ASSIGNMENT 15

1.(a) $T_3(x) = \underbrace{f(4) + f'(4)(x-4)}_{T_1(x) \text{ or } L_4(x)} + \frac{f''(4)}{2!}(x-4)^2 + \frac{f'''(4)}{3!}(x-4)^3$ ($2!=2$, $3!=6$)

$T_2(x)$

$f, f', \ldots$	at $a=4$
$f = \sqrt{x}$	$f(4)=2$
$f' = \frac{1}{2\sqrt{x}}$	$f'(4) = \frac{1}{4}$
$f'' = -\frac{1}{4\sqrt{x^3}}$	$f''(4) = -\frac{1}{4\sqrt{64}} = -\frac{1}{32}$
$f''' = \frac{3}{8}\frac{1}{\sqrt{x^5}}$	$f'''(4) = \frac{3}{8\sqrt{4^5}} = \frac{3}{256}$

$f' = \frac{1}{2}x^{-1/2}$

$\rightarrow f'' = -\frac{1}{4}x^{-3/2} = -\frac{1}{4\sqrt{x^3}}$

$f''' = (-\frac{1}{4})(-\frac{3}{2})x^{-5/2} = \frac{3}{8}\frac{1}{\sqrt{x^5}}$

so:

$$T_1(x) = 2 + \frac{1}{4}(x-4)$$

$$T_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{32\cdot 2}(x-4)^2$$

$$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$$

$$T_3(x) = T_2(x) + \frac{3/256}{6}(x-4)^3$$

$$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$$

(b) $\sqrt{4.5}$ using T_1 — $T_1(4.5) = 2 + \frac{1}{4}(4.5-4) = 2.125$

T_2 .. $T_2(4.5) = 2 + \frac{1}{4}(0.5) - \frac{1}{64}(0.5)^2$ → 2.125 correct

$= 2.12109375$

correct

T_3 ... $T_3(4.5) = 2 + \frac{1}{4}(0.5) - \frac{1}{64}(0.5)^2 + \frac{1}{512}(0.5)^3$

$= 2.121337891$

correct

2.(a) $L_1(x) = f(1) + f'(1)(x-1) = 2 + (1)(x-1) = x+1$

f, f'	at $a=1$
$f = x^2 + \frac{1}{x}$	$f(1) = 2$
$f' = 2x - \frac{1}{x^2}$	$f'(1) = 1$

$f(1.4) \approx L_1(1.4) = 1.4 + 1 = \underline{2.4}$

(b) $L_2(x) = f(2) + f'(2)(x-2) = \frac{9}{2} + \frac{15}{4}(x-2) = \frac{15}{4}x - 3$

f, f'	at $a=2$
$f = x^2 + \frac{1}{x}$	$f(2) = 4 + \frac{1}{2} = \frac{9}{2}$
$f' = 2x - \frac{1}{x^2}$	$f'(2) = 4 - \frac{1}{4} = \frac{15}{4}$

$f(1.4) \approx L_2(1.4) = \frac{15}{4}(1.4) - 3 = \underline{2.25}$

(c) $x=1 \longrightarrow f(1) = 2$
$x=2 \longrightarrow f(2) = 9/2$
} slope $= \frac{\frac{9}{2} - 2}{2-1} = \frac{5}{2}$

$y - 2 = \frac{5}{2}(x-1)$

ie $y = S(x) = 2 + \frac{5}{2}(x-1) = \frac{5}{2}x - \frac{1}{2}$

$f(1.4) \approx S(1.4) = \frac{5}{2}(1.4) - \frac{1}{2} = \underline{3}$

(d) $T_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2$

$f(1) = 2$, $f'(1) = 1$ from part (a), $\frac{f''(1)}{2!}(x-1)^2 = \frac{4}{2}(x-1)^2$

$f'' = 2 + \frac{2}{x^3} \rightarrow f''(1) = 4$

so $T_2(x) = 2 + (x-1) + 2(x-1)^2$

$f(1.4) \approx T_2(1.4) = 2 + (0.4) + 2(0.4)^2 = \underline{2.72}$

(e) $T_2(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2$

$f(2) = 9/2$, $f'(2) = 15/4$ from part (b), $\frac{f''(2)}{2!} = 9/4$

$f'' = 2 + \frac{2}{x^3} \rightarrow f''(2) = 2 + \frac{2}{8} = \frac{9}{4}$

so $T_2(x) = \frac{9}{2} + \frac{15}{4}(x-2) + \frac{9}{8}(x-2)^2$

$f(1.4) \approx T_2(1.4) = \frac{9}{2} + \frac{15}{4}(-0.6) + \frac{9}{8}(-0.6)^2$

