ASSIGNMENT 12

1. (a) $y' = a + be^x$

(b) $y' = a(e^{bx} + xe^{bx} \cdot b)$

(c) $y' = \frac{1}{ax+b} \cdot a$

(d) $y' = \frac{1}{ax} \cdot a + \frac{1}{bx} \cdot b = \frac{2}{x}$

can simplify! $y = \ln a + \ln x + \ln b + \ln x$,

so $y' = 0 + \frac{1}{x} + 0 + \frac{1}{x} = \frac{2}{x}$

(e) $y' = a\left(1 \cdot \ln(bx) + x \cdot \frac{1}{bx} \cdot b\right)$

$= a(\ln(bx) + 1)$

(f) $y' = \frac{1}{\cancel{a}x} \cdot \cancel{a} \cdot \ln(bx) + \ln(ax) \cdot \frac{1}{\cancel{b}x} \cdot \cancel{b}$

$= \frac{\ln(bx) + \ln(ax)}{x}$

2. $(g \circ f)(1) = g(f(1)) = g(3) = 2$

$(g \circ f)'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot f'(1)$

$= (-1) \cdot \frac{1}{2} = -\frac{1}{2}$

3. $x = 1 \longrightarrow y = \frac{e+1}{1} = e+1$ ... point is $(1, e+1)$

$$y' = \frac{e^x \cdot x - (e^x + 1)}{x^2} \rightarrow y'(1) = \frac{e - (e+1)}{1} = -1$$

so slope $= -1$

tangent: $y - (e+1) = -1(x-1)$

$$y - e - 1 = -x + 1 \rightarrow y = -x + e + 2$$

4.(a) $y' = 6(x^3 - x - 1)^5 (3x^2 - 1)$

(b) $y' = \frac{1}{2}(x^4 - 14x)^{-1/2}(4x^3 - 14) + 2(\sqrt{x} + 1) \cdot \frac{1}{2\sqrt{x}}$

(c) $y' = 1 \cdot e^{x^2+x} + x \cdot e^{x^2+x}(2x+1)$

(d) $y' = e^x + ex^{e-1} + 0$

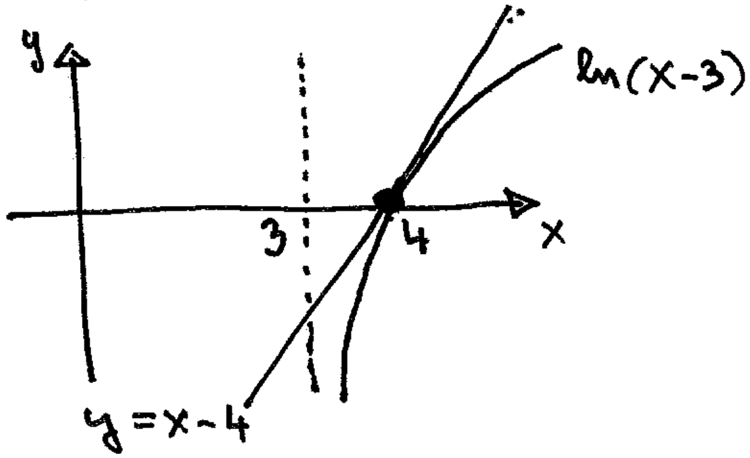
(e) $y' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{2}(e^x)^{-1/2} \cdot e^x$

5. (a) $x = 4 \rightarrow y = \ln(4-3) = \ln 1 = 0$

$y' = \frac{1}{x-3} \rightarrow$ slope at $x=4$ is $y'(4) = 1$

tangent: $y - 0 = 1(x-4)$, ie $y = x - 4$

(b)



6.(a) $\ln x \ldots\ x>0$

denominator .... $\ln x + 1 \neq 0$, $\ln x \neq -1$

so $x \neq e^{-1}$

domain: $x>0$ and $x \neq e^{-1}$

or $(0, e^{-1}), (e^{-1}, \infty)$

(b) $g(1) = \dfrac{1}{\ln 1 + 1} = 1$ ($\ln 1 = 0$!)

$g'(x) = (-1)(\ln x + 1)^{-2} \cdot \dfrac{1}{x}$

$g'(1) = (-1)(1)\left(\dfrac{1}{1}\right) = -1$

7. (a) $y' = \dfrac{a\frac{1}{x}(c\ln x + d) - (a\ln x + b)\cdot c\frac{1}{x}}{(c\ln x + d)^2}$

(b) $f'(x) = \ln(1+e^x) + x \cdot \dfrac{1}{1+e^x} \cdot e^x$

(c) $f(x) = \frac{1}{2}\ln x + \ln(\sqrt{x}+1)$, so

$f'(x) = \dfrac{1}{2}\cdot\dfrac{1}{x} + \dfrac{1}{\sqrt{x}+1}\cdot\dfrac{1}{2\sqrt{x}}$

(d) $f(x) = \ln(x^2 e^x) - \ln(x^3+1)$

$= \ln(x^2) + \ln(e^x) - \ln(x^3+1)$

$= 2\ln x + x - \ln(x^3+1)$, so

$f'(x) = \dfrac{2}{x} + 1 - \dfrac{1}{x^3+1}\cdot 3x^2$

8. (a) $e^{xy^2}(1\cdot y^2+x\cdot 2y\cdot y')-1-2yy'=0$

$$2xye^{xy^2}y'-2yy'=1-y^2e^{xy^2}$$

$$y'=\frac{1-y^2e^{xy^2}}{2xye^{xy^2}-2y}$$

(b) $\cos x+\cos x\sin y+\sin x\cos y\cdot y'=-\sin(xy)(y+xy')$

$$\sin x\cos y\, y'+\sin(xy)\,x y'=-\sin(xy)\,y-\cos x-\cos x\sin y$$

$$y'=\frac{-y\sin(xy)-\cos x-\cos x\sin y}{\sin x\cos y+\sin(xy)\cdot x}$$

(c) $(-1)(x^2+y^2)^{-2}(2x+2yy')=1 \quad \Big|\cdot(-1)\cdot(x^2+y^2)^2$

$$2x+2yy'=-(x^2+y^2)^2$$

$$y'=\frac{-(x^2+y^2)^2-2x}{2y}$$