LS

ASSIGNMENT 3

1. (a) $= 1.6^{2-(-3)} = 1.6^5$

(b) $= \frac{21.6^8}{21.6^6} = 21.6^2$

(c) $= \frac{1.6^8}{1.6^8} = 1$

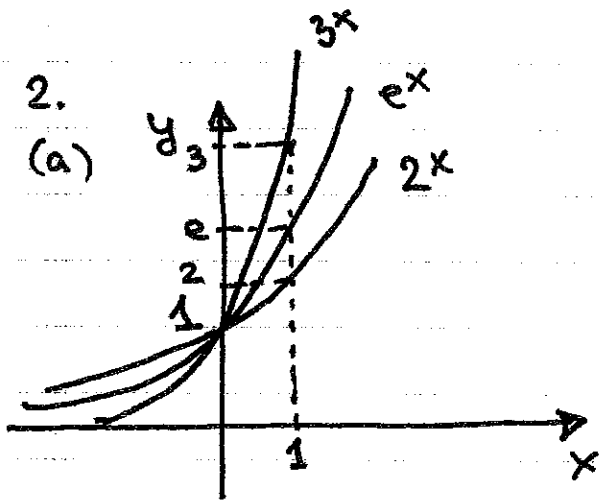
(d) $= 3.3^{0.44+0.44} \cdot \underbrace{3.4^{-0.46+0.23\cdot 2}}$

$= 3.3^{0.88}$ $\qquad = 3.4^{-0.46+0.46} = 3.4^0 = 1$

(e) $= \frac{(3.79)^{7/2}}{(3.79^4)^{1/2}} = \frac{3.79^{7/2}}{3.79^2} = 3.79^{\frac{7}{2}-2} = 3.79^{3/2}$

(f) $= 3^3 \cdot (3^2)^2 \cdot 3^3 = 3^{3+4+3} = 3^{10}$

2. (a)



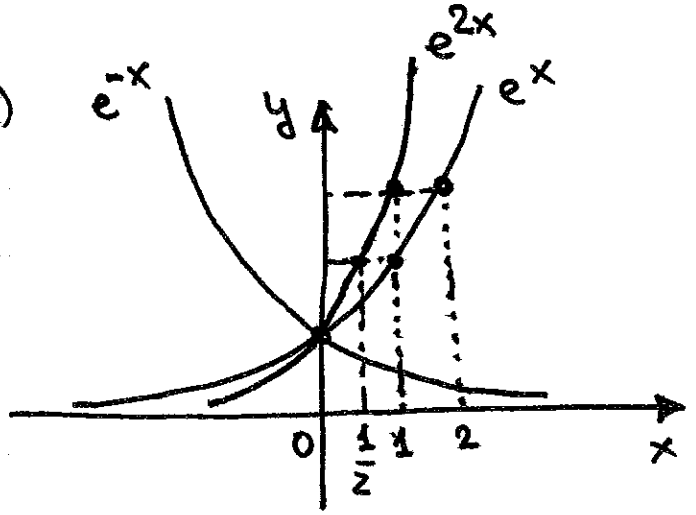
(y axis not to scale)

as a increases:

a^x approaches ∞ faster as $x \to \infty$

a^x approaches 0 faster as $x \to -\infty$

(b)

