Assignment 20

Section 7.1 (geese) 6.1 (elephants) Euler's Method

1. (a) What is Euler's Method used for?

To find an approximation of a solution of an initial value problem

2. In this exercise we get familiar with the notation used in Euler's method. We consider a pure-time differential equation $f'(t) = G(t)$, with initial condition $f(t_0) = y_0$. The step size is denoted by Δt.

If $f'(t) = e^{-2t} + t^3$, what is $G(t)$?

$G(t) = e^{-2t} + t^3$

If $f'(t) = \sec(2t - 4)$, what is $G(t)$?

$G(t) = \sec(2t-4)$

If $f'(t) = \ln(t^2 + 1)$, and $f(2) = -2$, what are $G(t)$, t_0 and y_0?

$G(t) = \ln(t^2+1)$, $t_0 = 2$, $y_0 = -2$

If $f'(t) = 3\sin(4t)$, and $f(\pi) = 1$, what are $G(t)$, t_0 and y_0?

$G(t) = 3\sin(4t)$, $t_0 = \pi$, $y_0 = 1$

Given that $f(2) = -2$ and $\Delta t = 0.1$, find the values of t_0, t_1, t_2, t_3, and t_4 for which we compute the approximations.

$t_0 = 2$, $t_1 = 2.1$, $t_2 = 2.2$, $t_3 = 2.3$, $t_4 = 2.4$

Given that $f(0) = 5$ and $\Delta t = 0.25$, find the values of t_0, t_1, t_2, t_3, and t_4 for which we compute the approximations.

$t_0 = 0$ (that's given)

$t_1 = t_0 + \Delta t = 0 + 0.25 = 0.25$

$t_2 = t_1 + \Delta t = 0.5$

$t_3 = 0.75$

$t_4 = 1$

Continued on next page

$G(t)$

3. (a) Given that $f'(t) = \ln(t^2+1)$, $f(2) = -2$, and $\Delta t = 0.1$, write down the formulas [look at algorithm 7.1.1 on page ~~467 (geese) 6.1 on~~ page 413 (elephants)] for Euler's Method.

$t_0 = 2$ $\qquad t_{n+1} = t_n + \Delta t$

$y_0 = -2$ $\qquad y_{n+1} = y_n + G(t_n)\Delta t = y_n + \ln(t_n^2+1)\Delta t$

(b) Compute t_1 and y_1.

$t_1 = t_0 + \Delta t = 2.1$

$y_1 = y_0 + \ln(t_0^2+1)\Delta t$

$= -2 + \ln(5)\cdot 0.1 \approx -1.839$

Compute t_2 and y_2.

$t_2 = t_1 + \Delta t = 2.2$

$y_2 = y_1 + \ln(t_1^2+1)\Delta t$

$= -1.839 + \ln(2.1^2+1)\cdot 0.1 \approx -1.670$

Compute t_3 and y_3.

$t_3 = t_2 + \Delta t = 2.3$

$y_3 = y_2 + \ln(t_2^2+1)\Delta t$

$= -1.670 + \ln(2.2^2+1)\cdot 0.1 \approx -1.494$

(c) What is the meaning of y_1 in (b)?

approximation of $f(2.1)$

What is the meaning of y_2 in (b)?

approximation of $f(2.2)$

What is the meaning of y_3 in (b)?

approximation of $f(2.3)$

Continued on next page

assignment 20

4. Consider the initial value problem $f'(t) = 3t^2$, $f(0) = 4$.

(a) Compute the first three steps of Euler's Method with step size $\Delta t = 0.5$.

$$t_0 = 0, \quad y_0 = 4 \qquad t_{n+1} = t_n + \Delta t$$

$$y_{n+1} = y_n + G(t_n)\Delta t = y_n + 3t_n^2 \cdot \Delta t$$

$$t_1 = t_0 + \Delta t = 0 + 0.5 = \underline{0.5} \rightarrow y_1 = y_0 + 3t_0^2(0.5)$$

$$= 4 + 3(0)^2(0.5) = 4$$

$$t_2 = t_1 + \Delta t = \underline{1} \rightarrow y_2 = y_1 + 3t_1^2(0.5) = 4 + 3(0.5)^2(0.5) = 4.375$$

$$t_3 = t_2 + \Delta t = \underline{1.5} \rightarrow y_3 = y_2 + 3t_2^2(0.5) = 4.375 + 3(1)^2(0.5) = 5.875$$

