ASSIGNMENT 51

1. (a) $\sinh 0.2 = \frac{e^{0.2} - e^{-0.2}}{2} \approx 0.20134$

$\sinh 3 = \frac{e^{3} - e^{-3}}{2} \approx 10.01787$

(b) $p = 0.267(10) + \frac{1.3}{0.5} \cdot \frac{\sinh 0.2}{\sinh 3}$

$\approx 2.67 + 2.6 \cdot \frac{0.20134}{10.01787} \approx 2.72226$

(c) $p = 0.267pi + \frac{1.3}{r} \cdot \frac{(e^{0.4r} - e^{-0.4r})/2}{(e^{3} - e^{-3})/2}$ ← cancel by 1/2

$= 0.267pi + \frac{1.3}{r} \cdot \frac{e^{0.4r} - e^{-0.4r}}{e^{3} - e^{-3}}$

ignore this part, the question is not clear

2. (a) $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are functions of t

parameters are: $\hat{v}_0, a, f_m, \hat{d}, \hat{d}_{lag}$

(b) $\cos(2\pi f_m(t + \hat{d}))$ ← $\hat{d}$ is a shift, so it does not affect the period

$\text{period} = \frac{2\pi}{2\pi f_m} = \frac{1}{f_m}$

(c) same as (b)

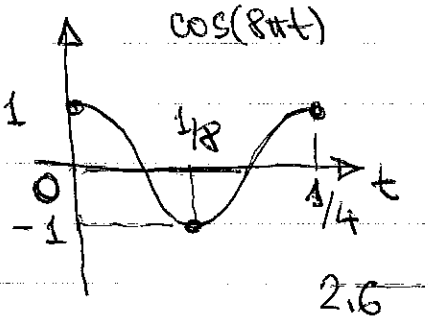
(d) $\hat{\lambda}_1 = 0 + 2.6\cos(2\pi \cdot 4(t + \pi))$

amplitude = 2.6 ; period = 1/4 ; phase = $-\pi$

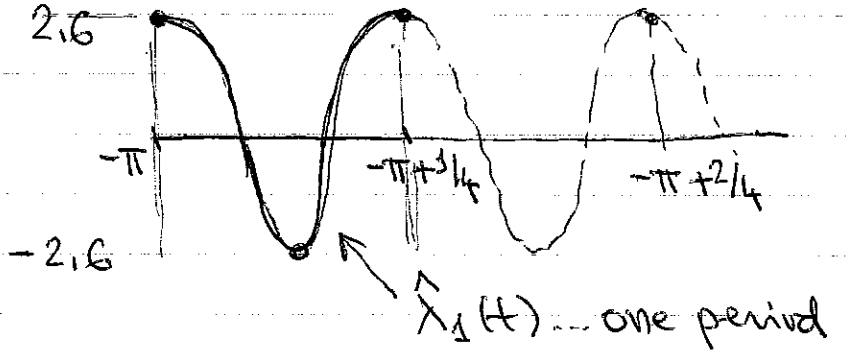
to graph, consider the sequence of transformations:

$\cos t \longrightarrow \cos(8\pi t) \longrightarrow \cos(8\pi(t+\pi)) \longrightarrow 2.6\cos(8\pi(t+\pi))$

main period:

