Name: ____________________

Math 1LS3 **Assignment 23**

Practice with integration

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1. Find the following integrals.

(a) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$

(b) $\int_1^2 \frac{12}{x^2}\,dx = 12\int_1^2 x^{-2}\,dx = 12\,\frac{x^{-1}}{-1}\Big|_1^2 = -12\cdot\frac{1}{x}\Big|_1^2$

$= -12\left(\frac{1}{2}-1\right) = 6$

(c) $\int \left(3 - \frac{7}{\sqrt[4]{x}}\right) dx = \int 3\,dx - 7\int x^{-1/4}\,dx = 3x - 7\cdot\frac{x^{3/4}}{3/4} + C$

$= 3x - \frac{28}{3}x^{3/4} + C$

(d) $\int_0^4 (7-2x)^2\,dx = \int_0^4 (49 - 28x + 4x^2)\,dx = 49x - 14x^2 + 4\frac{x^3}{3}\Big|_0^4$

$= 49(4) - 14(4^2) + 4\frac{(4^3)}{3} = \frac{172}{3}$

(e) $\int \frac{x^3-3x+1}{x}\,dx = \int \left(x^2 - 3 + \frac{1}{x}\right) dx = \frac{x^3}{3} - 3x + \ln|x| + C$

(f) $\int \frac{(x^2-3)^2}{7x^3}\,dx = \frac{1}{7}\int \frac{x^4-6x^2+9}{x^3}\,dx = \frac{1}{7}\int \left(x - \frac{6}{x} + \frac{9}{x^3}\right)dx$

$= \frac{1}{7}\left(\frac{x^2}{2} - 6\ln|x| + 9\frac{x^{-2}}{-2}\right) + C = \frac{1}{7}\left(\frac{x^2}{2} - 6\ln|x| - \frac{9}{2x^2}\right) + C$

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(g) $\int \frac{1}{23}\,dx = \frac{1}{23}x + C$

(h) $\int \frac{1}{23}\,dt = \frac{1}{23}t + C$

(i) $\int \frac{1}{23}\,du = \frac{1}{23}u + C$

NOTE: dt, du, dx tell us what the variable is

(j) $\int_1^{11} \ln 4\,dt = (\ln 4)\,t\Big|_1^{11} = (\ln 4)(11-1) = 10\ln 4$

(k) $\int (e^x - 2^x + 6^x)\,dx = e^x - \frac{2^x}{\ln 2} + \frac{6^x}{\ln 6} + C$

(l) $\int_0^1 4e^x\,dx = 4e^x\Big|_0^1 = 4(e^1 - e^0) = 4(e-1)$

(m) $\int_0^1 4e^t\,dt = 4e^t\Big|_0^1 = 4(e-1)$

(n) $\int_0^1 4e^v\,dv = 4e^v\Big|_0^1 = 4(e-1)$

NOTE: definite integral is a real number so does not matter what variable is used

(o) $\int (4e^2 - 24x)\,dx = 4e^2x - 12x^2 + C$

constant

(p) $\int \frac{14^x - 3}{7}\,dx = \frac{1}{7}\int (14^x - 3)\,dx = \frac{1}{7}\left(\frac{14^x}{\ln 14} - 3x\right) + C$

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Math 1LS3 assignment 23

(q) $\int \cos x \, dx = \sin x + C$

radians

(r) $\int_0^1 \cos 3x \, dx = \frac{1}{3}\sin 3x \Big|_0^1 = \frac{1}{3}\sin 3 - \cancel{\frac{1}{3}\sin 0} = \frac{1}{3}\sin 3$

0

(s) $\int \cos(-4x) \, dx = -\frac{1}{4}\sin(-4x) + C$

(t) $\int \sec^2 x \, dx = \tan x + C$

(u) $\int_0^{1/3} \sec^2 3x \, dx = \frac{1}{3}\tan 3x \Big|_0^{1/3} = \frac{1}{3}\tan 1 - \cancel{\frac{1}{3}\tan 0} = \frac{1}{3}\tan 1$

