ASSIGNMENT 52

1. (a) P_0 is a function of τ

since $e^{\text{anything}} > 0$, the denominator is always > 1
so never zero $\rightarrow$ domain is all real numbers

because of context: domain for τ is $\tau > 0$

(b) P_0 is a quotient of two positive numbers, so $P_0 > 0$
as well $\quad P_0 = \dfrac{1}{\underset{\text{(see (a))}}{\text{quantity larger than } 1}} < 1$

(c) $\qquad e^{\beta(\Delta G - \tau \Delta A)}$ term:

as $\tau \rightarrow \infty$, $\tau\Delta A \rightarrow \infty$ (since $\Delta A > 0$)
then $\quad -\tau\Delta A \rightarrow -\infty$
$\Delta G - \tau\Delta A \rightarrow -\infty$ (no matter what ΔG is)
$\beta(\Delta G - \tau\Delta A) \rightarrow -\infty$ (since $\beta > 0$)
$e^{\beta(\Delta G - \tau\Delta A)} \rightarrow e^{-\infty} = 0$

thus, P_0 approaches $\dfrac{1}{1+0} = 1$

2. (a) $\displaystyle \lim_{d \to \infty} \frac{1}{(1+d/\alpha)^\gamma} = \frac{1}{\infty} = 0$ $\quad$ since $1 + \frac{d}{\alpha} \rightarrow \infty$
and $(1+\frac{d}{\alpha})^\gamma \rightarrow \infty$
(as $\gamma > 0$)

(b) $\displaystyle \lim_{d \to \infty} \frac{1}{(1+d/\alpha)^\gamma} = \frac{1}{0} = \infty$ $\quad$ since $1 + \frac{d}{\alpha} \rightarrow \infty$
and $\gamma < 0$, $(1+\frac{d}{\alpha})^\gamma \rightarrow 0$

IF $\gamma < 0$ this is not a fraction!

(c) $\lim_{d \to 0} \frac{1}{(1+\frac{d}{\gamma})^{\gamma}} = \frac{1}{1} = 1$

(d) because $\lim_{d \to 0} f(d) = 1 \neq \pi = f(0)$

we see that f is not continuous at $d=0$.

3. (a) $V = \frac{N_e V_e + N_i V_i}{N_e + N_i} = \underbrace{\frac{N_i}{N_e + N_i}}_{\text{slope}} V_i + \underbrace{\frac{N_e V_e}{N_e + N_i}}_{\text{intercept}}$

line

(b) $\lim_{N_i \to \infty} \frac{N_e V_e + N_i V_i}{N_e + N_i} \begin{array}{l} | \div N_i \\ | \div N_i \end{array}$

$= \lim_{N_i \to \infty} \frac{\cancel{\frac{N_e V_e}{N_i}}^{\to 0} + V_i}{\cancel{\frac{N_e}{N_i}}_{\to 0} + 1} = V_i$

(c) $\lim_{V_e \to -\infty} \frac{N_e V_e + N_i V_i}{N_e + N_i} = \frac{N_e(-\infty) + \text{number}}{\text{number} > 0} = -\infty$

(from the context we know that $N_e, N_i > 0$)

4. (a) $b_{t+1} = \frac{2{,}7 a b_t^{\alpha+1}}{1{,}3 + b_t^2} = \underbrace{\frac{2{,}7 a b_t^{\alpha}}{1{,}3 + b_t^2}}_{\substack{\text{per capita} \\ \text{production}}} \cdot b_t$

(b) $b^* = \frac{2{,}7 a (b^*)^{\alpha+1}}{1{,}3 + (b^*)^2}$

$$b^* - \frac{2{,}7a(b^*)^{\alpha+1}}{1{,}3+(b^*)^2} = 0$$

$$b^*\left[1 - \frac{2{,}7a(b^*)^{\alpha}}{1{,}3+(b^*)^2}\right] = 0$$

so $b^* = 0$ is one equilibrium (makes sense : no bacteria to start with means no bacteria in future)

remaining equilibria:

