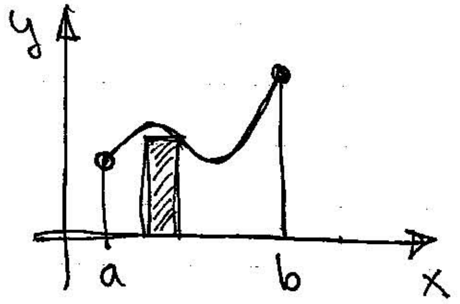
ASSIGNMENT 22

1.(a) Take a continuous function $f(x)$ on $[a,b]$



which satisfies $f(x) \geq 0$ on $[a,b]$. Divide $[a,b]$ into equal subintervals and build a rectangle over each subinterval, whose height is given by some rule (say, the value of $f(x)$ at right end). The sum of areas of these rectangles approximates the area of the region under $f(x)$ on $[a,b]$.

(b) Riemann sum is a common name for any of the sums we obtain when we use approximating rectangles for the area under a curve.

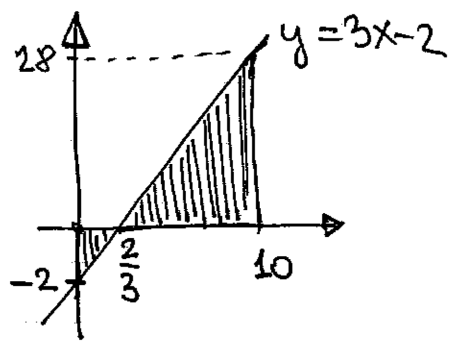
(c) The area under a curve defined by $f(x) \geq 0$ on an interval is obtained by making the approximating rectangles thinner and thinner (ie, by dividing the interval into more and more subintervals). More precisely, we take the limit of the approximating sums as the bases of rectangles approach zero.

(d) If $f(x) \geq 0$ then $\int_a^b f(x)\,dx$ = area under $f(x)$ on $[a,b]$

In general, $\int_a^b f(x)dx$ is the <u>difference</u> of areas (areas of regions above the x-axis minus areas of regions below the x-axis)

(e) As approximating rectangles get thinner, they approximate better the region under the curve.

2.(a)

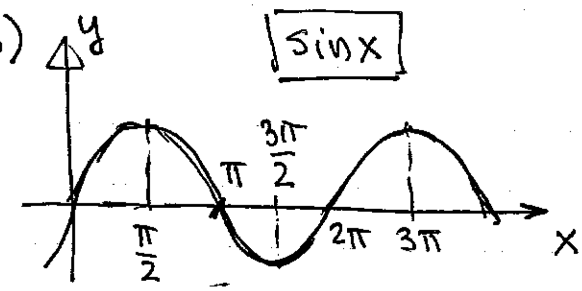


$$\int_0^{10} (3x-2)dx = -\frac{1}{2}\left(2\cdot\frac{2}{3}\right) + \frac{1}{2}\left(10-\frac{2}{3}\right)28$$

$$= -\frac{2}{3} + \frac{1}{2}\cdot\frac{28}{3}\cdot 28 = -\frac{2}{3} + \frac{392}{3}$$

$$= \underline{\underline{130}}$$

(b)

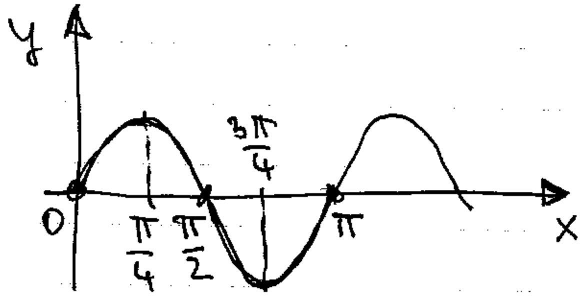


there are many answers

$$\int_0^{2\pi} \sin x \, dx = 0, \quad \int_{\pi/2}^{3\pi/2} \sin x \, dx = 0,$$

