ASSIGNMENT 26

1. (a) $f'(t) = k \cdot f(t) \rightarrow f(t) = f(0)\, e^{kt} = a e^{kt}$

(b) An initial value problem consists of a differential equation (or equations) and initial condition(s). A solution is any function which satisfies both the diff. equation(s) and initial condition(s).

(c) A cont. function has infinitely many antiderivatives which differ from each other by a constant. antiderivatives of x^{-10} are

$$\frac{x^{-9}}{-9} + C = -\frac{1}{9x^9} + C$$

(d) $\int 0\, dx = C$

(e) $\int c \cdot f(x)\, dx = c \cdot \int f(x)\, dx$ where c is a constant

as well, $\int_a^b c\, f(x)\, dx = c \int_a^b f(x)\, dx$

2. (a) No. Take, for instance $f(x) = x$ and $g(x) = 1$ then

$$\int f(x) g(x)\, dx = \int x\, dx = \frac{x^2}{2} + C$$

$$\left(\int f(x)dx\right)\left(\int g(x)dx\right) = \left(\int x\,dx\right)\left(\int 1\,dx\right)$$

$$= \left(\frac{x^2}{2} + C\right)(x + C)$$

so when $C = 0$, we get $\frac{x^2}{2}$ and $\frac{x^3}{2} \rightarrow$ not equal

(b) Since $e^{-x}>0$ on $[-3,7]$ the integral must be positive as well

(c) to compute a definite integral of a function which is positive (or zero), we can argue using areas (if the regions involved are simple enough so that we can calculate their areas)

(d) the definite integral is equal to the net area. The net area is the area of the region(s) above the x-axis minus the area of the region(s) below the x-axis.
(see page 444)

3. (a) the definite integral of a constant times a function is equal to the constant times the definite integral.

(b) the growth in the first 10 years is given by

$$\int_0^{10} 6.48\, e^{-0.09t}\, dt$$

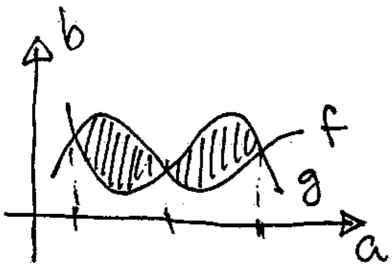
Using the formula for $\underset{\substack{\shortparallel \\ 72(1-e^{-0.09t})}}{L(t)}$ derived in the example, we get

$$L(t)\Big|_0^{10} = L(10)-L(0)$$

$$= 72(1-e^{-0.09\cdot 10}) - \underbrace{72(1-\cancel{e^{-0.09\cdot 0}}^{\,1})}_{0}$$

$$= 72(1-e^{-0.9})$$

$$\approx 42.73 \text{ cm}$$

(c) 

identify all bounded regions defined by the two functions

the area of each bounded region is

$$\int_{\text{intersection point}}^{\text{intersection point}} (\text{top function} - \text{bottom function})\, dx$$

4. (a) total change in pressure between times a and b

(b) distance covered between times a and b

(c) total length change between times a and b

(d) total change in velocity between times a and b

(e) total number of people infected with a flu between times a and b