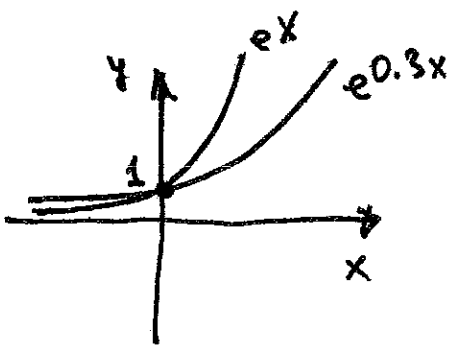


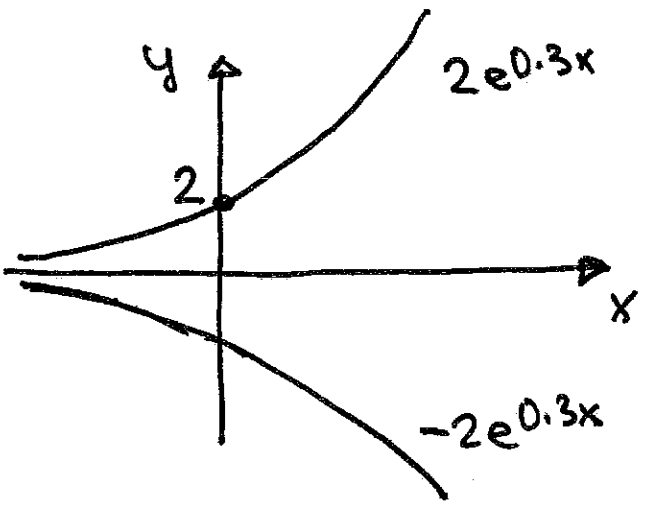
e^{-x}... symmetric to e^{x} with respect to y-axis

e^{2x}... horizontal compression by factor of 2

(c) $e^{x} \rightsquigarrow e^{0.3x} \rightarrow 2e^{0.3x} \rightsquigarrow -2e^{0.3x}$

horiz. expansion; vertical expansion; mirror across x-axis





3. (a) $= \log_{10} 10^{4} = 4$

(b) $= -0.33$

(c) $= 1$

(d) $= \log_{10} 100 = 2$

(e) $= \log_{10}(50 \cdot 2000) = \log_{10} 100000 = 5$

(f) $= \log_{10}(0.001 \cdot 0.1) = \log_{10} \underbrace{0.0001}_{=10^{-4}} = -4$

(g) $= \log_{10} 10^{-3} - \log_{10} 10^{-1} = -3 - (-1) = -2$

or:

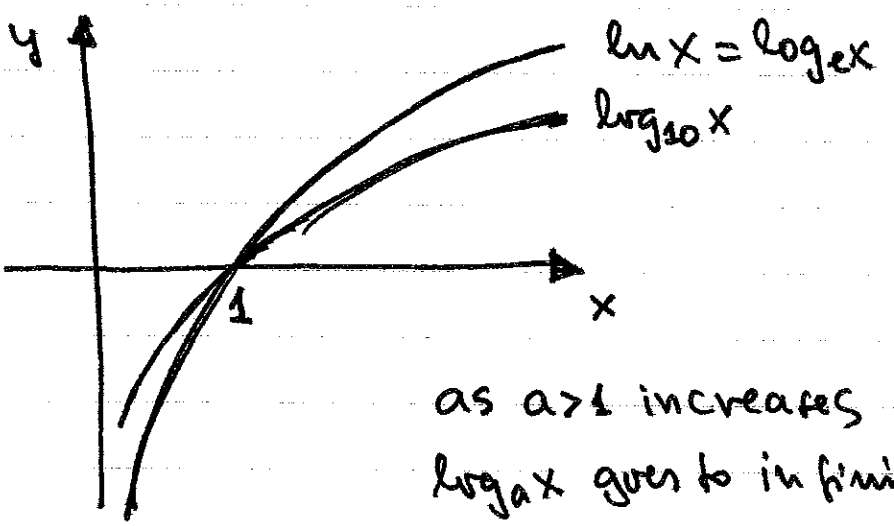
$= \log_{10} \frac{0.001}{0.1} = \log_{10} 0.01 = \log_{10} 10^{-2} = -2$

(h) $= 0.09$ (i) $= \ln(e^{-1}) = -1$

(j) cannot be simplified

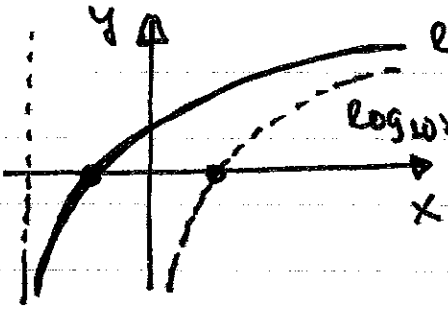
(k) $= \ln 11.4 + \ln e = \ln 11.4 + 1$

4.(a)

