assignment 25

1\. $\int x \ln x \, dx$

$$\boxed{\begin{array}{ll} u = \ln x & dv = x\,dx \\ du = \frac{1}{x}\,dx & v = \frac{x^2}{2} \end{array}}$$

$$= (\ln x)\left(\frac{x^2}{2}\right) - \int \left(\frac{x^2}{2}\right)\left(\frac{1}{x}\right) dx$$

$$= \frac{x^2 \ln x}{2} - \frac{1}{2}\int x\,dx = \frac{x^2 \ln x}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$$

2\. $\int x \sin x \, dx$

$$\boxed{\begin{array}{ll} u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{array}}$$

$$= (x)(-\cos x) - \int (-\cos x)\,dx$$

$$= -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C$$

3\. $\int_0^2 x e^{-3x} dx$

$$\boxed{\begin{array}{ll} u = x & dv = e^{-3x}dx \\ du = dx & v = -\frac{e^{-3x}}{3} \end{array}}$$

step 1.

$$= (x)\left(-\frac{e^{-3x}}{3}\right) - \int \left(-\frac{e^{-3x}}{3}\right) dx$$

$$= -\frac{xe^{-3x}}{3} + \frac{1}{3}\int e^{-3x}\,dx = -\frac{xe^{-3x}}{3} - \frac{e^{-3x}}{9} + C$$

step 2.

$$\int_0^2 x e^{-3x} dx = -\frac{xe^{-3x}}{3} - \frac{e^{-3x}}{9}\Bigg|_0^2$$

cont'd...

$$= \left(-\frac{(2)e^{-3(2)}}{3} - \frac{e^{-3(2)}}{9}\right) - \left(0 - \frac{1}{9}\right)$$ ②

$$= -\frac{7e^{-6}}{9} + \frac{1}{9}$$

common denomintar to subtract fractions

④ $\int x\cos(\pi x)\,dx$

$$\boxed{\begin{array}{ll} u = x & dv = \cos(\pi x)\,dx \\ du = dx & v = \frac{\sin(\pi x)}{\pi} \end{array}}$$

$$= (x)\left(\frac{\sin(\pi x)}{\pi}\right) - \int \frac{\sin(\pi x)}{\pi}\,dx$$

$$= \frac{x\sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2} + C$$

⑤ $\int \arcsin(2x)\,dx$

$$\boxed{\begin{array}{ll} u = \arcsin(2x) & dv = dx \\ du = \frac{1}{\sqrt{1-(2x)^2}}\,2dx & v = x \\ \quad = \frac{2\,dx}{\sqrt{1-4x^2}} & \end{array}}$$

$$= \arcsin(2x)(x) - \int (x)\left(\frac{2\,dx}{\sqrt{1-4x^2}}\right)$$

$$= x\arcsin(2x) - 2\int \frac{x\,dx}{\sqrt{1-4x^2}}$$

substitution:

$w = 1-4x^2$

$dw = -8x\,dx$

$-\frac{dw}{8} = x\,dx$

$$= x\arcsin(2x) - \cancel{2}\int \frac{1}{w^{1/2}}\left(\frac{-dw}{\cancel{8}\,4}\right)$$

$$= x\arcsin(2x) + \frac{1}{4}\int w^{-1/2}\,dw$$

$$\boxed{\frac{1}{w^{1/2}} = w^{-1/2}}$$

$$= x\arcsin(2x) + \frac{1}{4}\left(\frac{w^{1/2}}{1/2}\right) + C$$

$$= x\arcsin(2x) + \frac{(1-4x^2)^{1/2}}{2} + C$$

(6.) $\int_0^1 \arctan x \, dx$

$$\boxed{\begin{array}{ll} u = \arctan x & dv = dx \\ du = \frac{1}{1+x^2}\,dx & v = x \end{array}}$$

(3)

step 1.

$$= (\arctan x)(x) - \int x \cdot \frac{1}{1+x^2}\,dx$$

$$= x \arctan x - \int \frac{x}{1+x^2}\,dx \quad \frac{dz}{2}$$

substitution:

$z = 1+x^2$

$dz = 2x\,dx$

$\frac{dz}{2} = x\,dx$

$$= x \arctan x - \int \frac{1}{z} \frac{dz}{2}$$

$$= x \arctan x - \frac{1}{2} \int \frac{1}{z}\,dz$$

$$= x \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

step 2.

$$\int_0^1 \arctan x \, dx = x \arctan x - \frac{1}{2}\ln(1+x^2) \Big|_0^1$$

$$= \left(1 \cdot \arctan(1) - \frac{1}{2}\ln(1+1^2)\right) - (0 - 0)$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2} \quad \blacksquare$$

(7.) $\int \ln x \, dx$

$$\boxed{\begin{array}{ll} u = \ln x & dv = dx \\ du = \frac{1}{x}\,dx & v = x \end{array}}$$

$$= (\ln x)(x) - \int \cancel{x} \left(\frac{1}{\cancel{x}}\right) dx = x \ln x - \int dx$$

$$= x \ln x - x + C \quad \blacksquare$$

(4)

8. $\int_1^2 (2x+1)e^x dx$

$\boxed{\begin{array}{ll} u = 2x+1 & dv = e^x dx \\ du = 2dx & v = e^x \end{array}}$

step 1.

$= (2x+1)(e^x) - \int e^x (2dx) = (2x+1)e^x - 2\int e^x dx$

$= (2x+1)e^x - 2e^x + C$

step 2.

$\int_1^2 (2x+1)e^x dx = (2x+1)e^x - 2e^x \Big|_1^2$

$= \left[(2(2)+1)e^2 - 2e^2\right] - \left[(2(1)+1)e^1 - 2e^1\right]$

$= 3e^2 - e$ ■

9. $\int x^2 e^{-x} dx$

$\boxed{\begin{array}{ll} u = x^2 & dv = e^{-x} dx \\ du = 2x\,dx & v = -e^{-x} \end{array}}$

$= (x^2)(-e^{-x}) - \int (-e^{-x})(2x\,dx)$

$= -x^2e^{-x} + 2\int xe^{-x} dx$

By parts

$\boxed{\begin{array}{ll} U = x & dV = e^{-x} dx \\ dU = dx & V = -e^{-x} \end{array}}$

$= -x^2e^{-x} + 2\left[(x)(-e^{-x}) + \int e^{-x} dx\right]$

$= -x^2e^{-x} + 2\left[-xe^{-x} - e^{-x}\right] + C$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$ ■

10. $\int x^2 \cos x \, dx$ $\boxed{\begin{array}{ll} u = x^2 & dv = \cos x \, dx \\ du = 2x\,dx & v = \sin x \end{array}}$ 5

$= (x^2)(\sin x) - \int (\sin x)(2x\,dx)$

$= x^2 \sin x - 2 \int x \sin x \, dx$

by parts twice $\boxed{\begin{array}{ll} U = x & dV = \sin x \, dx \\ dU = dx & V = -\cos x \end{array}}$

$= x^2 \sin x - 2\left[(x)(-\cos x) - \int (-\cos x)\, dx\right]$

$= x^2 \sin x - 2\left[-x\cos x + \int \cos x \, dx\right]$

$= x^2 \sin x - 2\left[-x\cos x + \sin x\right] + c$

$= x^2 \sin x + 2x\cos x - 2\sin x + c$