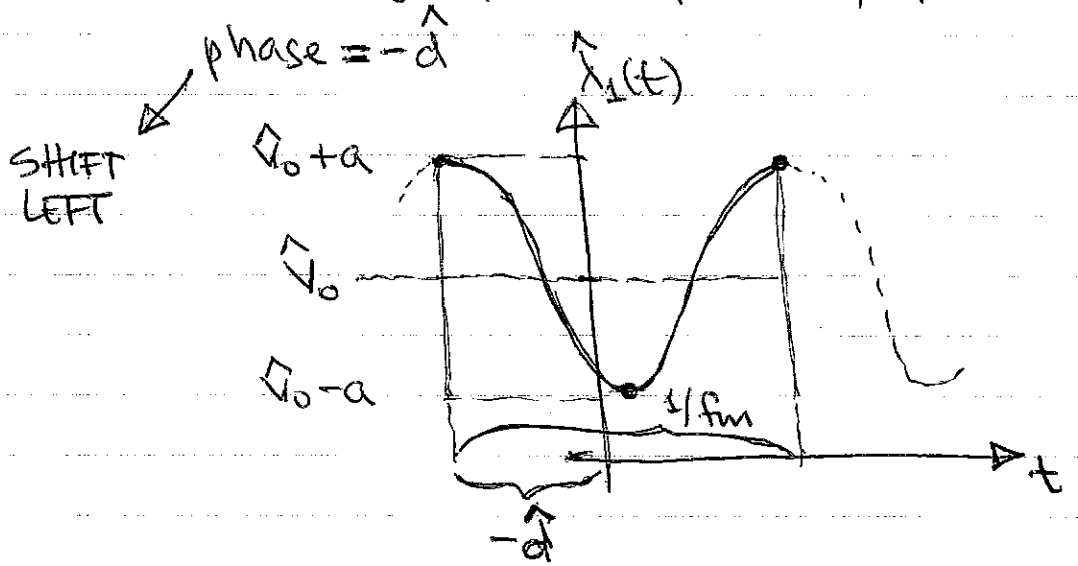


shift left by π and expand vertically by a factor of 2.6:



$\hat{\lambda}_1(t)$ ... one period

(e) $\hat{V}_0$ = average, a = amplitude, period $= \frac{2\pi}{2\pi f_m} = \frac{1}{f_m}$



(f) shift $\hat{\lambda}_1(t)$ to the right by the amount equal to $|\hat{d}_{lag}|$ ← absolute value of $\hat{d}_{lag}$

3.(a) first peak: $t=21$; second peak: $t=44$ → period $=23$

(check: third peak should be at $44+23=67$; looks like it is)

(b) minimum: 0.35; maximum: 1.8

average $= (1.8+0.35)/2 = 1.075$

amplitude $= 1.8 - 1.075 = 0.725$

(c) shift right $=21$

so $$Bmal(t) \approx 1.075 + 0.725 \cos\left(\frac{2\pi}{23}(t-21)\right)$$

(d) first peak: $t=8$; second peak: $t=32$ → period $=24$

(check: third peak: $32+24=56$)

minimum: 0.8, maximum: 1.2

average $=1$, amplitude $=0.2$

shift right $=8$

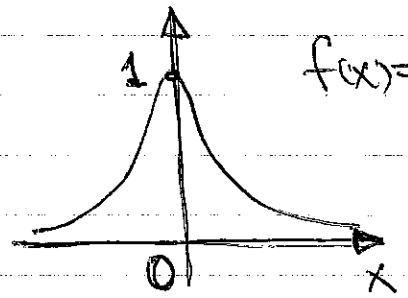
$$REV-ERB_N(t) = 1 + 0.2\cos\left(\frac{2\pi}{24}(t-8)\right)$$

(e) amplitude $=0.1$

shift right $=18$

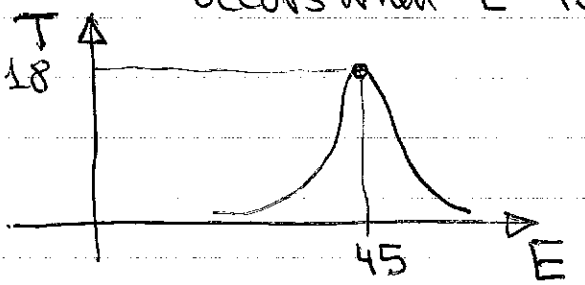
$$ROR_N = 1 + 0.1\cos\left(\frac{2\pi}{24}(t-18)\right)$$

4. (a)



