(1-x^2)}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{1}-\cancel{x^2}-2xh-h^2-\cancel{1}+\cancel{x^2}}{h}=\lim_{h\to 0}\frac{\cancel{h}(-2x-h)}{\cancel{h}}=-2x$$

(c)
$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}\cdot\frac{\sqrt{x+\Delta x}+\sqrt{x}}{\sqrt{x+\Delta x}+\sqrt{x}}$$
$$=\lim_{\Delta x\to 0}\frac{\cancel{x}+\Delta x-\cancel{x}}{\Delta x\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}=\lim_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

(d)
$$\lim_{\Delta x\to 0}\frac{\frac{2}{x+\Delta x}-\frac{2}{x}}{\Delta x}=\lim_{\Delta x\to 0}\frac{2x-2(x+\Delta x)}{x(x+\Delta x)}\cdot\frac{1}{\Delta x}$$
$$=\lim_{\Delta x\to 0}\frac{-2}{x(x+\Delta x)}=-\frac{2}{x^2}$$

(e)
$$\lim_{\Delta x\to 0}\frac{\frac{1}{\sqrt{x+\Delta x}}-\frac{1}{\sqrt{x}}}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sqrt{x}-\sqrt{x+\Delta x}}{\sqrt{x}\sqrt{x+\Delta x}}\cdot\frac{1}{\Delta x}$$
$$=\lim_{\Delta x\to 0}\frac{x-(x+\Delta x)}{\sqrt{x}\sqrt{x+\Delta x}\cdot\Delta x\left(\sqrt{x}+\sqrt{x+\Delta x}\right)}$$

multiply and divide by $\sqrt{x}+\sqrt{x+\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{-1}{\sqrt{x}\,\sqrt{x+\Delta x}\,\left(\sqrt{x} + \sqrt{x+\Delta x}\right)}$$

$$= \frac{-1}{\sqrt{x}\sqrt{x}\cdot 2\sqrt{x}} = -\frac{1}{2x\sqrt{x}} = -\frac{1}{2x^{3/2}}$$

(f) $$\lim_{\Delta x \to 0} \frac{\frac{x+\Delta x}{x+\Delta x+1} - \frac{x}{x+1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)(x+1) - x(x+\Delta x+1)}{(x+\Delta x+1)(x+1)\cdot \Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + \cancel{x\Delta x} + \cancel{x} + \Delta x - \cancel{x^2} - \cancel{x\Delta x} - \cancel{x}}{(x+\Delta x+1)(x+1)\,\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{(x+\Delta x+1)(x+1)} = \frac{1}{(x+1)^2}$$

2. (a) $$y' = 1 + 0 + (-1)x^{-2} = 1 - \frac{1}{x^2}$$

(b) $$y = \sqrt{12}\cdot x^{-1} + \frac{1}{\sqrt{12}}\cdot x$$

so $$y' = \sqrt{12}(-1)x^{-2} + \frac{1}{\sqrt{12}}\cdot 1 = \frac{-\sqrt{12}}{x^2} + \frac{1}{\sqrt{12}}$$

(c) $$y = \sqrt{3}\cdot x^{1/2} + \sqrt{3}\cdot x$$

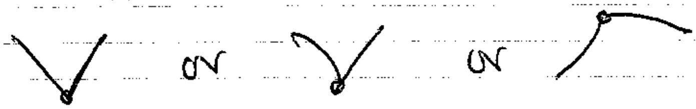
so $$y' = \sqrt{3}\cdot \frac{1}{2}x^{-\frac{1}{2}} + \sqrt{3}\cdot 1 = \frac{\sqrt{3}}{2\sqrt{x}} + \sqrt{3}$$

3. $$f'(x) = \frac{(2x-6x^2)\overbrace{(\quad)}^{12+x^2} - (x^2-2x^3-4)(2x)}{(12+x^2)^2}$$

so $$f'(2) = \frac{(-20)(16) - (-16)(4)}{16^2} = -1$$

4. (a) A function is differentiable at x if $f'(x)$ is a real number, where $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

(b) A corner is a shape obtained by curves, or lines, which look like:

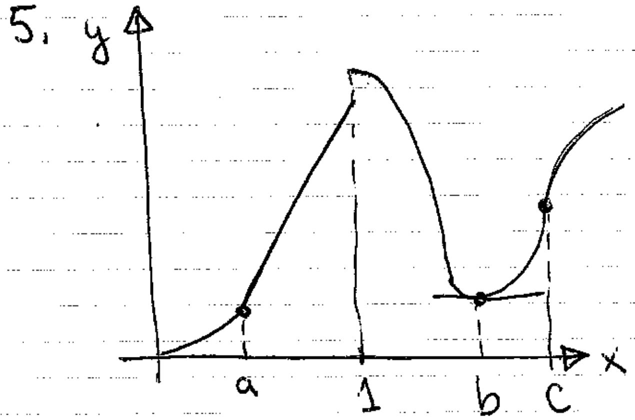


A function is not differentiable at a corner, because it does not have a tangent line there (the slope of the tangent is not defined).

(c) A point in the domain of a function $f(x)$ where $f'(x)=0$ or $f'(x)$ does not exist

(d) monkeys/ time

5.



a ... not diff. (corner)

1 ... not cont.

b ... derivative is zero

c ... vertical tangent so not diff.

