ASSIGNMENT 2

1. (a) $(m \circ p)(x) = m(p(x))$

(b) nothing! the product of an increasing and a decreasing function does not have to be an increasing or a decreasing function.
note: could be constant: $f(x) = \frac{1}{x}$ (decreasing), $g(x) = x$ increasing $\rightarrow f(x) \cdot g(x) = 1$ (neither increasing nor decreasing)

(c) to figure out whether a given function (its graph) has an inverse
HLT: if every horizontal line crosses the graph of $y = f(x)$ at most once, then $f(x)$ has an inverse function

(d) If f is a function with domain D and range R, then f^{-1} has domain R and range D and

$$f^{-1}(b) = a \quad \text{if} \quad f(a) = b$$

(e) $f^{-1}(f(x)) = x$ for all x in domain of f
$f(f^{-1}(x)) = x$ for all x in domain of f^{-1}

2. algebraic function: obtained from polynomials using elementary algebraic operations ($+, -, *, \div$) and root functions

transcendental: all other functions (ie not algebraic)

3.(a) shift right 13 units

(b) expand vertically by a factor of 2, then shift down 13 units

(c) expand vertically by a factor of 2, shift right 13 units

(d) expand vertically by a factor of 4, reflect across the x-axis, move left 13 units

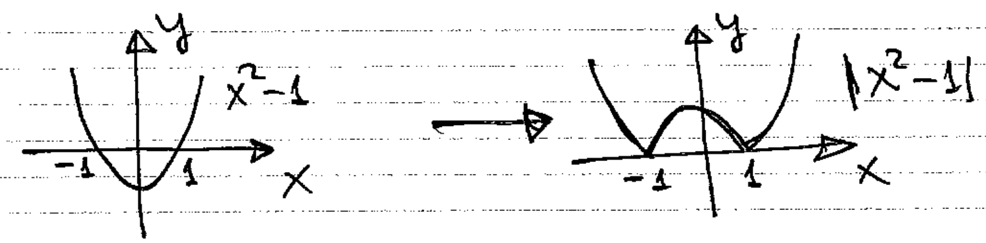
(e) compress horizontally by a factor of 3, move up 1 unit

(f) expand horizontally by a factor of 10, reflect across the x-axis

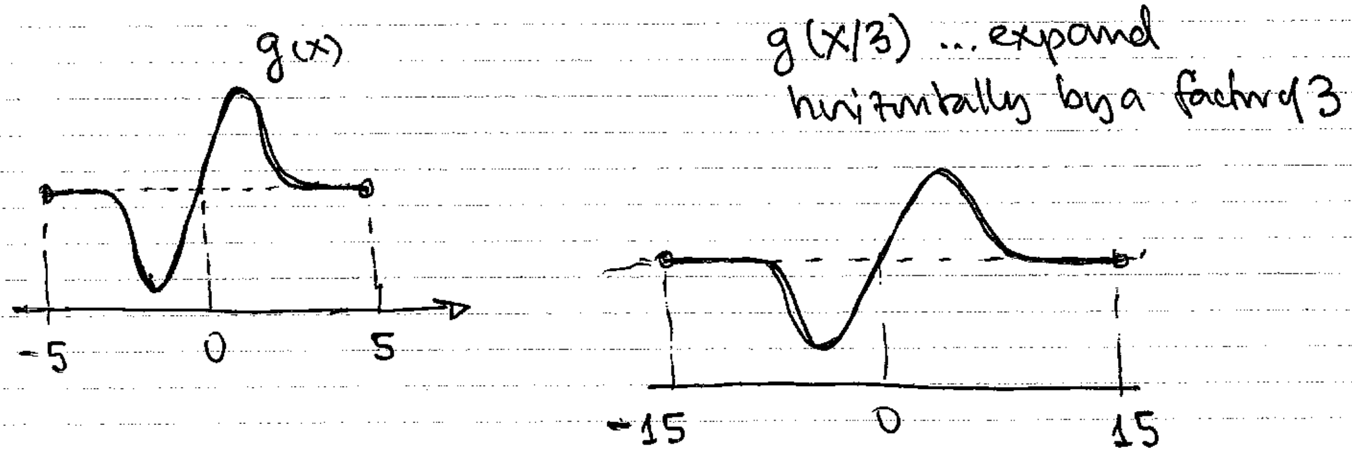
(g) reflect across the y-axis, compress vertically by a factor of 4

(h) reflect across the y-axis, expand vertically by a factor of 4, reflect across the x-axis

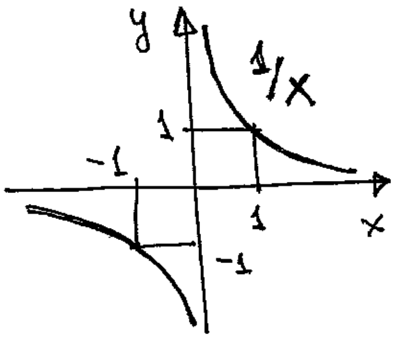
4.