$$1 - \frac{2{,}7a(b^*)^{\alpha}}{1{,}3+(b^*)^2} = 0$$

$$2{,}7a(b^*)^{\alpha} = 1{,}3+(b^*)^2$$

(c) $a=1, \alpha=2 \rightarrow$ $2{,}7(b^*)^2 = 1{,}3+(b^*)^2$

$$1{,}7(b^*)^2 = 1{,}3$$

$$b^* = \pm\sqrt{1{,}3/1{,}7} \approx \pm 0{,}87$$

makes no sense

thus $b^* = 0{,}87$ (million) is an equilibrium

5 (a) $\text{p.c.p.} = ae^{-mb_t} - de^{-nb_t} + 1$

(b) $b^* = b^*(ae^{-mb^*} - de^{-nb^*} + 1)$

$$b^*(\cancel{1} - ae^{-mb^*} + de^{-nb^*} - \cancel{1}) = 0$$

so $b^* = 0$ or $ae^{-mb^*} = de^{-nb^*}$

always equilibrium (no conditions on parameters needed)

$$e^{-mb^*+nb^*} = \frac{d}{a}$$

$$e^{(n-m)b^*} = \frac{d}{a}$$

so $(n-m)b^* = \ln(d/a)$

$$b^* = \frac{\ln(d/a)}{n-m}$$

by assumption, $n-m>0$; for b^* to be positive, $\ln\left(\frac{d}{a}\right)>0$

i.e. $\underline{\underline{\frac{d}{a}>1}}$ or $\underline{\underline{d>a}}$

6. (a) Simplify: $AIC = n(\log SS - \log n) + 2K + C$

Thus $AIC' = (\log SS - \log n) + n\left(0 - \frac{1}{n \ln 10}\right) + 0$

product rule

$$= \log\frac{SS}{n} - \frac{1}{\ln 10}$$

(b) $\lim_{n\to\infty}\left(n\log\left(\frac{SS}{n}\right) + \underbrace{2K+C}_{\text{constants}}\right) = \infty(-\infty) + 2K + C = -\infty$

as $n\to\infty$, $\frac{SS}{n}\to 0^+$ and so $\log\left(\frac{SS}{n}\right)\to-\infty$

7. (a) $S(d) = e^{-\alpha n d - \beta n d^2} = e^{-(\alpha n d + \beta n d^2)}$

by assumption, α, β, n, d are positive

so $\alpha n d + \beta n d^2 > 0$

$-(\alpha n d + \beta n d^2) < 0$

$e^{-(\alpha n d + \beta n d^2)} < e^0 = 1$

as well, $e^{\text{anything}} > 0$ $\rightarrow$ so range is $\underline{\underline{(0,1)}}$

(b) S is a composition of two continuous functions: polynomial and exponential function.

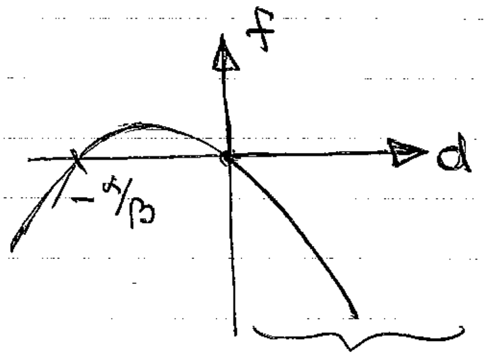
(c) $f(d) = -\alpha n d - \beta n d^2$ ... parabola, looks like $\cap$

intercepts
$$\begin{cases} -\alpha n d - \beta n d^2 = 0 \\ d(-\alpha n - \beta n d) = 0 \end{cases}$$

$\rightarrow d = 0$

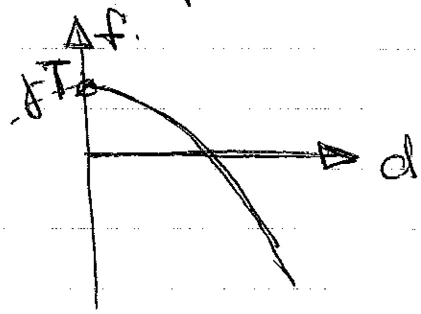
$\rightarrow -\alpha n - \beta n d = 0$

$$d = \frac{-\alpha n}{\beta n} = -\frac{\alpha}{\beta}$$



relevant part of the graph

(d) Shift the graph in (c) γT units up



(e) $S(d) = e^{-\alpha n d - \beta n d^2 + \gamma T}$

$$\rightarrow S'(d) = \underbrace{e^{-\alpha n d - \beta n d^2 + \gamma T}}_{>0} (-\alpha n - 2\beta n d) = 0$$

$$-\alpha \cancel{n} - 2\beta \cancel{n} d = 0$$

$$d = -\frac{\alpha}{2\beta}$$ is the only critical point