LS

ASSIGNMENT 1

PAGE 1

1. (a) see Figure 1.2.3 on page 6 in your textbook (geese) or Figure 0.1.1., page 2 (elephants)

Math model is a way to describe an applied problem using mathematics. It consists of the following stages:

* observation, question or problem
* mathematical formulation, using dynamical systems or differential equations or other objects from math
* solution
* interpretation in terms of the application
* if needed: improvement of the model, adjustment, refinement, etc.

(b) discrete-time: values given at equally spaced intervals (graph is a collection of dots)

continuous time: values given for all values of the variable (continuous curve)

(c) $(1,4)$ open $[1,4]$ closed $(0,\infty)$ open

$(-\infty,7]$ closed $(-\infty,7)$ open

(d) one order of magnitude is one power of 10

37 → one order of magn. is 370

2 orders 3,700

3 orders 37,000

$\boxed{\text{4 orders}}$ 370,000

(e) M is inversely proportional to K

2(a) Assume 1 year ≈ 365 days

$365 \cdot 24 \cdot 60 \cdot 60 = 31{,}536{,}000$ seconds

if 1 year ≈ 365.25 days

$365{,}25 \cdot 24 \cdot 60 \cdot 60 = 31{,}557{,}600$ seconds

(b) $\underbrace{\underbrace{\underbrace{72 \cdot 60}_{\text{one hour}} \cdot 24}_{\text{one day}} \cdot 365}_{\text{one year}} \cdot 50 = 1{,}892{,}160{,}000$

(c) volume $V = \frac{4\pi}{3}\,\text{radius}^3$

diameter = 15 inches $\rightarrow$ radius = 7.5 inches

$= 7{,}5 \cdot 2{,}54 = 19{,}05$ cm

$$V = \frac{4\pi}{3}(19{,}05)^3 \approx 28{,}958.33 \text{ cm}^3$$

mass = density · volume $= 3.4 \frac{g}{cm^3} \cdot 28{,}958.33 \text{ cm}^3$

$= 98{,}458.32\,g \approx 98.4$ kg

(d) 650 kg $= 650{,}000$ g

650 kg $= 650 \cdot 2.204$ lb $= 1432.6$ lb

table on page 7

(e) $95 \frac{km}{h} = 95 \cdot \frac{\frac{1}{1{,}609}}{1} \frac{\text{miles}}{h}$

$= 59{,}04$ mph

3 (a) A parameter is a variable which changes from experiment to experiment, but is constant within an experiment

exercise 1: dependent : wombat density
independent: altitude
parameter : rainfall

(b) A relation is a set of all pairs of values of independent and dependent variables. A function is a <u>unique</u> assignment of a number y (or $f(x)$) to a number x

g... dependent
m. independent
$g(m)$ d is parameter

(c) To identify those curves that are the graphs of functions

(d) Natural domain: largest set of values where f is defined. Given domain: subset of the natural domain that has to be explicitly stated

(e) Set of all points $(x, f(x))$, where x is in the domain of f

4. (a) $f_1, f_2, f_3, f_4, f_5, f_9, f_{10}$

(b) f_6, f_7

(c) f_6, f_7

(d) f_1, f_5, f_9 keep in mind that it says increasing for <u>all</u> real numbers
($\sqrt{x}$ is increasing, but only for $x \geq 0$
x^2 is increasing only for $x > 0$)

(e) f_4, f_{10}

(f) linear means $f(x) = mx + b$: f_1, f_2, f_3

f_{10} is not linear, but consists of two linear pieces: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

(g) positive means >0 for all x (so if a function touches x-axis it is not called positive; it's called non-negative

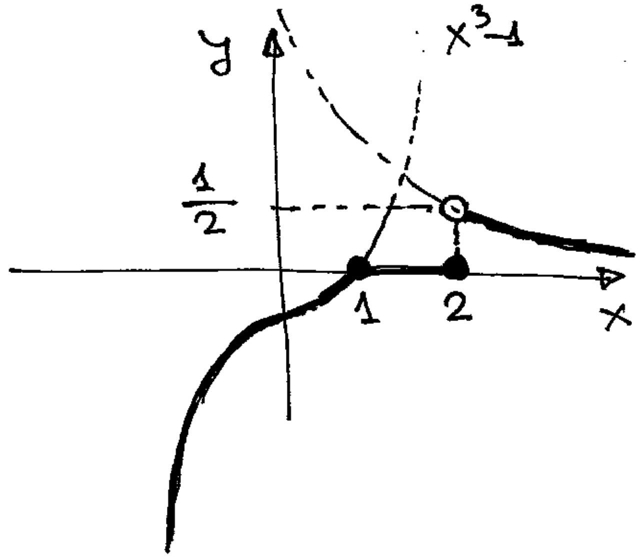
f_3, f_7

(h) f_2, f_5, f_6, f_9

(i) f_3, f_4, f_7, f_{10}

(j) f_3

5.