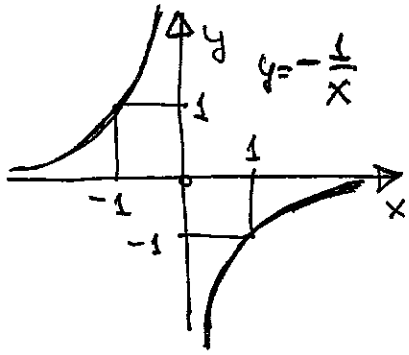
5.



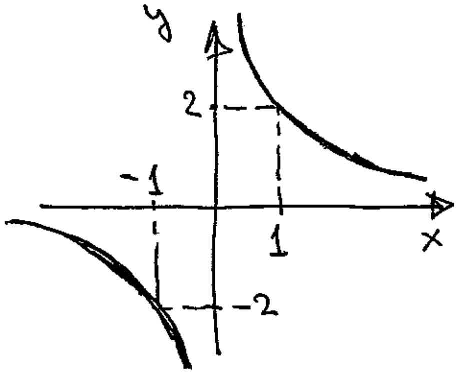
6. (a)



(b) reflect w.r.to x-axis



(c) stretch vertically by factor of 2

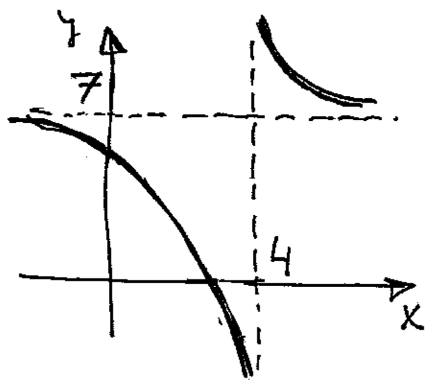


$y = \frac{2}{x}$

(d)

$$\frac{1}{x} \rightsquigarrow \frac{1}{x-4} \rightsquigarrow \frac{1}{x-4} + 7$$

so 4 units right
7 units up



7. (a) $(f \circ g)(x) = f(g(x)) = f(x^2-1) = 2(x^2-1)+3$

$$= 2x^2+1$$

$$(g \circ f)(x) = g(f(x)) = g(2x+3) = (2x+3)^2 - 1$$
$$= 4x^2 + 12x + 8$$

$f \circ g \neq g \circ f$

(b) $g(h(x)) = g\left(\frac{1}{x-2}\right) = \left(\frac{1}{x-2}\right)^2 - 1 = \frac{1}{(x-2)^2} - 1$

$h(g(x)) = h(x^2-1) = \frac{1}{x^2-1-2} = \frac{1}{x^2-3}$

$g \circ h \neq h \circ g$

(c) $(f \circ h)(3) = f(h(3)) = f(1) = 5$

8. (a) $f(g(x)) = f\left(\frac{1}{(x^2-1)^3}\right) = \frac{1}{\frac{1}{(x^2-1)^3}} = (x^2-1)^3$

$g(f(x)) = g\left(\frac{1}{x}\right) = \left(\left(\frac{1}{x}\right)^2 - 1\right)^{-3} = \frac{1}{\left(\frac{1}{x^2}-1\right)^3}$

(b) $g(f(x)) = g(\sqrt{2-x}) = \sqrt{\sqrt{2-x} - 2}$

(c) $f(g(x)) = f(4) = 12 - 4^2 = -4$

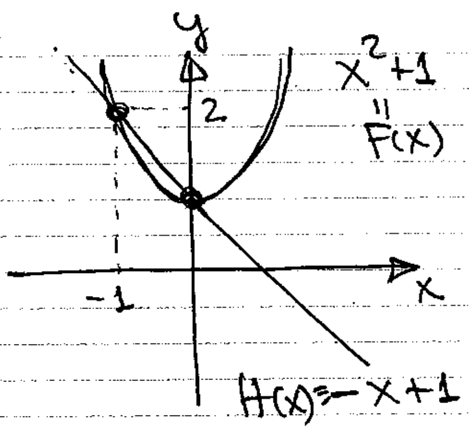
$g(f(x)) = g(12 - x^2) = 4$

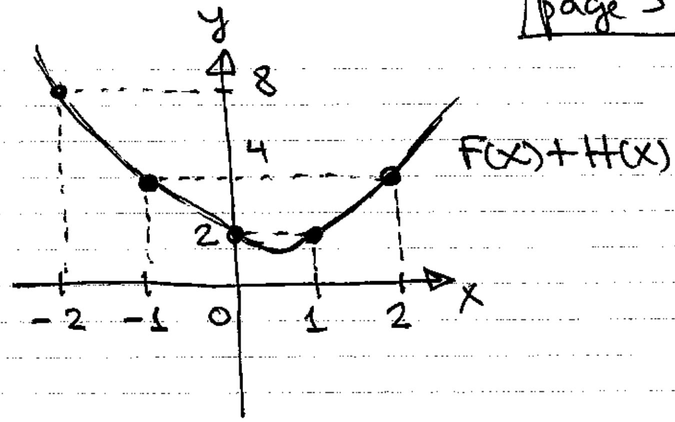
9. (a)

