1.(a) $\int_0^\pi (\sin x - 2\cos x\,dx) = (-\cos x - 2\sin x)\big|_0^\pi = 1 - (-1) = 2$

(b) $\int_1^9 x^{-1/2}\,dx = 2x^{1/2}\big|_1^9 = 2\sqrt{9} - 2\sqrt{1} = 4$

(c) $\int_0^1 \frac{6}{1+x^2}\,dx = 6\arctan x\big|_0^1 = 6\cdot\frac{\pi}{4} - 0 = \frac{3\pi}{2}$

(d) $\int_0^1 11e^x\,dx = 11e^x\big|_0^1 = 11e - 11$

(e) $\int_{-2}^{-1} x^{-1}\,dx = \ln|x|\big|_{-2}^{-1} = \ln 1 - \ln 2 = -\ln 2$

(f) $\int_1^2 \frac{x^2+1}{x}\,dx = \int_1^2 \left(x + \frac{1}{x}\right)dx = \left(\frac{x^2}{2} + \ln|x|\right)\Big|_1^2$

$= (2 + \ln 2) - (\frac{1}{2} + \ln 1) = \frac{3}{2} + \ln 2$

2.(a) $= \left\{ \begin{array}{l} u = x^2 + x + 2 \\ du = (2x+1)dx \end{array} \right\} = \int \frac{du}{u^2} = -\frac{1}{u} + C$

$\qquad\qquad\qquad = -\frac{1}{x^2 + x + 2} + C$

(b) $= \left\{ \begin{array}{l} u = 3 - e^x \\ du = -e^x dx \end{array} \right\} = \int \frac{-du}{u} = -\ln|u| + C = -\ln|3 - e^x| + C$

(c) $= \left\{ \begin{array}{l} u = 1 + \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}dx \rightarrow dx = 2\sqrt{x}\,du \end{array} \right\} = \int \frac{u^2}{\sqrt{x}} 2\sqrt{x}\,du$

$= 2\int u^2\,du = \frac{2u^3}{3} + C = \frac{2}{3}(1 + \sqrt{x})^3 + C$

(d) $= \left\{ \begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array} \right\} = \int_0^{\ln 2} \frac{u^4}{1}\,du = \frac{u^5}{5}\Big|_0^{\ln 2} = \frac{(\ln 2)^5}{5}$

when $x=1 \rightarrow u = \ln 1 = 0$
when $x=2 \rightarrow u = \ln 2$

3.(a) $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx = \arctan x \Big|_0^1$

$\qquad\qquad = \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

(b) $\displaystyle\int_0^1 \frac{x}{1+x^2}\,dx = \left\{ \begin{array}{l} U = 1+x^2 \\ du = 2x\,dx \end{array} \right\}$

$\qquad = \displaystyle\int_1^2 \frac{x}{U}\frac{du}{2x} = \frac{1}{2}\int_1^2 \frac{1}{U}\,du$

$\qquad = \frac{1}{2}\ln|U|\Big|_1^2 = \frac{1}{2}\ln 2 - \frac{1}{2}\ln 1^{\;0} = \frac{1}{2}\ln 2$

(c) $\displaystyle\int_0^1 \frac{x^2}{1+x^2}\,dx = $

$\qquad\qquad\qquad\qquad x^2+1 \overline{\smash{\big)}\,x^2}$
$\qquad\qquad\qquad\qquad\quad \underline{x^2+1}$
$\qquad\qquad\qquad\qquad\qquad -1$

$\qquad = \displaystyle\int_0^1 \left(1 - \frac{1}{1+x^2}\right)dx$

$\qquad = (x - \arctan x)\Big|_0^1$

$\qquad = (1 - \arctan 1) - (0 - \underbrace{\arctan 0}_{0}) = 1 - \frac{\pi}{4}$

4. $\displaystyle\int_{e}^{e^4} \frac{dx}{x\sqrt{\ln x}} = \left\{ \begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array} \right\}$

$= \displaystyle\int_{\ln e}^{\ln e^4} \frac{1}{\sqrt{u}}\, du = \int_{1}^{4} u^{-1/2} du = 2\sqrt{u}\,\Big|_{1}^{4}$

$= 2\sqrt{4} - 2\sqrt{1} = 2$

5.

$A = \displaystyle\int_{1}^{3} \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

~~$A = \left(\ln|x| - \frac{x^{-1}}{-1}\right)\Big|$~~ (scratched out)
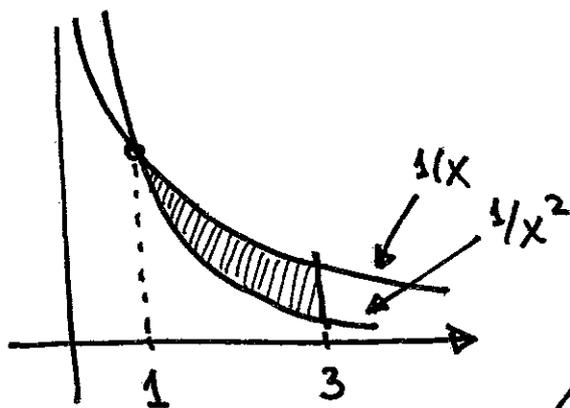
$A = \left(\ln|x| - \dfrac{x^{-1}}{-1}\right)\Big|_{1}^{3} = \left(\ln|x| + \dfrac{1}{x}\right)\Big|_{1}^{3}$