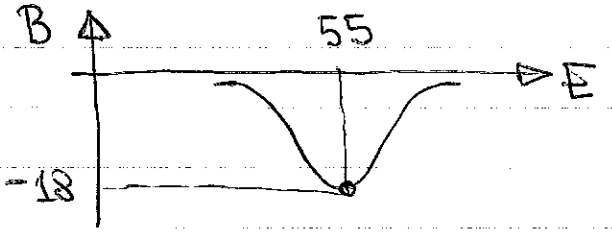
$$f(x) = \frac{1}{1+x^2}$$

top curve: maximum = 18
occurs when $E = 45$



so $$T(E) = 18 \cdot \frac{1}{1+(E-45)^2}$$

(b)



$$B(E) = -18 \cdot \frac{1}{1+(E-55)^2}$$

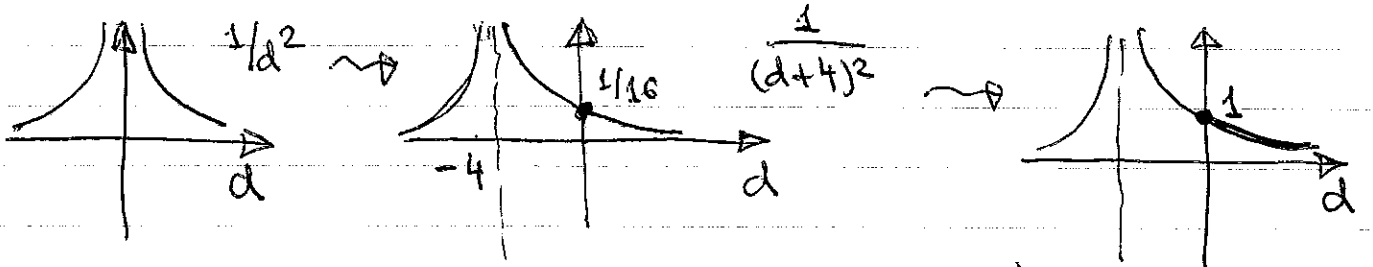
(NOTE: 18, 45 and 55 are estimates)

5. (a) $f(0) = \pi$; $f(5) = \dfrac{1}{(1+5/4)^5}$

(b) $$f(d) = \begin{cases} \pi & \text{if } d = 0 \\ \dfrac{1}{(1+d/4)^2} & \text{if } d > 0 \end{cases}$$

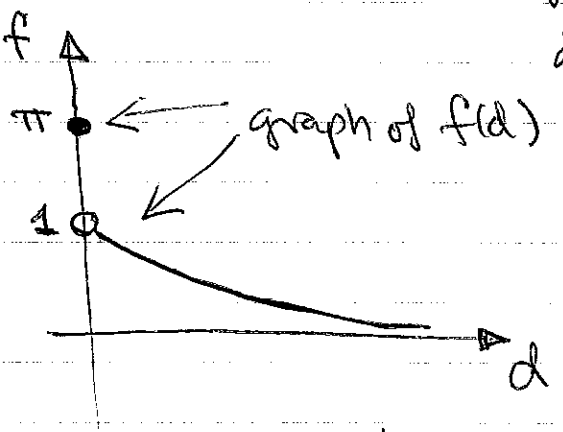
$$= \frac{1}{\left(\frac{1}{4}(4+d)\right)^2} = \frac{1}{\frac{1}{16}(4+d)^2} = 16 \cdot \frac{1}{(d+4)^2}$$

SIMPLIFY TO SEE HOW TO TRANSFORM



graph of $\frac{16}{(d+4)^2}$

we need only $d>0$ part



6. (a) when $\log r = -5$, $\log k = 0.1$; when $\log r = 1$, $\log k = 2.4$ (GIVEN; ESTIMATE)

so slope $= \frac{2.4 - 0.1}{1-(-5)} = \frac{2.3}{6} \approx 0.38$

point-slope equation:

$$\log k - 0.1 = 0.38(\log r - (-5))$$

$$\log k = 0.38 \log r + 2$$

(b) $$10^{\log k} = 10^{0.38 \log r + 2}$$

$$k = 10^{\log r^{0.38}} \cdot 10^2$$

$$k = 100 \cdot r^{0.38}$$