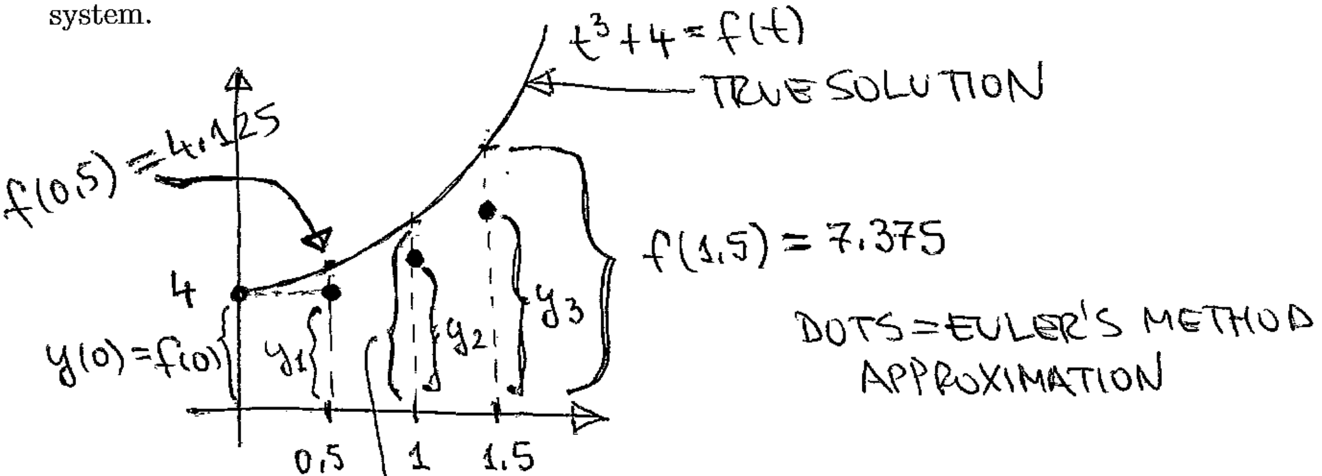
(b) Solve the given initial value problem algebraically.

$$f'(t) = 3t^2 \rightarrow f(t) = t^3 + C$$

$$f(0) = 4 \rightarrow 4 = (0)^3 + C, \text{ so } C = 4$$

$$\boxed{f(t) = t^3 + 4}$$

(c) Plot the function in (b) and the values you obtained in (a) in the same coordinate system.



Continued on next page

$G(t)$ $\quad t_0 = 0$

$p_0 = 1000$

5. Consider the initial value problem $P'(t) = 0.4P(t)$, $P(0) = 1000$.

(a) Compute the first four steps of Euler's Method with step size $\Delta t = 1$.

$t_{n+1} = t_n + \Delta t, \quad p_{n+1} = p_n + 0.4 p_n \underbrace{\Delta t}_{1} = p_n + 0.4 p_n = \underline{1.4\, p_n}$

$t_1 = 1 \qquad p_1 = 1.4 \cdot p_0 = 1400$

$t_2 = 2 \qquad p_2 = 1.4\, p_1 = 1960$

$t_3 = 3 \qquad p_3 = 1.4\, p_2 = 2744$

$t_4 = 4 \qquad p_4 = 1.4\, p_3 = 3841.6$

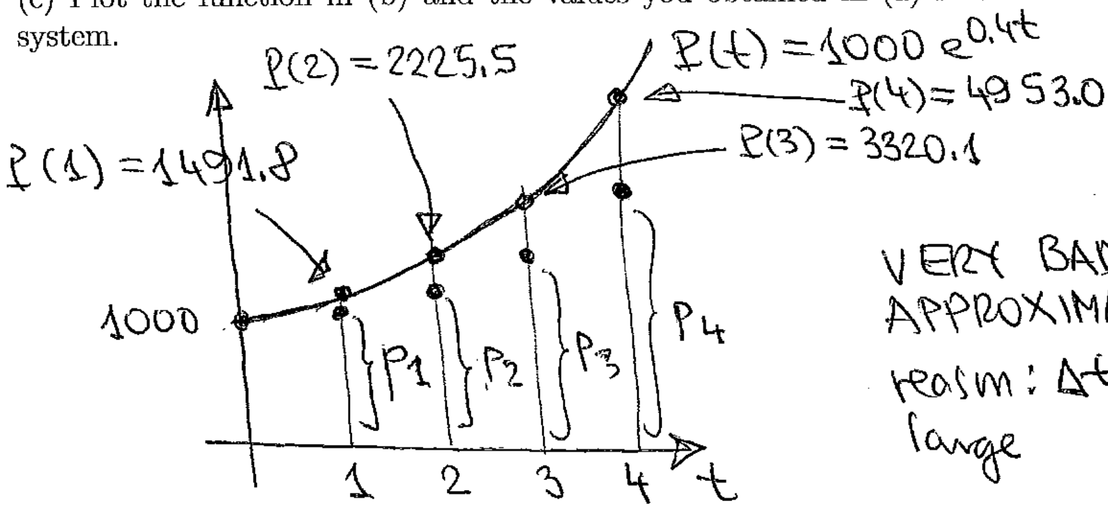
(recall from before: $p_n = 1000 \cdot 1.4^n$)

(b) Show that $P(t) = 1000e^{0.4t}$ is the solution of the given initial value problem.

$P'(t) = 1000 \cdot e^{0.4t} \cdot 0.4$

$0.4\, P(t) = 0.4 \cdot 1000\, e^{0.4t}$ equal

(c) Plot the function in (b) and the values you obtained in (a) in the same coordinate system.



VERY BAD APPROXIMATION!
reason: Δt is too large

THE END