ASSIGNMENT 19

1.(a)

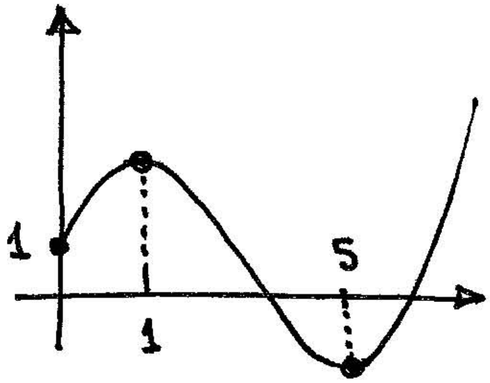
		1		5	
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

incr. on $(-\infty, 1)$
$(5, \infty)$

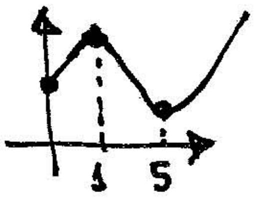
decr. on $(1, 5)$

(b) max. at $x = 1$

(c)



many possibilities could be



etc.

2.(a) $f(x) = x^{-1/2} + x^{-1} + x^{-3/2}$

so $\displaystyle\int f(x)\,dx = \frac{x^{1/2}}{1/2} + \ln|x| + \frac{x^{-1/2}}{-1/2} + C$

$\displaystyle = 2\sqrt{x} + \ln|x| - \frac{2}{\sqrt{x}} + C$

(b) $\displaystyle\int y\,dx = \frac{\pi^x}{\ln\pi} + \frac{x^{\pi+1}}{\pi+1} + \pi^{\pi}x + C$

(c) $\displaystyle\int f(x)\,dx = -3\cos x + 4\sin x + 5x + C$

(d) $\int f(x)\,dx = e^x + \frac{x^{-1}}{-1} + C = e^x - \frac{1}{x} + C$

(e) $\int y\,dx = \sec x - x + C$

(f) $f(x) = \dfrac{x - 2x^2 + 1}{x^{1/2}} = x^{1/2} - 2x^{3/2} + x^{-1/2}$

so $\int f(x)\,dx = \dfrac{x^{3/2}}{3/2} - 2\,\dfrac{x^{5/2}}{5/2} + \dfrac{x^{1/2}}{1/2} + C$