and so on

now $\boxed{y = \sin 2x}$

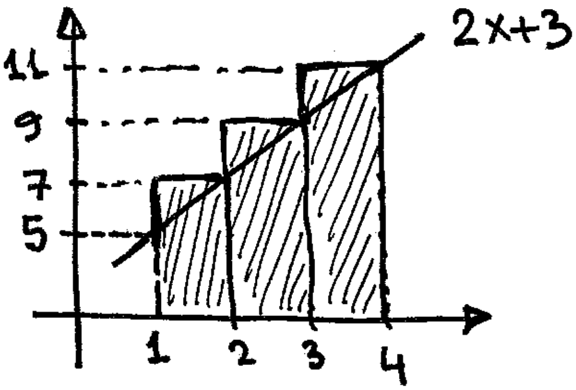


$$\int_0^{\pi} \sin 2x \, dx = 0,$$

or $$\int_{\pi/4}^{3\pi/4} \sin 2x \, dx = 0,$$

and so on

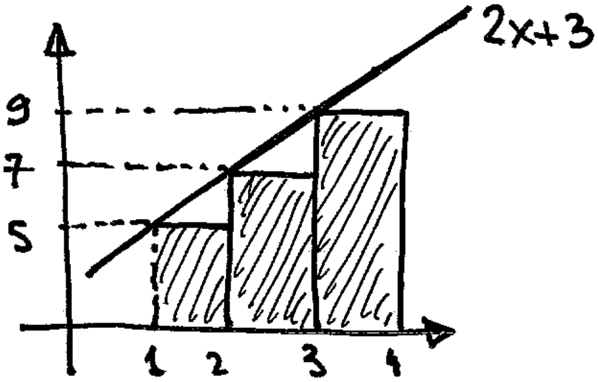
3. (a)



area ≈ $1\cdot 7 + 1\cdot 9 + 1\cdot 11$

$= 27$

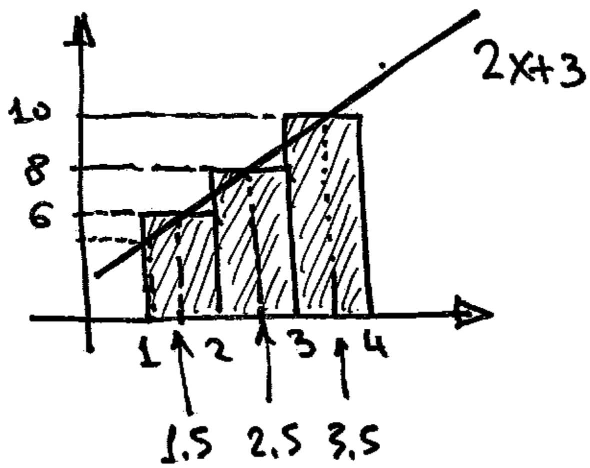
(b)



aren ≈ $1\cdot 5 + 1\cdot 7 + 1\cdot 9$

$= 21$

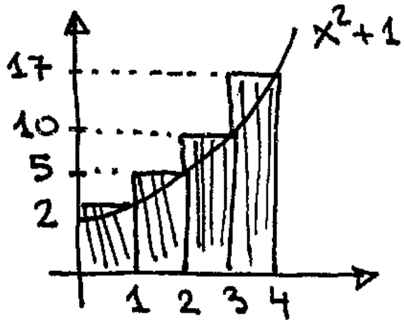
(c)



area ≈ $1\cdot 6 + 1\cdot 8 + 1\cdot 10$

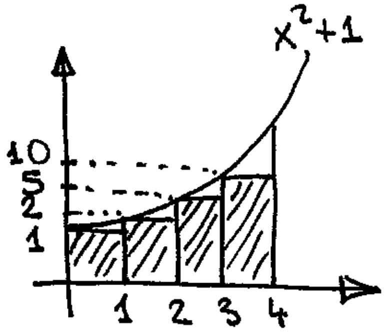
$= 24$

4. (a)



$$\text{area} = 1\cdot 2 + 1\cdot 5 + 1\cdot 10 + 1\cdot 17$$
$$= 34$$

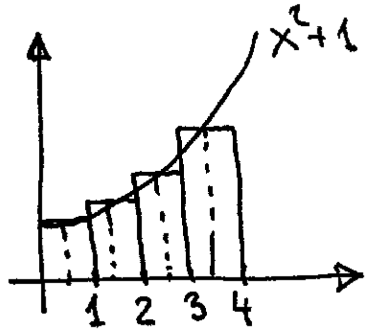
(b)



heights of the four rectangles are: 1, 2, 5, 10

$$\text{area} = 1\cdot 1 + 1\cdot 2 + 1\cdot 5 + 1\cdot 10$$
$$= 18$$

(c)