x	$F(x)$	$H(x)$	$F(x)+H(x)$
-2	5	3	8
-1	2	2	4
0	1	1	2
1	2	0	2
2	5	-1	4

$F(x) = x^2 + 1$

$H(x) = -x + 1$



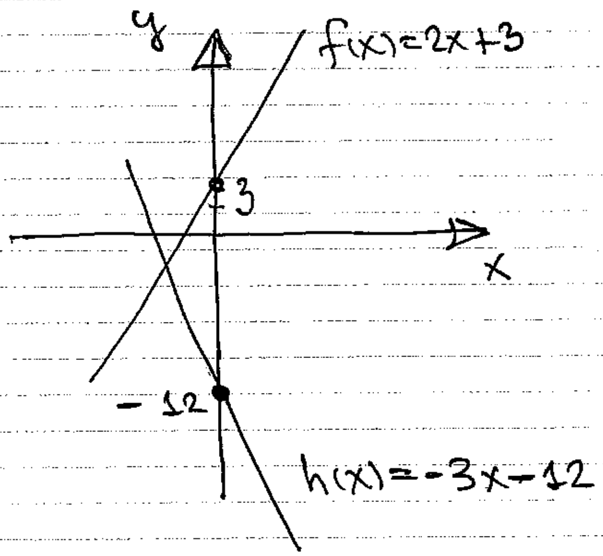


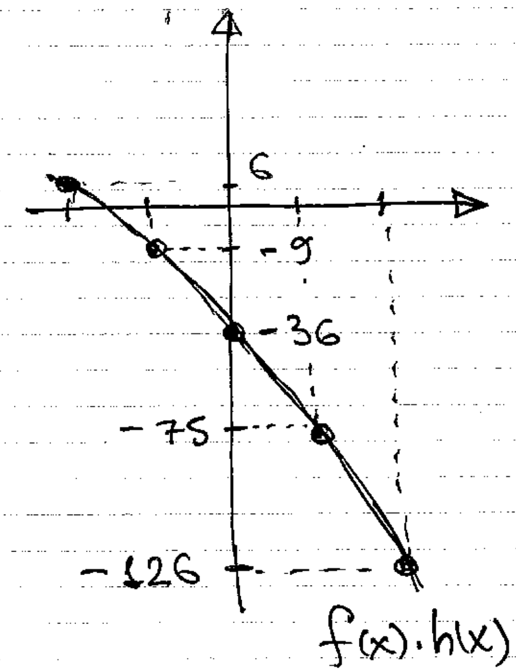
(b)

x	f(x)	h(x)	f(x) · h(x)
-2	-1	-6	6
-1	1	-9	-9
0	3	-12	-36
1	5	-15	-75
2	7	-18	-126

$f(x) = 2x+3$

$h(x) = -3x-12$





10. $f_1, f_2, f_5, f_6, f_8, f_9$

11. (a) $y=\sqrt{x}+4 \rightarrow \sqrt{x}=y-4, \quad x=(y-4)^2$

$\longrightarrow f^{-1}(x)=(x-4)^2$

(b) $y=\dfrac{1-x}{2+x} \longrightarrow 2y+xy=1-x$

$xy+x=1-2y$

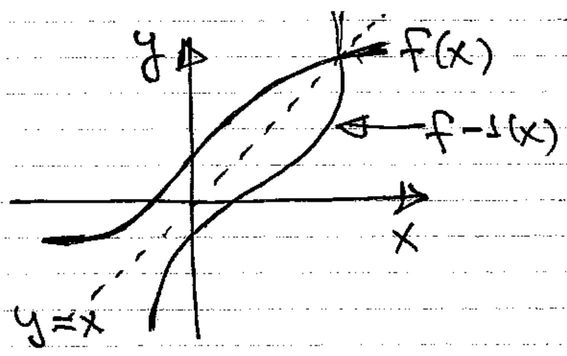
$x(y+1)=1-2y \rightarrow x=\dfrac{1-2y}{y+1}$

$f^{-1}(x)=\dfrac{1-2x}{x+1}$

(c) $y=(x-4)^7 \rightarrow x-4=y^{1/7}, \quad x=y^{1/7}+4$

$f^{-1}(x)=x^{1/7}+4$

(d) reflect across $y=x$



12. (a) no value of m is repeated

m^{-1} = time when there was certain number of monkeys

for instance $m^{-1}(41)=4$

ie there were 41 monkeys in year 4 (=2004)

(b) $m^{-1}(65)$ = year when the monkey count was 65
$= 6$ (ie in 2006)

(c) $C(m(t))$ is the number of coconuts eaten by all monkeys in year t