ASSIGNMENT 13

1(a) $f'(x) = \cos(2x)\cdot 2 + 2\cos x$

(b) $y' = 2\cos x(-\sin x) - \sin(x^2)\cdot 2x$

(c) $f'(x) = \sec^2(x^2-5x+7)\cdot(2x-5)$

(d) $f'(x) = e^{\tan(x^2-5x+7)}\cdot\sec^2(x^2-5x+7)\cdot(2x-5)$

(e) $y' = 2x\cdot\sin\frac{1}{x} + x^2\cdot\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)$

2 (a) $z' = \dfrac{1}{1+x^2} + \dfrac{1}{1+(x^2)^2}\cdot 2x$

(b) $z' = \dfrac{1}{\sqrt{1-x^2}} + 2\arcsin x\cdot\dfrac{1}{\sqrt{1-x^2}}$

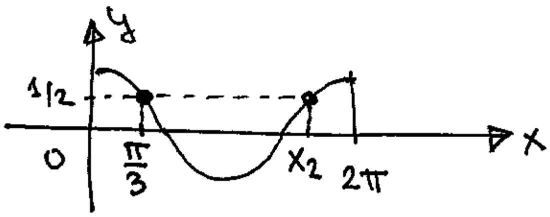
(c) $z' = \dfrac{1}{\sqrt{1-(x^3-11x+4)^2}}\cdot(3x^2-11)$

(d) $z' = \cos(\arctan x)\cdot\dfrac{1}{1+x^2}$

$+\sec^2(\arcsin x)\cdot\dfrac{1}{\sqrt{1-x^2}}$

3. $y' = 1 - 2\cos x = 0$

$\rightarrow \cos x = \frac{1}{2}$



$\cos\frac{\pi}{3} = \frac{1}{2}$

$x_2 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

answer: $x = \frac{\pi}{3} + 2\pi k,\ \frac{5\pi}{3} + 2\pi k$

[note: you can use unit circle instead of a graph]

4. use quotient rule:

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

5.

$$(\sec x)' = \left(\frac{1}{\cos x}\right)' = \left((\cos x)^{-1}\right)'$$

$$= (-1)(\cos x)^{-2} \cdot (-\sin x)$$

$$= \frac{\sin x}{\cos x \cdot \cos x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \cdot \tan x$$

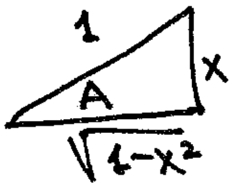
6. $\sin(\arcsin x) = x \quad \Big|\,'$

$\cos(\arcsin x) \cdot (\arcsin x)' = 1$

$\rightarrow (\arcsin x)' = \dfrac{1}{\cos(\arcsin x)} = \sqrt{1-x^2}$

simplify: $\cos(\underbrace{\arcsin x}_{A}) = \cos A = \sqrt{1-x^2}$

$A = \arcsin x$, so $\sin A = x$



7. $\tan(\arctan x) = x \quad \Big|\,'$

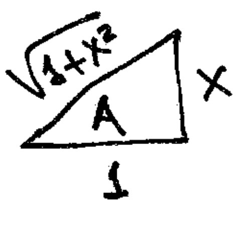
$\sec^2(\arctan x) \cdot (\arctan x)' = 1$

$\rightarrow (\arctan x)' = \dfrac{1}{\sec^2(\arctan x)} = \dfrac{1}{1+x^2}$

simplify: $\sec^2(\underbrace{\arctan x}_{A}) = \sec^2 A$

$A = \arctan x$

$\tan A = x$



so $\cos A = \dfrac{1}{\sqrt{1+x^2}}$

$\sec A = \sqrt{1+x^2}$

8.(a)

$$y' = -2.3\sin t - 1.9\cos t$$
$$\left.\begin{aligned} y'' &= -2.3\cos t + 1.9\sin t \\ -y &= -2.3\cos t + 1.9\sin t \end{aligned}\right\} \text{ equal!}$$

(b) for sint and cost, 4th derivative is the same as the original function!

so if $y = 2.3\cos t - 1.9\sin t$

$$y^{(4)} = 2.3\cos t - 1.9\sin t$$

(c)

$$y' = -2\sin 2t - 2\cos 2t$$
$$\left.\begin{aligned} y'' &= -4\cos 2t + 4\sin 2t \\ -4y &= -4\cos 2t + 4\sin 2t \end{aligned}\right\} \text{ yes!}$$

(d)

$$y = \sin(0.5x)$$
$$y' = 0.5\cos(0.5x)$$
$$y'' = 0.5\,(-\sin(0.5x))\cdot 0.5$$
$$= -0.25\sin(0.5x)$$

.... $y'' = -0.25y$ ie $y'' + 0.25y = 0$

END