ASSIGNMENT 53

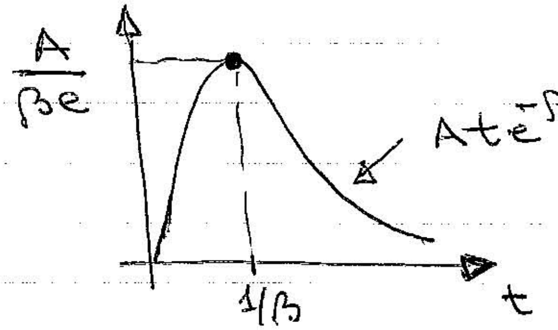
1.(a) $f(t) = Ate^{-\beta t}$

$$f'(t) = Ae^{-\beta t} + Ate^{-\beta t}(-\beta) = A\underbrace{e^{-\beta t}}_{\neq 0}(1-\beta t) = 0$$

so $1-\beta t = 0 \rightarrow t = 1/\beta$

(if $t < 1/\beta$ then $f'(t) > 0 \rightarrow f'$ is incr.
if $t > 1/\beta$ then $f'(t) < 0 \rightarrow f'$ is decr.
so $t = 1/\beta$ is rel. max.)

max value: $f\left(\frac{1}{\beta}\right) = A\frac{1}{\beta}e^{-\beta\frac{1}{\beta}} = \frac{A}{\beta}e^{-1}$



(b) hard to estimate: let $t = 5$, and $\max = 0.0027$

$$\frac{1}{\beta} = 5 \rightarrow \beta = \frac{1}{5} = 0.2$$

$$\frac{A}{\beta e} = 0.0027$$

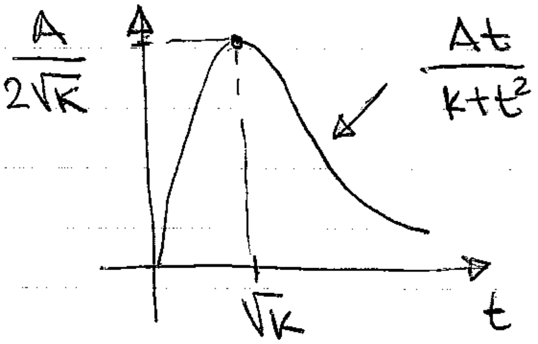
$$A = \underbrace{\beta}_{0.2} e\, 0.0027 \approx 0.001468$$

thus $\boxed{C(t) = 0.001468\, t\, e^{-0.2t}}$

(c) again, find max. of $c(t) = \frac{At}{k+t^2}$ first

$$c'(t) = \frac{A(K+t^2) - At(2t)}{(K+t^2)^2} = \frac{AK - At^2}{(K+t^2)^2} = 0$$

$$AK - At^2 = 0 \longrightarrow t^2 = K, \quad t = \pm\sqrt{K}$$



max. value

$$C(\sqrt{K}) = \frac{A\sqrt{K}}{K+K} = \frac{A}{2\sqrt{K}}$$

as in (b) ... $t=5 \rightarrow \sqrt{K}=5$ so $K=25$

$$\max = 0{,}0027 \longrightarrow \frac{A}{2\sqrt{K}} = 0{,}0027$$

$$A = 2\sqrt{25}(0{,}0027) = 0{,}027$$

so $\boxed{C(t) = \dfrac{0{,}027t}{25+t^2}}$

2.

$$I_2(t) = 1 + ae^{-2t} + be^{-2{,}4t}$$

$$I_2'(t) = -2ae^{-2t} - 2{,}4be^{-2{,}4t} = 0 \quad \ldots \quad \circledast$$

$$I_2'(t) = \underbrace{-2e^{-2t}}_{\neq 0}\left(a + 1{,}2be^{-0{,}4t}\right) = 0$$

$$a + 1{,}2be^{-0{,}4t} = 0$$

$$e^{-0{,}4t} = \frac{-a}{1{,}2b}$$

$$-0.4t = \ln\left(\frac{-a}{1{,}2b}\right)$$

$$t = -\frac{1}{0.4}\ln\left(\frac{-a}{1{,}2b}\right)$$

Thus, there is one critical point, as long as the parameters a and b are chosen so that $\frac{-a}{1,2b} > 0$.
If $\frac{-a}{1,2b} \le 0$ then there are no critical points