$= \frac{2}{3}x^{3/2} - \frac{4}{5}x^{5/2} + 2x^{1/2} + C$

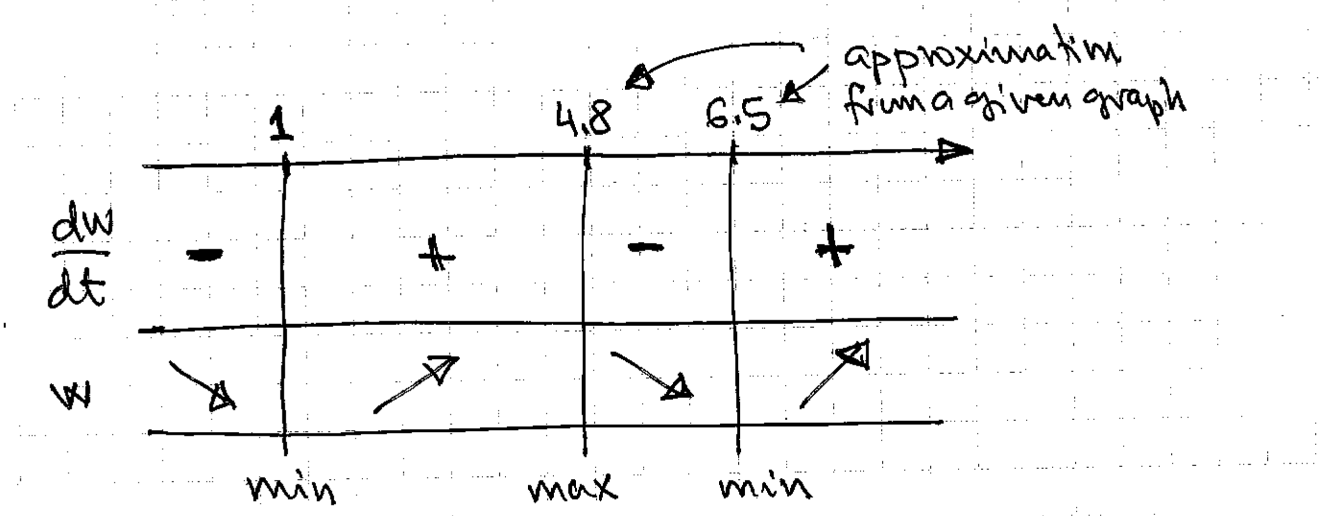
3. (a) $= \tan x + 3\sin x + C$

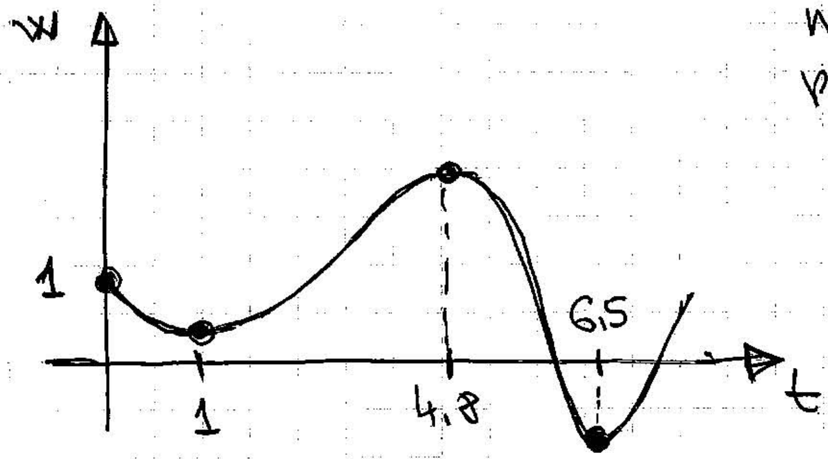
(b) $= \int\left(\frac{1.4}{5}x^{-1/2} + 2x^{-1/5}\right)dx$

$= 0.28 \cdot \dfrac{x^{1/2}}{1/2} + 2 \cdot \dfrac{x^{4/5}}{4/5} + C = 0.56\sqrt{x} + \frac{5}{2}x^{4/5} + C$

(c) $= 6\arctan x + C$

4.





many possibilities!!

5.

(a)

LEFT SIDE

$$y' = e^{1-\sqrt{1+x^2}} \cdot \left(0 - \cancel{\tfrac{1}{2}}(1+x^2)^{-1/2} \cdot \cancel{2}x\right)$$

$$= -x(1+x^2)^{-1/2}\, e^{1-\sqrt{1+x^2}}$$

RIGHT SIDE

$$-x(1+y)\sqrt{1+x^2} = -x\left(\cancel{1} + e^{1-\sqrt{1+x^2}} - \cancel{1}\right) \cdot \sqrt{1+x^2}$$

$$= -x \cdot \sqrt{1+x^2} \cdot e^{1-\sqrt{1+x^2}}$$

inital condition: $y(0) = e^{1-\sqrt{1-(1)^2}} = e^0 - 1 = 0$ ✓

(b) $f'(t) = \dfrac{4}{1+t}$ by the chain rule.