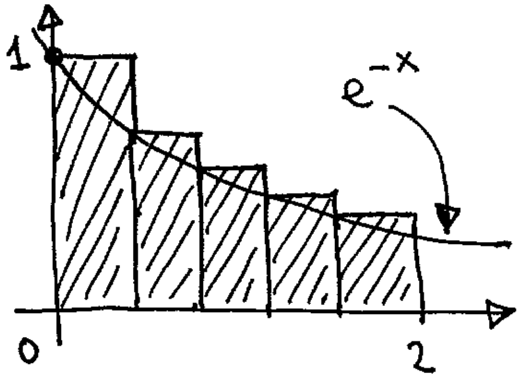


the heights are:

$$(0.5)^2+1 = 1.25$$
$$(1.5)^2+1 = 3.25$$
$$(2.5)^2+1 = 7.25$$
$$(3.5)^2+1 = 13.25$$

$$\text{area} = 25$$

5.

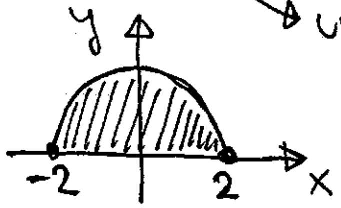


left sum > area under e^{-x}

since e^{-x} is decreasing

6. $y=\sqrt{4-x^2} \longrightarrow y^2=4-x^2,\ x^2+y^2=4$

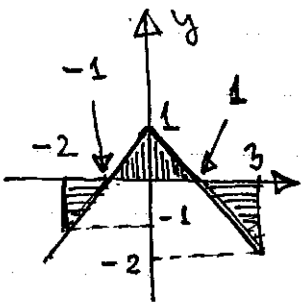
$\searrow$ upper semi-circle, radius 2



$$\int_{-2}^{2}\sqrt{4-x^2}\,dx = \frac{1}{2}\text{ area of the circle of radius 2}$$

$$=\frac{1}{2}\pi\cdot 2^2 = 2\pi$$

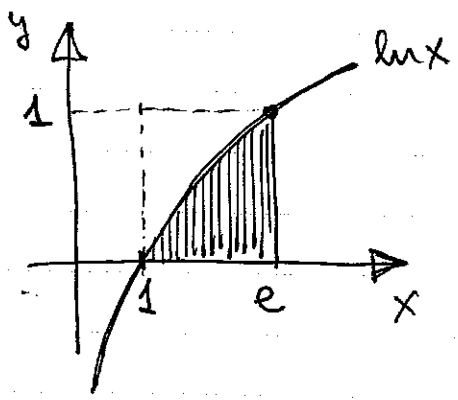
7. $f(x)=1-|x|$



$$\int_{-2}^{3}(1-|x|)\,dx = -\frac{1}{2}\cdot 1\cdot 1+\frac{1}{2}\cdot 2\cdot 1 - \frac{1}{2}\cdot 2\cdot 2$$

$$=-\frac{1}{2}+1-2=-\frac{3}{2}$$

8.



$\int_{1}^{e}\ln x\,dx$ = area of the shaded region $<$ area of the rectangle on $[1,e]$ of height 1

$$=(e-1)\cdot 1 = e-1$$

OR: $\ln x \leq 1$ on $[1,e]$

thus $\displaystyle\int_1^e \ln x\,dx \leq \underbrace{\int_1^e 1\cdot dx}_{=e-1}$

(although $\ln x \leq 1$, in the final answer we use $< e-1$, since the shaded region is smaller than the rectangle)

9. Given was the rate of change of the amount of chemical p, ie

$$\frac{dp}{dt}$$

was given. dp/dt tells us how p changes <u>at time t</u>.

To find the <u>total change</u> in p on some time interval, we add up the changes over small subintervals.