Name: ______________________

Math 1LS3	Assignment 55

Math functions in context – Integration

1. Find the following integrals. Keep in mind that the notation for the integral tells you what the variable is.

(a) $\int ae^{bx}\,dx = a \cdot \frac{1}{b} e^{bx} + C$

(b) $\int ae^{bt}\,dt = a \cdot \frac{1}{b} e^{bt} + C$

(c) $\int Me^{-(b+c)t}\,dt = M \cdot (-1) \cdot \frac{1}{b+c} e^{-(b+c)t} = -\frac{M}{b+c} e^{-(b+c)t} + C$

(d) $\int \left(\gamma b^2 t^2 - abt\right)\,dt = \gamma b^2 \frac{t^3}{3} - ab\frac{t^2}{2} + C$

(e) $\int (N_e V_e + N_i V_i)\,dN_i = N_e V_e N_i + V_i \frac{N_i^2}{2} + C$

(f) $\int \frac{N_e V_e + N_i V_i}{4}\,dV_i = \frac{1}{4}\left(N_e V_e V_i + N_i \frac{V_i^2}{2}\right) + C$

(g) $\int \frac{7}{aMt}\,dt = \frac{7}{aM} \ln|t| + C$

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(h) $\int \frac{Kl(\gamma+1)^2}{D^4}\,dK = \frac{l(\gamma+1)^2}{D^4}\cdot\frac{K^2}{2} + C$

(i) $\int \frac{Kl(\gamma+1)^2}{D^4}\,d\gamma = \frac{Kl}{D^4}\int(\gamma^2+2\gamma+1)\,d\gamma$

$= \frac{Kl}{D^4}\left(\gamma^3/3 + \gamma^2 + \gamma\right) + C$

(j) $\int \frac{Kl(\gamma+1)^2}{D^4}\,dD = Kl(\gamma+1)^2\int D^{-4}\,dD$

$= Kl(\gamma+1)^2\cdot\frac{D^{-3}}{-3} = -\frac{Kl(\gamma+1)^2}{3D^3} + C$

(k) $\int b\left(e^{-at}+Ne^t\right)dt = b\left(-\frac{1}{a}e^{-at} + Ne^t\right) + C$

(l) $\int b\left(e^{-xt}+xe^t\right)dt = b\left(-\frac{1}{x}e^{-xt} + xe^t\right) + C$

(m) $\int b\left(e^{-xt}+xe^t\right)dx = b\left(-\frac{1}{t}e^{-xt} + e^t\cdot\frac{x^2}{2}\right) + C$

(n) $\int b\left(e^{-xt}+xe^t\right)db = \left(e^{-xt}+xe^t\right)\cdot\frac{b^2}{2} + C$

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2. According to Von Bertalanffy model, the rate of growth of pacific salmon is given by $dL/dt = 12.6e^{-0.15t}$, where L is in centimetres and t is in years.

(a) Given that $L(0) = 0$ (i.e., at the moment of fertilization it is assumed that the length is zero), find a formula for the length $L(t)$ of pacific salmon.

$$L(t) = \int 12{,}6\, e^{-0{,}15t} = \frac{12{,}6}{-0{,}15} e^{-0{,}15t} + C$$

$$L(t) = -84 e^{-0{,}15t} + C$$

$$L(0) = 0 \rightarrow 0 = -84 \cdot e^{0} + C \Rightarrow C = 84$$

$$L(t) = -84\, e^{-0{,}15t} + 84$$

(b) Use (a) to find how much the salmon grows between the ages of 2 and 3.

length at 3 yrs old − length at 2 yrs old

$= L(3) - L(2)$

$= (-84 \cdot e^{-0{,}15(3)} + 84) - (-84 \cdot e^{-0{,}15(2)} + 84)$

$\approx 8{,}67$ cm

(c) Use the definite integral to compute how much the salmon grows between the ages of 2 and 5.

recall: total change from a to b $= \int_a^b$ rate of change

so $\int_2^5 \frac{dL}{dt} \cdot dt = \int_2^5 12{,}6\, e^{-0{,}15t}\, dt$

$= -84 e^{-0{,}15t} \Big|_2^5 \approx 22{,}55$ cm

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→ from this we conclude that $P(0)=0$