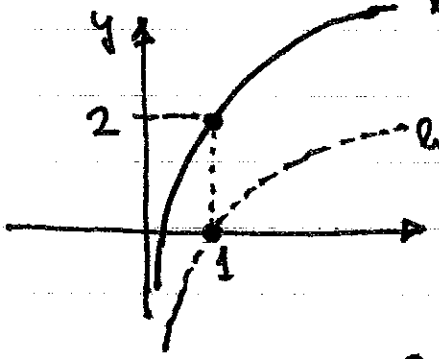


as $a > 1$ increases $\log_a x$ goes to infinity slower as $x \to \infty$

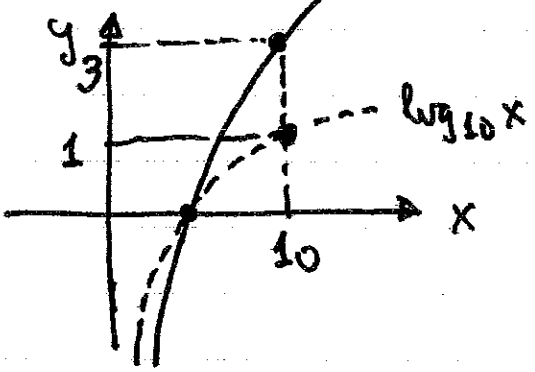
(b)



$\log_{10}(x+2)$ = $\log_{10}x$ shifted 2 units to the left

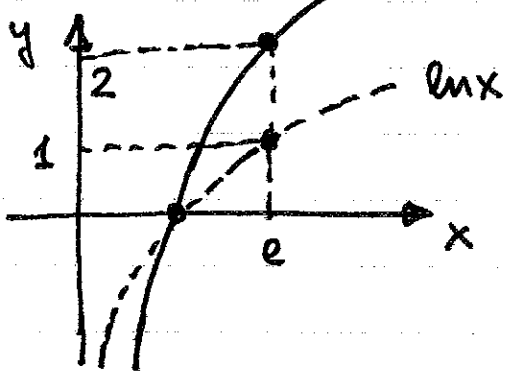


$\log_{10}x + 2$ = $\log_{10}x$ moved up 2 units

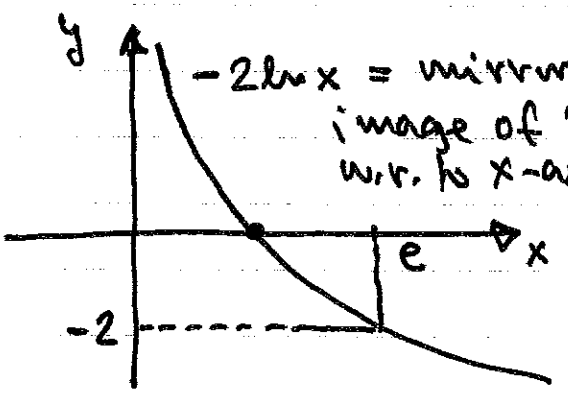


$3\log_{10}x$ = $\log_{10}x$ stretched by the factor of 3

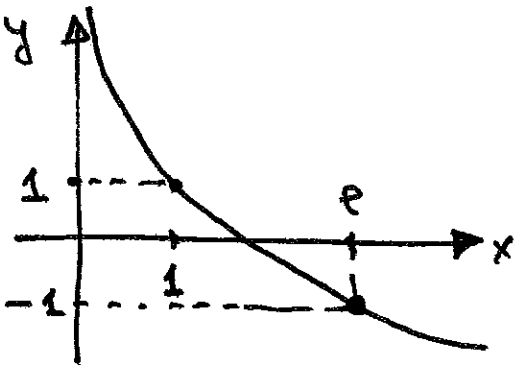
(c)