3. Write $R_0 = \frac{\sqrt{\beta K_f}}{\sqrt{1+\frac{d_f}{1-e^{-aK}}}} = \sqrt{\beta K_f} \cdot \left(1 + \frac{d_f}{1-e^{-aK}}\right)^{-1/2}$
$= \sqrt{\beta K_f} \cdot \left(1 + d_f(1-e^{-aK})^{-1}\right)^{-1/2}$

$$R_0' = \sqrt{\beta K_f} \cdot \left(-\frac{1}{2}\right)\left(1 + \frac{d_f}{1-e^{-aK}}\right)^{-3/2} \cdot d_f \cdot$$
$$\cdot (-1)(1-e^{-aK})^{-2}(-e^{-aK})(-a)$$

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$$= \underbrace{\sqrt{\beta K_f}}_{\oplus ve} \cdot \frac{1}{2} \cdot \underbrace{\left(1 + \frac{d_f}{1-e^{-aK}}\right)^{-3/2}}_{\text{square root so } \oplus ve} \underbrace{a}_{\text{assumed } \oplus ve} \underbrace{e^{-aK}}_{\oplus} \underbrace{(1-e^{-aK})^{-2}}_{\text{square so } \oplus ve}$$

Thus $R_0' > 0$, ie R_0 is increasing

4. (a) $S(d) = e^{-\alpha n d - \beta n d^2}$

$$S'(d) = e^{-\alpha n d - \beta n d^2}(-\alpha n - 2\beta n d)$$
$$= \underbrace{e^{-\alpha n d - \beta n d^2}}_{\oplus}(-1)\underbrace{(\alpha n + 2\beta n d)}_{\oplus \text{ since } \alpha, \beta, n, d > 0}$$

$S'(d) < 0 \to S(d)$ is decreasing

Makes sense: stronger dosage means more cells are killed, so survival rate decreases

(b) $S(d) = e^{-\alpha n d + \gamma T}$

$S'(d) = \underbrace{e^{-\alpha n d + \gamma T}}_{\oplus} (-\underbrace{\alpha n}_{\oplus}) < 0 \Rightarrow$ S(d) is decreasing

(c) $S(d) = e^{-\beta n d^2 + \gamma T}$

$S'(d) = \underbrace{e^{-\beta n d^2 + \gamma T}}_{\oplus} (-\underbrace{2\beta n d}_{\oplus}) < 0 \Rightarrow$ S(d) is decreasing

(d) $S(n) = e^{-\alpha n d - \beta n d^2 + \gamma T}$

$S'(n) = e^{-\alpha n d - \beta n d^2 + \gamma T} \underbrace{(-\alpha d - \beta d^2)}$

$= -(\alpha d + \beta d^2) = \text{negative}$

So S(n) decreases.

As we increase the number of treatments, the survival rate will decline.

5.
$$p = 0.267 p_i + \frac{1.3}{r} \cdot \frac{e^{0.4r} - e^{-0.4r}}{2\sinh\alpha}$$
$$= 0.267 p_i + \frac{1.3}{2\sinh\alpha} \cdot \frac{e^{0.4r} - e^{-0.4r}}{r}$$

$$p' = \frac{1.3}{2\sinh\alpha} \cdot \frac{(0.4e^{0.4r} + 0.4e^{-0.4r})r - (e^{0.4r} - e^{-0.4r}) \cdot 1}{r^2}$$

$$= \underbrace{\frac{1.3}{2\sinh\alpha}}_{\oplus} \cdot \underbrace{\frac{1}{r^2}}_{\oplus} \cdot \left(\underbrace{e^{0.4r}}_{\oplus} (0.4r - 1) + \underbrace{e^{-0.4r}}_{\oplus} \underbrace{(0.4r + 1)}_{\oplus} \right)$$

If $r > 2.5$ then

$$0.4r - 1 > 0.4(2.5) - 1 = 0$$

so $p' > 0$, ie p is increasing

6. (a)

$$f(d) = \left(1 + \frac{d}{\alpha}\right)^{-\gamma}$$

$$f'(d) = (-\gamma)\left(1 + \frac{d}{\alpha}\right)^{-\gamma-1} \cdot \frac{1}{\alpha}$$

$$= -\frac{\gamma}{\alpha}\left(1 + \frac{d}{\alpha}\right)^{-\gamma-1}$$

(b)

$$f''(d) = -\frac{\gamma}{\alpha}(-\gamma-1)\left(1 + \frac{d}{\alpha}\right)^{-\gamma-2}\left(\frac{1}{\alpha}\right)$$

$$= \frac{(-\gamma)(-1)(\gamma+1)}{\alpha^2}\left(1 + \frac{d}{\alpha}\right)^{-\gamma-2}$$

$$= \frac{\gamma(\gamma+1)}{\alpha^2} \cdot \frac{1}{(1 + d/\alpha)^{\gamma+2}}$$