3. Most human papillomavirus (HPV) infections in young women are temporary and have very little long-term effects. Assume that $P(t)$ is the percent of women initially infected with the HPV who no longer have the virus at time t (t is measured in years). The *rate of change* of $P(t)$ is modelled by the function

$$p(t) = 0.5 - 0.21t^{1.5} + 0.5e^{-1.2t}$$

for $0 \le t \le 2$

↑ rate of change

(a) For what percent of young women will the virus be gone within one year?

$$P(1) - \underbrace{P(0)}_{=0} = \int_0^1 p(t)\,dt = \left(0.5t - 0.21\frac{t^{2.5}}{2.5} + 0.5\frac{1}{-1.2}e^{-1.2t}\right)\Big|_0^1$$

$$= \left(0.5t - 0.084t^{2.5} - 0.417e^{-1.2t}\right)\Big|_0^1$$

$$= \left(0.5 - 0.084 - 0.417e^{-1.2}\right) - (-0.417)$$

so $P(1) \approx 0.707$ i.e, about 70.7%

(b) For what percent of young women will the virus be gone within two years?

$$\int_0^2 p(t)\,dt = P(2) - P(0)$$

→ $P(0)$ zero because of the context of the question

$$P(2) = \left(0.5t - 0.084t^{2.5} - 0.417e^{-1.2t}\right)\Big|_0^2$$

≈ 0.904 i.e, about 90.4%

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4. Assume that $P(t)$ is the precent of women initially infected with the HPV who no longer have the virus at time t (t is measured in years). Another model for the *rate of change* of $P(t)$ is given by

this means that $P(0)=0$

$$p(t) = 1.7e^{-1.5t} - \frac{0.0016}{(t+0.01)^2}$$

(a) For what percent of young women will the virus be gone within one year?

$$P(1) - \underbrace{P(0)}_{0} = \int_0^1 p(t)\,dt = 1.7 \cdot \frac{1}{-1.5} e^{-1.5t} - 0.0016 \cdot \frac{(t+0.01)^{-1}}{-1} \Big|_0^1$$

$$\Rightarrow P(1) = \left(-1.133\, e^{-1.5t} + \frac{0.0016}{t+0.01}\right)\Big|_0^1$$

$$= \left(-1.133\, e^{-1.5} + \frac{0.0016}{1.01}\right) - \left(-1.133 + \frac{0.0016}{0.01}\right)$$

≈ 0.722 ie, about $\underline{72.2\,\%}$

(b) What percent of women will still have the virus after 10 years?

$$P(10) - \cancel{P(0)}^{\,0} = \int_0^{10} p(t)\,dt = \left(-1.133\, e^{-1.5t} + \frac{0.0016}{t+0.01}\right)\Big|_0^{10}$$

$$P(10) = \left(-1.133\, e^{-15} + \frac{0.0016}{10.01}\right) - \left(-1.133 + \frac{0.0016}{0.01}\right)$$

$\approx 97.3\,\%$

So about 2.7% of women will still have the virus after 10 years

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5. The number of new cases infected by a strain H2T1 of influenza A virus is given by $dv/dt = 245.1(t-1)^2$, where t is time in months; the time $t = 1$ represents January 2013. It is known that in January 2013 there were 200 cases of flu.

(a) Find a formula for $v(t)$.

$$v(t) = \int \frac{dv}{dt}\,dt = \int 245.1\,(t-1)^2\,dt$$

$$\rightarrow \quad v(t) = 245.1\,\frac{(t-1)^3}{3} + C = 81.7(t-1)^3 + C$$

$$v(1) = 200 \rightarrow \quad 200 = 81.7\cdot(1-1)^3 + C$$

$$C = 200$$

$$v(t) = 81.7\,(t-1)^3 + 200$$

(b) According to this model, how many people will be infected by the end of the year?

$$\text{number} = v(12) - v(1)$$

$$= 108{,}942.7 - 200 = 108{,}742.7$$

$$\text{or} \quad \text{number} = \int_1^{12} \frac{dv}{dt}\,dt = \dots 108{,}742.7$$

total (including initial 200) $= v(12) = 108{,}942.7$
(ignore decimals)

(c) Explain why this model cannot be used for long term predictions (i.e., 50 or 100 years from now).

assumes permanent increase in the rate
(and thus in the number of infected people)

for example $v(5 \text{ years}) = v(60) = 1.68\cdot 10^7 \approx 16.8$ million

still OK → $v(10 \text{ years}) = v(120) = 1.37\cdot 10^8 \approx 137$ million

BUT... → $v(50 \text{ years}) \approx \underline{17.6}$ billion

THE END