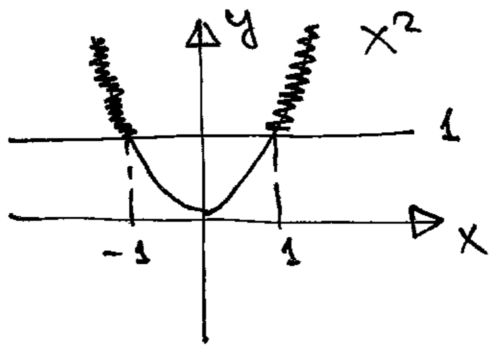


domain: all x

range: $(-\infty, 0]$ and $(0, 1/2)$

combine: $(-\infty, 1/2)$

6. $x^2 - 1 \geq 0 \to x^2 \geq 1$



(identify x where x^2 lies above, or touches (horizontal line) 1)

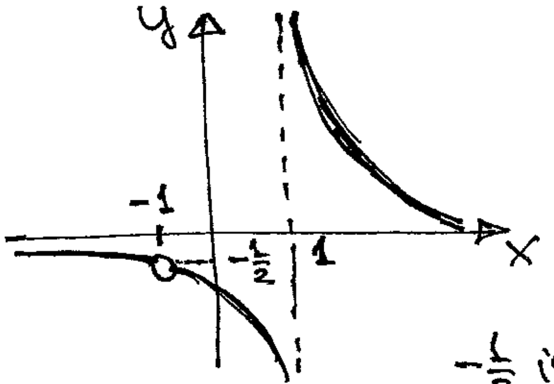
$x \leq -1$ or $x \geq 1$

7 (a) $x \neq 1$

(b) $x^2 - 1 = 0 \to x = \pm 1$ ... domain: all $x \neq \pm 1$

(c) $x^2 + 1 = 0$ has no solutions ... domain: all x

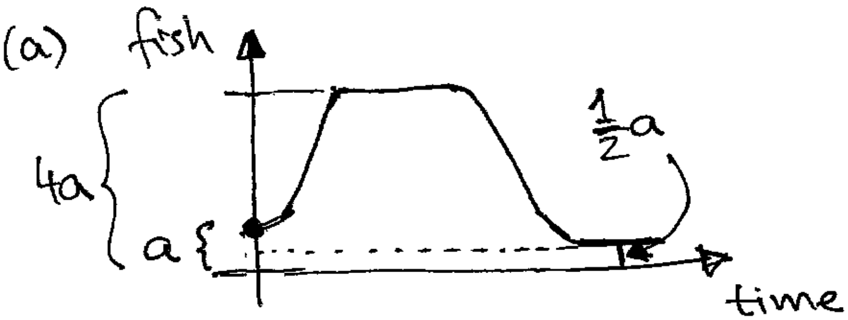
(d) $f_2(x) = \dfrac{x+1}{(x+1)(x-1)} = \begin{cases} \dfrac{1}{x-1} & \text{if } x \neq -1 \\ \text{not defined} & x = -1 \end{cases}$



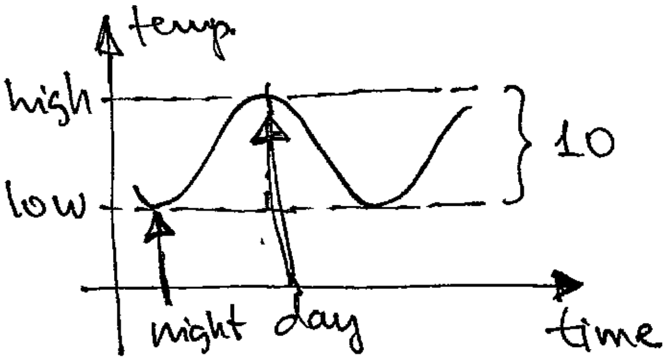
range: all real numbers except $-\frac{1}{2}$ and 0

$-\frac{1}{2}$ is what we got when we substitute -1 into $\frac{1}{x-1}$

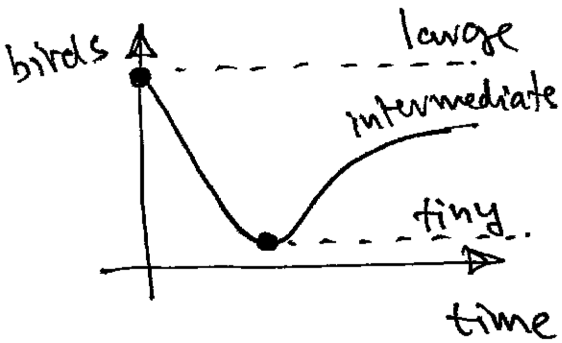
8(a)



(b)



(c)



9. Average height increases with the age of a tree until tree is about 30 years old. Then, the height starts decreasing.

10.(a) $f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3a^2 + 5a + 4$

(b) $f(a) + f(1) = (3a^2 - a + 2) + (4) = 3a^2 - a + 6$

(c) $f(1+h) - f(1) = \left(3(1+h)^2 - (1+h) + 2\right) - (4)$

$= 3h^2 + 5h$

11. (a) $f\left(\frac{a}{4}\right) = \dfrac{2}{\frac{a}{4}} = 2 \cdot \frac{4}{a} = \frac{8}{a}$

(b) $f\left(\frac{22}{a}\right) = \dfrac{2}{\frac{22}{a}} = 2 \cdot \frac{a}{22} = \frac{a}{11}$

(c) $\dfrac{f(3+h) - f(3)}{h} = \dfrac{\frac{2}{3+h} - \frac{2}{3}}{h}$

$= \dfrac{2(3) - 2(3+h)}{(3+h)(3)} \cdot \dfrac{1}{h} = \dfrac{-2h}{3(3+h)} \cdot \dfrac{1}{h}$

$= \dfrac{-2}{3(3+h)}$