$= \underline{2.655}$

Compare (a)–(e) with a precise value

$f(1.4) = 2.674285714$

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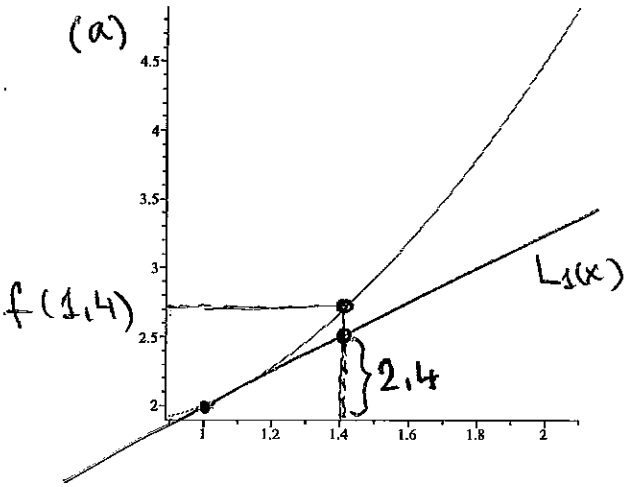
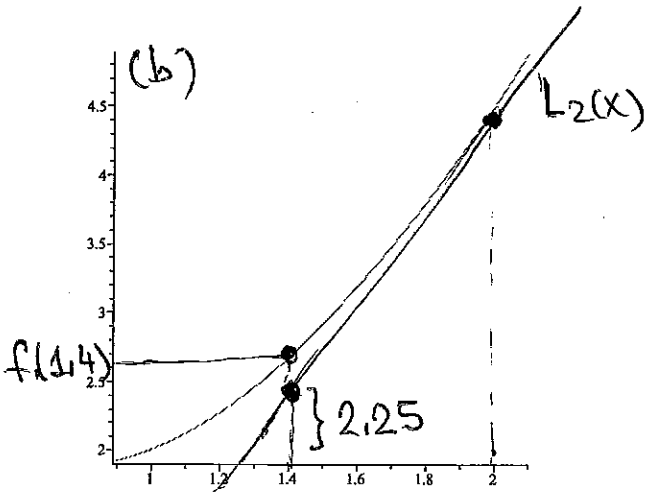
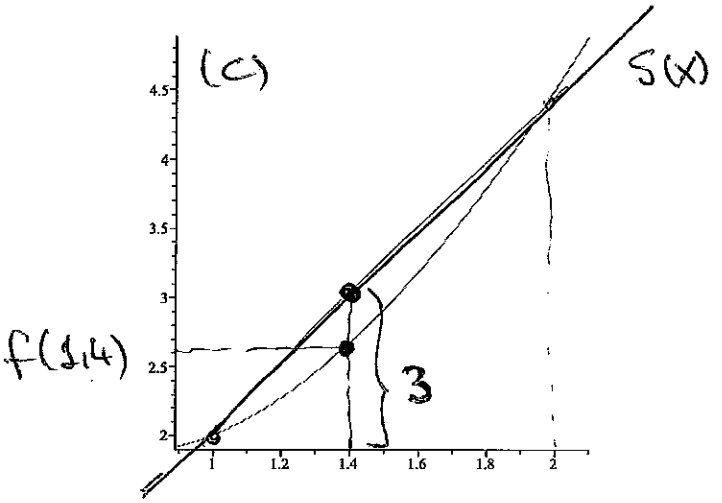
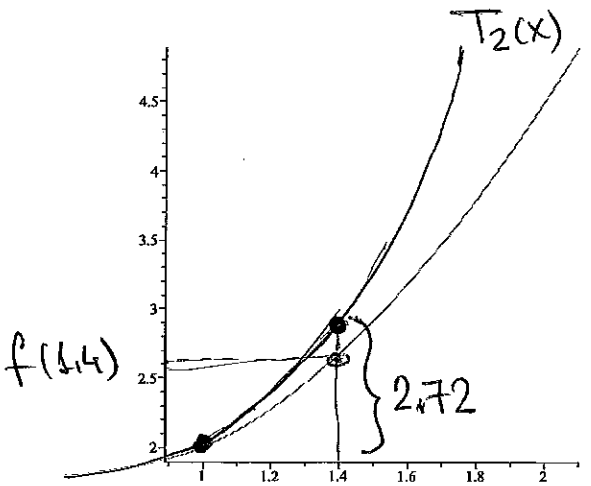
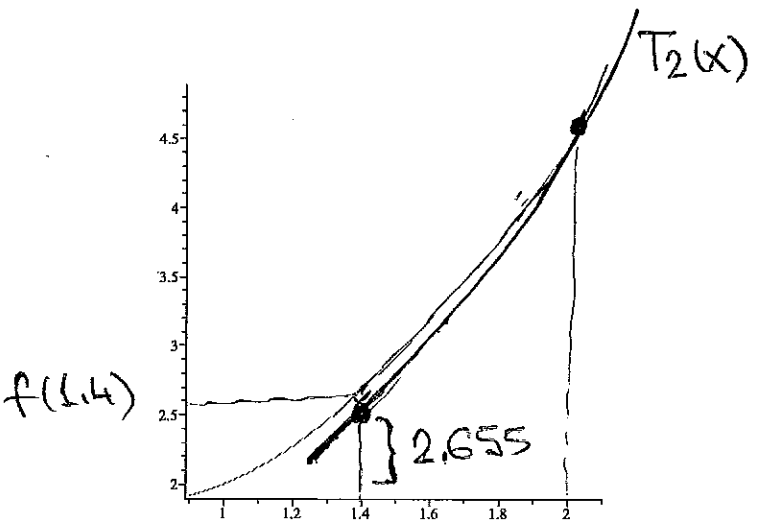
(f) Sketch each approximation (a)-(e).











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