answer: YES

(c) $f'(t) = \frac{1}{2}(t^2+3)^{-1/2} \cdot 2t = t(t^2+3)^{-1/2}$

$t(f(t))^{-1} = t(\sqrt{t^2+3})^{-1} = t(t^2+3)^{-1/2}$ equal

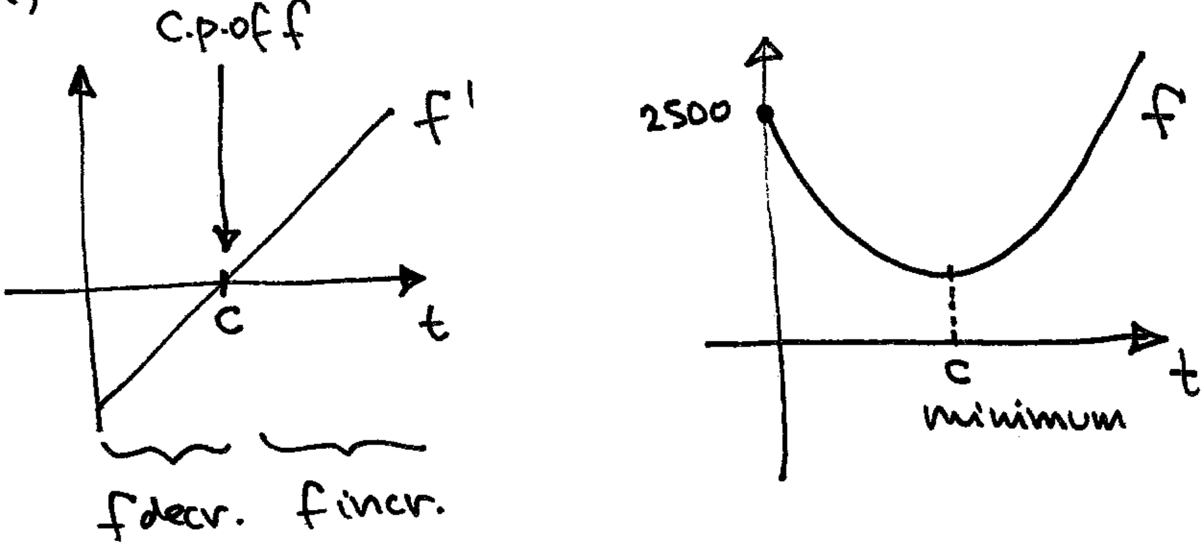
$f(1) = \sqrt{(1)^2+3} = 2$ ✓

6. (a) $T'(t) = k \cdot \dfrac{1}{T(t)^2}$, $T(0)=0$

(b) $S'(t) = kS(t)(15{,}000 - S(t))$, $S(0)=1$

