Assignment 54

1. Need to show that $P_0' > 0$; write

$$P_0 = \left(1 + e^{\beta(\Delta G - \tau\Delta A)}\right)^{-1}$$

and differentiate with respect to τ using the chain rule (β, ΔG and ΔA are constants)

assumed positive

$$P_0' = (-1)\left(1 + e^{\beta(\Delta G - \tau\Delta A)}\right)^{-2} \cdot$$

$$\cdot \left(0 + e^{\beta(\Delta G - \tau\Delta A)}\right) \cdot \beta(-\Delta A)$$

$$= \frac{\beta\Delta A \cdot e^{\beta(\Delta G - \tau\Delta A)}}{\underbrace{\left(1 + e^{\beta(\Delta G - \tau\Delta A)}\right)^2}_{\text{always } \oplus}} \longrightarrow \text{always } \oplus$$

β, ΔA are $\oplus$ve by assumption, so $P_0' > 0$

2. (a) Rewrite V as

variable!

$$V = \frac{N_e V_e + N_i V_i}{N_e + N_i} = \frac{N_e}{N_e + N_i} V_e + \frac{N_i V_i}{N_e + N_i}$$

so V is a line of slope $\frac{N_e}{N_e + N_i}$ (which is $\oplus$ve)

and vertical intercept $= \frac{N_i V_i}{N_e + N_i}$

thus $V' = \frac{N_e}{N_e + N_i}$ ← derivative of V as a function of V_e

so we can use notation $V' = \frac{dV}{dV_e}$

so V is an increasing function of the membrane potential V_e (ie, as V_e increases, so does V)

OR we could have computed V' straight from
$$V = \frac{N_eV_e + N_iV_i}{N_e+N_i}$$ using quotient rule, keeping in mind that V_e is the variable and N_e, N_i, V_i are constants:

$$V' = \frac{(N_eV_e + N_iV_i)'(N_e+N_i) - (N_eV_e+N_iV_i)\underbrace{(N_e+N_i)'}_{\text{constant}}}{(N_e+N_i)^2}$$

$$= \frac{(N_e+0)\cdot(N_e+N_i) - \cancel{(N_eV_e+N_iV_i)\cdot 0} \to 0}{(N_e+N_i)^2}$$

$$= \frac{N_e}{N_e+N_i}$$

(b) Use quotient rule: (N_e is like x now!)

this time $V' = \dfrac{dV}{dN_e}$

$$V' = \frac{(N_eV_e + N_iV_i)'(N_e+N_i) - (N_eV_e+N_iV_i)(N_e+N_i)'}{(N_e+N_i)^2}$$

$$= \frac{V_e(\cancel{N_e}+N_i) - (\cancel{N_eV_e}+N_iV_i)\cdot 1}{(N_e+N_i)^2}$$

$$= \frac{V_eN_i - V_iN_i}{(N_e+N_i)^2}$$

V' is the rate of change, so:

as Ne increases by 1 unit, V changes by

$$\frac{VeNi - ViNi}{(Ne+Ni)^2}$$

(could be ⊕ve or ⊖ve) so V might increase or decrease — in this case increases

recall: $f'(x) = 3$ means that f changes by approximately 3 units per unit change in x (think of the rise over run, with run $= 1$)

(c) $$V' = \frac{VeNi - ViNi}{(\underbrace{Ne}_{\text{variable}}+Ni)^2} = \underbrace{Ni(Ve-Vi)}_{\text{constant}} \cdot (Ne+Ni)^{-2}$$

chain rule

$$V'' = Ni(Ve-Vi)(-2)(Ne+Ni)^{-3} \cdot 1$$

$$= -2 \cdot \frac{\overset{\oplus}{Ni}(Ve-Vi)}{(Ne+Ni)^3_{\oplus}}$$

given: $Ne, Ni > 0$
$Ve, Vi < 0$

V is CU when $V'' > 0$

$\rightarrow -2(V_e - V_i) > 0$

$Ve - Vi < 0$

ie $Ve < Vi$

3. $$\bar{t} = -\frac{1}{r}\cdot\left(\underbrace{\ln \frac{\beta_w \beta_h \Lambda_1 \xi_1}{\beta_w \xi_1 + \sigma + r}}_{\text{no } \gamma \text{ here, so constant}} - \gamma\right)$$

thus $$\bar{t}' = \frac{d\bar{t}}{d\gamma} = -\frac{1}{r}\cdot(0-1) = \frac{1}{r}$$

so the rate of change of $\bar{t}$ is equal to the constant $1/r$

4. (a) $$R = \frac{K(\gamma+1)^2}{d^4}\cdot l \longrightarrow \frac{dR}{dl} = \frac{K(\gamma+1)^2}{d^4} > 0$$

so as the length of the tube increases, so does the resistance;

OR as l increases by 1 unit, R increases by approximately $\frac{K(\gamma+1)^2}{d^4}$ units

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(b) $$R = Kl(\gamma+1)^2 \cdot d^{-4} \longrightarrow R' = \frac{dR}{dd} = Kl(\gamma+1)^2\cdot\frac{-4}{d^5}$$

so $R' < 0$

ie, as the diameter of the tube increases, the resistance decreases

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or: as d increases by 1 unit, the resistance R decreases by approximately $\frac{4Kl(\gamma+1)^2}{d^5}$ units