$= \left(\ln 3 + \dfrac{1}{3}\right) - \left(\ln\overset{0}{1} + 1\right) = \ln 3 - \dfrac{2}{3}$

6. (a)
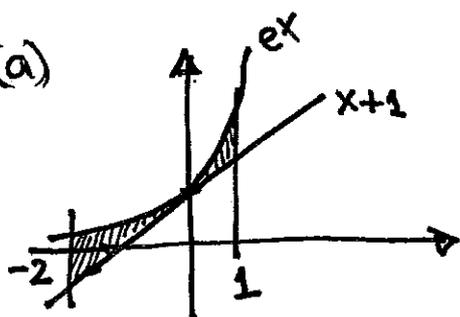
$A = \displaystyle\int_{-2}^{1} \left(e^x - (x+1)\right) dx$

(b) $\quad A = (e^X - \frac{x^2}{2} - x)\Big|_{-2}^{1}$

$= (e - \frac{1}{2} - 1) - (e^{-2} - \cancel{2} + \cancel{2})$

$= e - e^{-2} - \frac{3}{2} \approx \underline{\underline{1.08}}$

7.
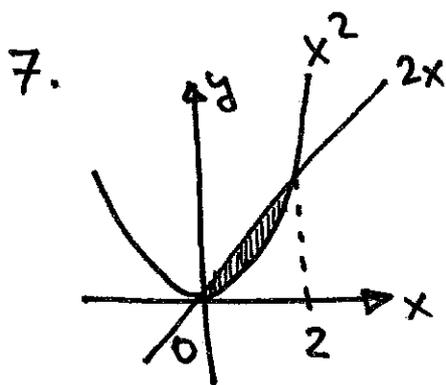
$x^2 = 2x \longrightarrow x^2 - 2x = x(x-2) = 0$

$\longrightarrow x = 0, 2$ are

pts. of intersection

$A = \int_{0}^{2} (2x - x^2)\, dx$

$= (x^2 - \frac{x^3}{3})\Big|_{0}^{2} = 4 - \frac{8}{3} = \underline{\underline{\frac{4}{3}}}$

8. (a)
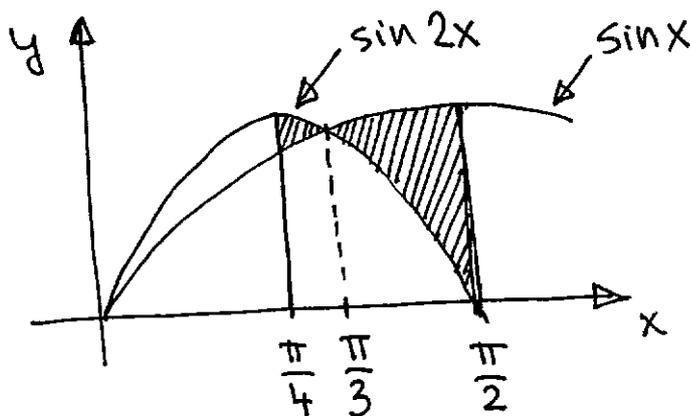
$$\sin 2x = \sin x$$
$$2\sin x \cos x = \sin x$$
$$\rightarrow \quad \sin x (2\cos x - 1) = 0$$

$$\sin x = 0 \rightarrow x = 0, \pi, 2\pi, \dots$$

$$2\cos x - 1 = 0 \rightarrow \cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}$$

(we need solutions between $\pi/4$ and $\pi/2$)

$$A = \int_{\pi/4}^{\pi/3} (\sin 2x - \sin x)\, dx + \int_{\pi/3}^{\pi/2} (\sin x - \sin 2x)\, dx$$

(b)
$$= \left(-\tfrac{1}{2}\cos 2x + \cos x\right)\Big|_{\pi/4}^{\pi/3} + \left(-\cos x + \tfrac{1}{2}\cos 2x\right)\Big|_{\pi/3}^{\pi/2}$$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\cos\frac{2\pi}{3} = -\frac{1}{2}$

$$= \left(-\tfrac{1}{2}\cos\tfrac{2\pi}{3} + \cos\tfrac{\pi}{3}\right) - \left(-\tfrac{1}{2}\cos\tfrac{\pi}{2} + \cos\tfrac{\pi}{4}\right)$$

$$+ \left(-\cos\tfrac{\pi}{2} + \tfrac{1}{2}\cos\pi\right) - \left(-\cos\tfrac{\pi}{3} + \tfrac{1}{2}\cos\tfrac{2\pi}{3}\right)$$

$$= \left(-\tfrac{1}{2}\left(-\tfrac{1}{2}\right) + \tfrac{1}{2}\right) - \left(\tfrac{\sqrt{2}}{2}\right)$$

$$+ \left(-\tfrac{1}{2}\right) - \left(-\tfrac{1}{2} + \tfrac{1}{2}\left(-\tfrac{1}{2}\right)\right)$$

$$= \tfrac{3}{4} - \tfrac{\sqrt{2}}{2} - \tfrac{1}{2} + \tfrac{3}{4} = 1 - \tfrac{\sqrt{2}}{2}$$

END