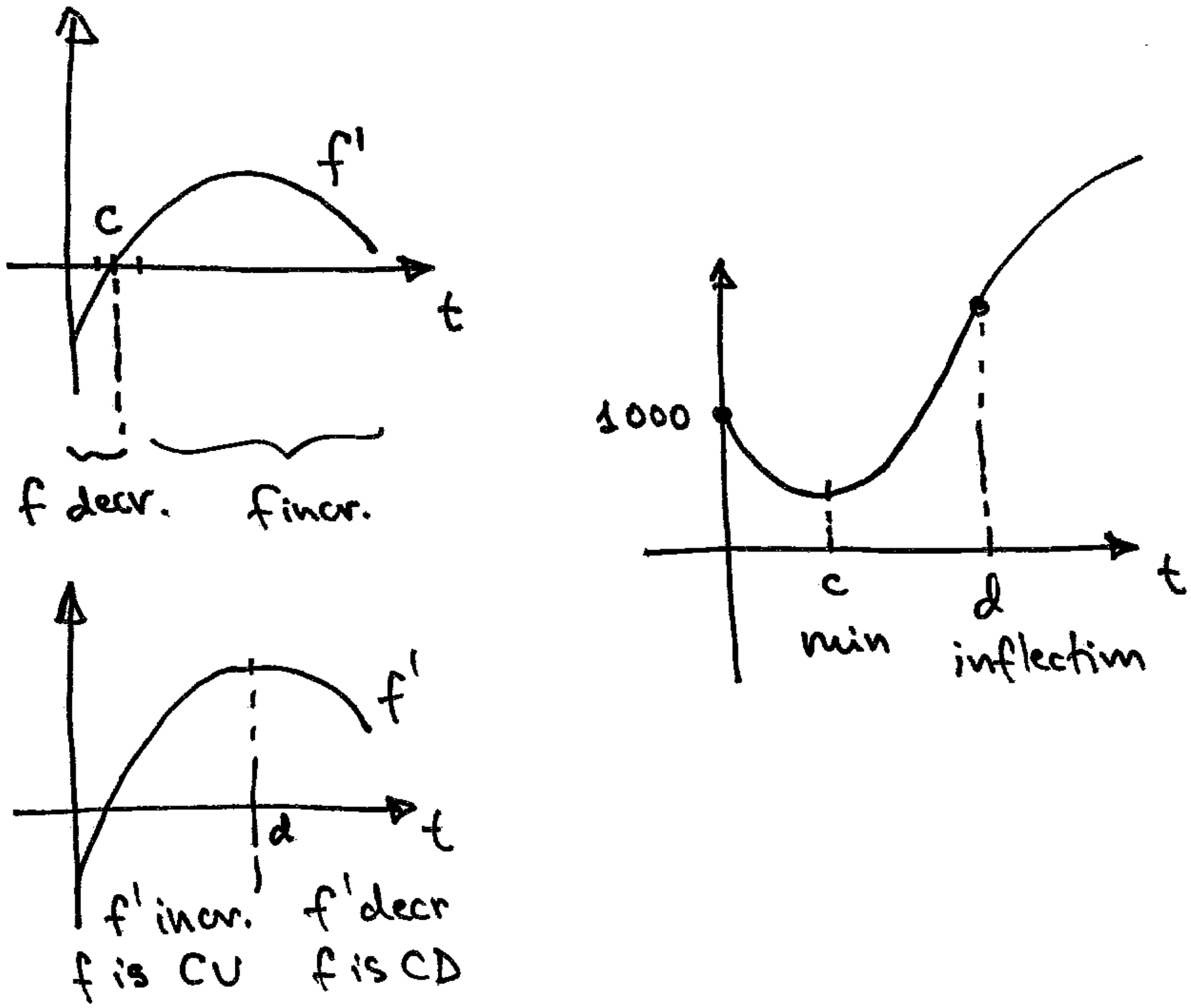
(c) $T'(t) = k(300 - T(t))$, $T(0) = 20°C$

7. (a)



(antiderivative of a line is a parabola)

(b)



8. (a) If $f(t)$ is unknown function:

$f'(t) =$ function of t (measured rate of change)
is pure-time

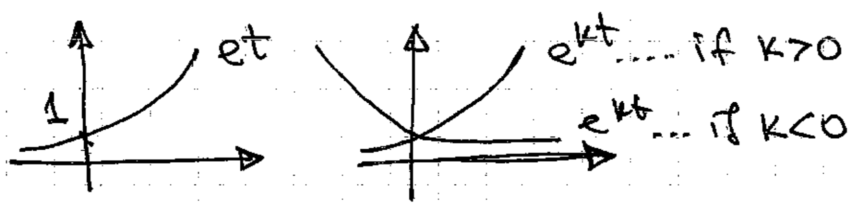
$f'(t) =$ function of f, no explicit appearance of t
is autonomous