6. slope $= f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$$= \lim_{h\to 0} \frac{1+h-2\sqrt{1+h}-(\cancel{1}-2\sqrt{1})}{h} = \lim_{h\to 0} \frac{2+h-2\sqrt{1+h}}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2-(2\sqrt{1+h})^2}{h(2+h+2\sqrt{1+h})}$$

multiply and divide by $2+h+2\sqrt{1+h}$

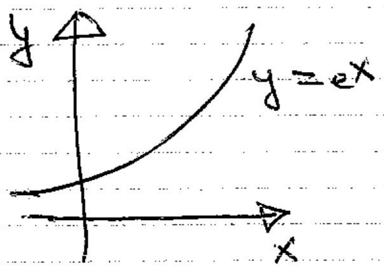
$$= \lim_{h\to 0} \frac{\cancel{4}+\cancel{4h}+h^2-\cancel{4}-\cancel{4h}}{h\,(2+h+2\sqrt{1+h}\,)}$$

$$= \lim_{h\to 0} \frac{h}{2+h+2\sqrt{1+h}} = 0$$

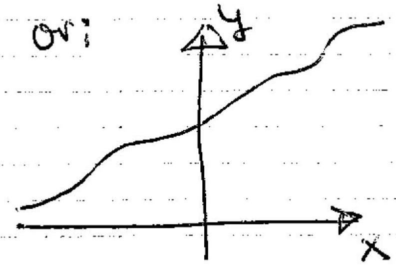
when $x=1$, $f(1) = 1-2\sqrt{1} = -1$

tangent: $y-(-1) = 0\,(x-1) \Rightarrow y=-1$

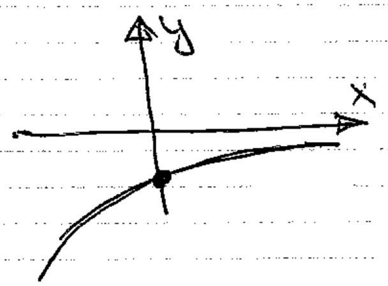
7. (a) positive for all x → above x-axis
derivative positive → all slopes are positive (f. is increasing)



or:



(b) need a function which is below x-axis and increasing:



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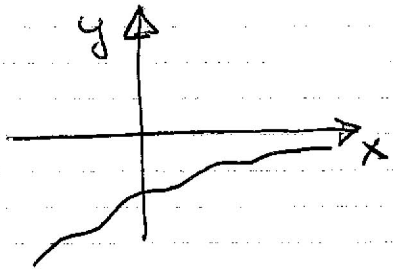
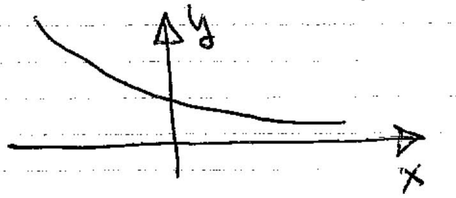


8. (a) FALSE for example

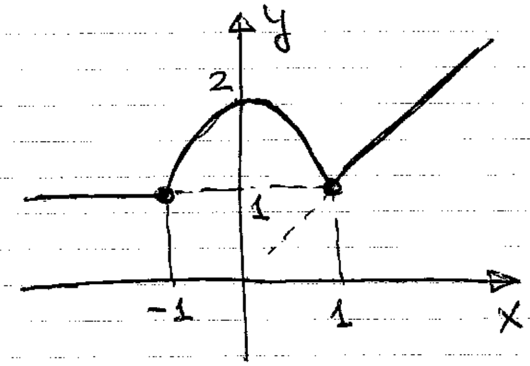


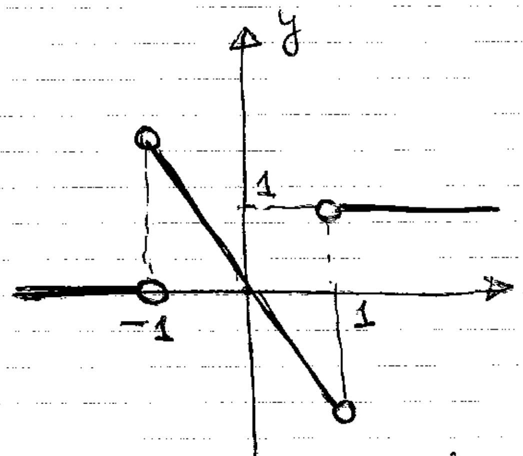
is a positive function
but its derivative (slopes)
is (are) negative

(b) FALSE; for instance, $y=x$ is increasing, $y'=1$ is not increasing

9. $y' = 3x^2 - 2x - 1 = 0 \to (3x+1)(x-1)=0 \to x = -1/3$
$\to x = 1$

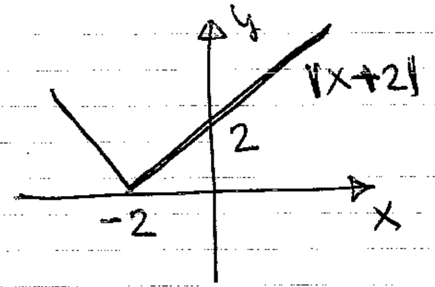
10.





f' is not defined at $x = \pm 1$ (corners)

11.



$|x+2|$ is not diff. at $x=-2$ (corner)