0

(v) $\int \sec^2(-4x) \, dx = -\frac{1}{4}\tan(-4x) + C$

(w) $\int \sec x \tan x \, dx = \sec x + C$

(x) $\int \sec 4x \tan 4x \, dx = \frac{1}{4}\sec 4x$

(y) $\int (\sin 2x - \cos 3x + 4) \, dx = -\frac{1}{2}\cos 2x - \frac{1}{3}\sin 3x + 4x + C$

(z) $\int_0^1 (10\sin(\pi x) - 2) \, dx = \left(10 \cdot \frac{-1}{\pi} \cdot \cos(\pi x) - 2x\right)\Big|_0^1$

$= \left(-\frac{10}{\pi}(-1) - 2\right) - \left(-\frac{10}{\pi} - 0\right) = \frac{20}{\pi} - 2$

(aa) $\int_0^1 \frac{1}{1+x^2}\,dx$ $= \arctan x \Big|_0^1 = \underbrace{\arctan 1}_{\pi/4} - \underbrace{\arctan 0}_{0} = \frac{\pi}{4}$

(ab) $\int \left(2 - \frac{4}{1+x^2}\right) dx$ $= 2x - 4\arctan x + C$

(ac) $\int \frac{1}{1+4x^2}\,dx$ $= \int \frac{1}{1+(2x)^2}\,dx = \frac{1}{2}\arctan 2x + C$

(ad) $\int \frac{5}{1+6x^2}\,dx$ $= 5\cdot\int \frac{1}{1+(\sqrt{6}x)^2}\,dx = 5\cdot\frac{1}{\sqrt{6}}\arctan(\sqrt{6}x) + C$

(ae) $\int 45(1-x^2)^{-1/2}\,dx$ $= 45\int \frac{1}{\sqrt{1-x^2}}\,dx = 45\arcsin x + C$

(af) $\int \left(\frac{3}{\sqrt{1-x^2}} + \sin x\right) dx$ $= 3\arcsin x - \cos x + C$

(ag) $\int \frac{1}{\sqrt{1-9x^2}}\,dx$ $= \int \frac{1}{\sqrt{1-(3x)^2}}\,dx = \frac{1}{3}\arcsin 3x + C$

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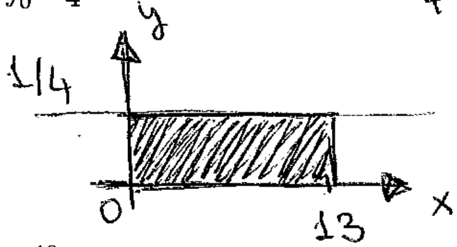
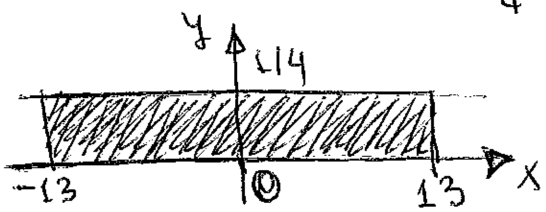
2. Evaluate each integral by interpreting it as area or as a difference of areas.

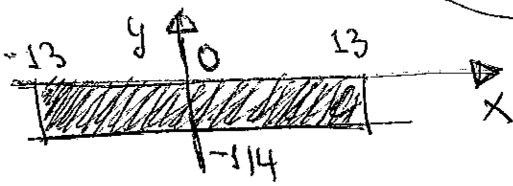
(a) $\int_0^{13} \frac{1}{4}\,dx$ $= \text{area} = \frac{1}{4}\cdot 13 = 13/4$



(b) $\int_{-13}^{13} \frac{1}{4}\,dx$ $= \text{area} = 26\cdot\frac{1}{4} = 13/2$