(b) $f'(t) =$ constant

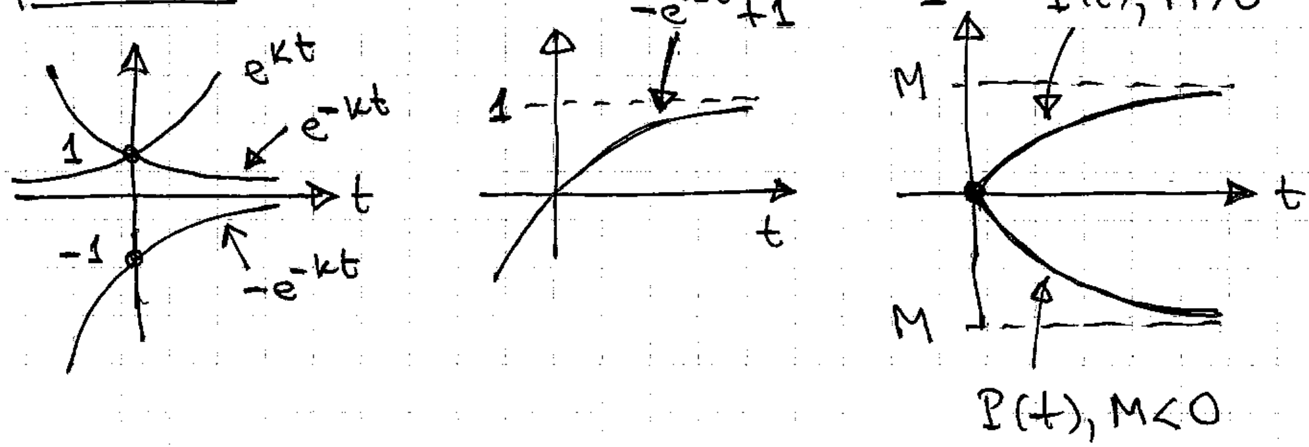
(c) $f'(t) = t^2 \cdot f(t)$ ← not pure-time

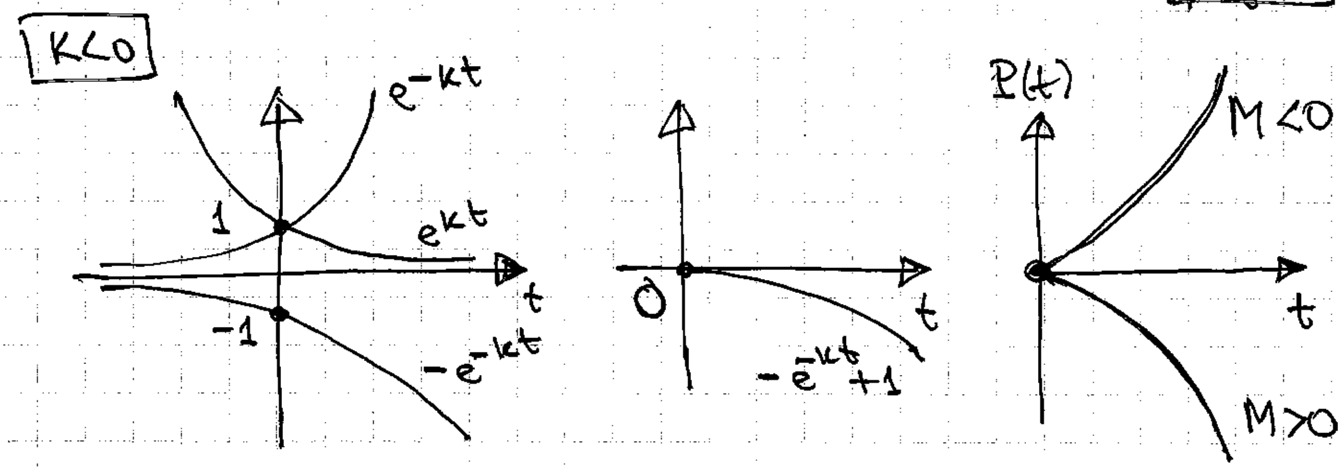
↑ not autonomous

(d) $P(t) = M - Me^{-kt} = M(1 - e^{-kt})$



K>0 case





9. $P'(t) = 0.02\, P(t)$

guess: $P(t) = C \cdot e^{0.02t}$

check: $P'(t) = C \cdot e^{0.02t}\,(0.02)$

$0.02\, P(t) = 0.02 \cdot C\, e^{0.02t}$ equal

so $P(t) = C\, e^{0.02t}$

$P(0) = 240 \longrightarrow 240 = C \cdot e^{0}, \quad C = 240$

so $\underline{\underline{P(t) = 240\, e^{0.02t}}}$