$2\ln x$ = $\ln x$ stretched by the factor of 2



$1 - 2\ln x = \underbrace{-2\ln x}_{\text{we have it}} + 1$



move $-2\ln x$ one unit up

5. (a) $4e^x = 7 \rightarrow e^x = 7/4 \rightarrow x = \ln(7/4)$

(b) $e^{5x-1} = \frac{64}{4} = 16 \rightarrow 5x - 1 = \ln 16$

$$x = \frac{\ln 16 + 1}{5}$$

(c) $0.5^{x^2} = 0.5^3 \rightarrow x^2 = 3, \quad x = \pm\sqrt{3}$

(d) $\ln(2-3x) = 3.5 \rightarrow 2 - 3x = e^{3.5}$

so $3x = 2 - e^{3.5}, \quad x = \frac{2 - e^{3.5}}{3}$

(e) $\ln(x+6) = \ln x^2 \rightarrow x + 6 = x^2$

ie $x^2 - x - 6 = 0 \rightarrow (x-3)(x+2) = 0$

$\rightarrow x = 3$ ✓

$\rightarrow x = -2$ not a solution since $\ln x$ is not defined!

(f) $\ln(x-4) = \ln 7 - \ln 3.5 = \ln \frac{7}{3.5} = \ln 2$

$\rightarrow x-4=2\,,\quad x=6$

6. It is given that

$$P(t) = 3.2 \cdot 10^4 \cdot 1.32^t$$

1 millim: $10^6 = 3.2 \cdot 10^4 \cdot 1.32^t$

$$\rightarrow 1.32^t = \frac{10^6}{3.2 \cdot 10^4} = \frac{10^2}{3.2} = 31.25$$

$$\ln(1.32^t) \overset{\ln}{=} (31.25)$$

$$t \cdot \ln 1.32 = \ln 31.25$$

$$\text{so } t = \frac{\ln 31.25}{\ln 1.32} = 12.398$$

so $P_{12} < 1$ millim, $\underline{\underline{P_{13} > 1 \text{ millim}}}$

1 billim: $10^9 = 3.2 \cdot 10^4 \cdot 1.32^t$

$$\rightarrow 1.32^t = 31250$$

$$\text{as above ... } t = \frac{\ln(31250)}{\ln 1.32} = 37.279$$

so $P_{37} < 1$ billim, $\underline{\underline{P_{38} > 1 \text{ billim}}}$

7. $2400 = 1200\, e^{1.32t} \rightarrow 2 = e^{1.32t}$

so $\ln 2 = \ln(e^{1.32t}) = 1.32\, t$

i.e. $t = \frac{\ln 2}{1.32} \approx 0.525$ time units

8. $P(t) = P(0)\, e^{rt}$ $\quad \frac{2}{3} P(0)$ lost

$\frac{1}{3} P(0) = P(0)\, e^{r \cdot 1}$ $\quad \rightarrow \frac{1}{3} P(0)$ left

$e^r = 1/3$, i.e. $r = \ln(1/3) = \ln 1 - \ln 3 = -\ln 3$

so $P(t) = P(0)\, e^{(-\ln 3)\, t}$

half life: $\frac{1}{2} P(0) = P(0)\, e^{(-\ln 3)t}$

$(-\ln 3)\, t = \ln(1/2) = \ln 1 - \ln 2 = -\ln 2$

so $t = \frac{-\ln 2}{-\ln 3} = 0.631$ years

9. 100 $\underbrace{\qquad}_{2 \text{ hours}}$ 250 $\underbrace{\qquad}_{2 \text{ hours}}$?

in 2 hours, population increased 2.5 times $\rightarrow$ in another 2-hour interval has to increase 2.5 times

answer: $250 \cdot 2.5 = 625$

10.(a) $0.45 \cdot 24.5 = 24.5\, e^{-0.0032t}$

$-0.0032t = \ln 0.45$

reach 45% ... $t = \dfrac{\ln 0.45}{-0.0032} = 249.5$

reach 44% ... $t = \dfrac{\ln 0.44}{-0.0032} = 256.6$

time difference: $256.6 - 249.5 =$ **7.1**

(b) reach 10% ... $t = \dfrac{\ln 0.1}{-0.0032} = 719.6$

reach 9% ... $t = \dfrac{\ln 0.09}{-0.0032} = 752.5$

time difference: $752.5 - 719.6 =$ **32.9**

(c) because M(t) is not linear!