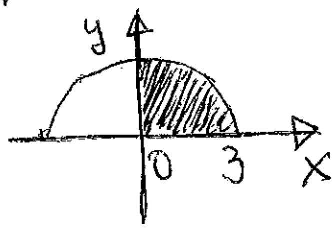


(c) $\int_{-13}^{13} \left(-\frac{1}{4}\right) dx$ $\longrightarrow = -\text{area} = -26\cdot\frac{1}{4} = -13/2$

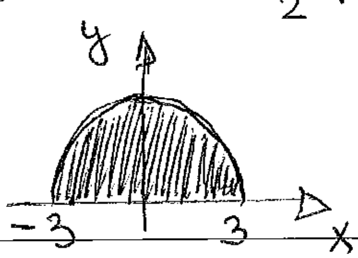


(d) $\int_0^{3} \sqrt{9-x^2}\,dx$ $\longrightarrow y=\sqrt{9-x^2} \longrightarrow x^2+y^2=9$

$$\int_0^3 \sqrt{9-x^2}\,dx = \frac{1}{4}\pi(3)^2 = \frac{9\pi}{4}$$



(e) $\int_{-3}^{3} \sqrt{9-x^2}\,dx$ $= \frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$



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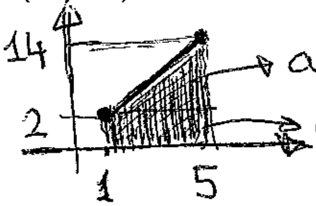
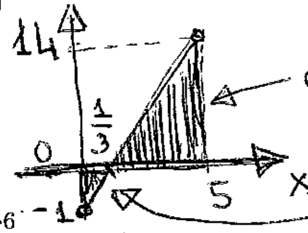
(f) $\int_1^5 (3x-1)\,dx = 8 + 24 = 32$



area $= \frac{1}{2} \cdot 4 \cdot 12 = 24$

area $= 4 \cdot 2 = 8$

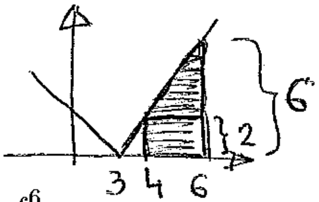
(g) $\int_0^5 (3x-1)\,dx = -\frac{1}{6} + \frac{98}{3} = \frac{65}{2}$



area $= \frac{1}{2} \cdot (5 - \frac{1}{3}) \cdot 14 = \frac{14 \cdot 7}{3} = \frac{98}{3}$

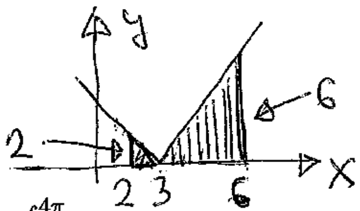
area $= \frac{1}{2} \cdot \frac{1}{3} \cdot 1 = \frac{1}{6}$

(h) $\int_4^6 |2x-6|\,dx$



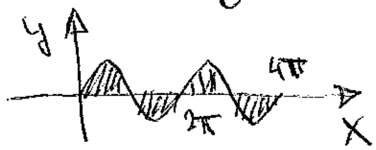
$\int_4^6 |2x-6| =$ area of rectangle + area of triangle $= 4 + 4 = 8$

(i) $\int_2^6 |2x-6|\,dx =$ sum of areas $= \frac{1}{2} \cdot 1 \cdot 2 + \frac{1}{2} \cdot 3 \cdot 6 = 10$



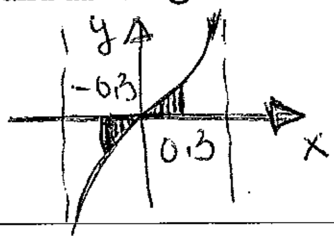
(j) $\int_0^{4\pi} \sin x\,dx = 0$

difference of areas



(k) $\int_{-0.3}^{0.3} \tan x\,dx = 0$

difference of areas



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