LS

ASSIGNMENT 8

1 (a) $a = \text{avg. rate of change} = \dfrac{f(t_0+\Delta t) - f(t_0)}{\Delta t}$

$\boxed{f(t) = t^2 - 1}$

$\Delta t = 0.5 \ldots \quad a = \dfrac{f(1.5+0.5) - f(1.5)}{0.5}$

$= \dfrac{f(2) - f(1.5)}{0.5} = \dfrac{3 - 1.25}{0.5} = 3{,}50$

$\Delta t = 0.1 \ldots \quad a = \dfrac{f(1.6) - f(1.5)}{0.1} = 3.1$

$\Delta t = 0.01 \ldots \quad a = \dfrac{f(1.51) - f(1.5)}{0.01} = 3.01$

secant needs to connect

$(t_0, f(t_0)) = (1.5, 1.25)$

and $(t_0 + 0.1, f(t_0 + 0.1)) = (1.6, 1.56)$

slope computed above $= 3.1$

point-slope equation $\quad y - 1.25 = 3.1(t - 1.5)$

$y = 3.1t - 3.1 \cdot 1.5 + 1.25$

$y = 3.1t - 3.4$

~~(b)~~ $\cancel{\Delta t = 0.5 \ldots \quad a = \dfrac{f(2) - f(1.5)}{0.5} \approx 69.03}$

$\cancel{\Delta t = 0.1 \ldots \quad a = \dfrac{f(1.6) - f(1.5)}{0.1} \approx 44.47}$

$\cancel{\Delta t = 0.01 \ldots \quad a \approx 40.58}$

$$\boxed{f(t) = e^{3t}}$$

(b) $\Delta t = 0.5 \ldots \quad a = \dfrac{f(0.5) - f(0)}{0.5} \approx 6.96$

$\Delta t = 0.1 \ldots \quad a = \dfrac{f(0.1) - f(0)}{0.1} \approx 3.50$

$\Delta t = 0.01 \ldots \quad a \approx 3.05$

secant line : slope $= 3.50$ point $(t_0, f(t_0)) = (0,1)$

$$y - 1 = 3.5(t - 0)$$

$$y = 3.5t + 1$$

2. (a) $a = \dfrac{P(5) - P(4)}{5-4} = \dfrac{1.37^5 - 1.37^4}{1} \approx 1.30342$

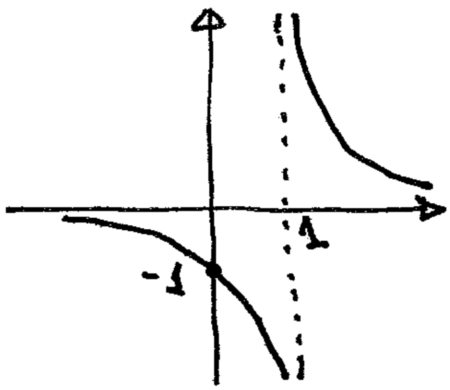
(b) $a = \dfrac{P(4.5) - P(4)}{4.5 - 4} \approx 1.20105$

(c) $a = \dfrac{P(4.1) - P(4)}{4.1 - 4} \approx 1.12664$

3. (a) $a = \dfrac{P(3) - P(2)}{3-2} \approx 21.95258$

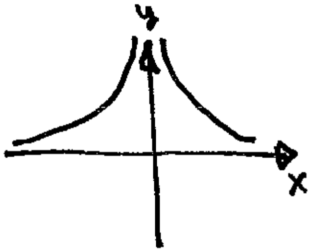
(b) $a = \dfrac{P(2.1) - P(2)}{0.1} \approx 15.70148$

4. (a)



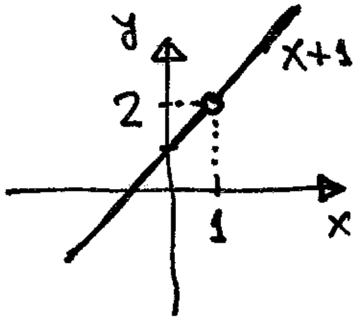
$$\lim_{x \to 0} \frac{1}{x-1} = -1$$

(b)



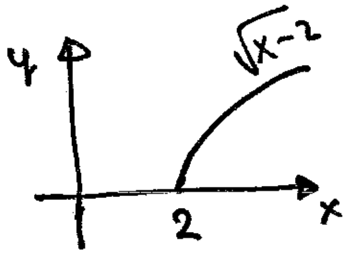
$$\lim_{x \to 0} \frac{1}{x^2} = +\infty$$

(c) $\frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} = (x+1)$, if $x \neq 1$



$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$

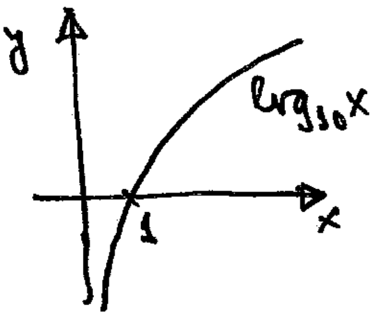
(d)



$$\lim_{x \to 2^+} \sqrt{x-2} = 0$$

(note: $\lim_{x \to 2^-} \sqrt{x-2}$ dne!)

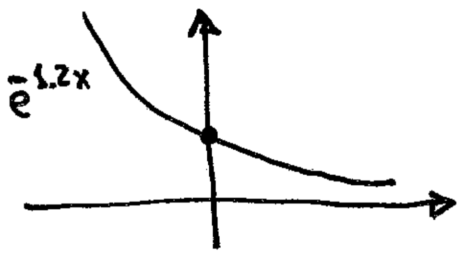
5. (a)



$$\lim_{x \to 0^+} \log_{10} x = -\infty$$

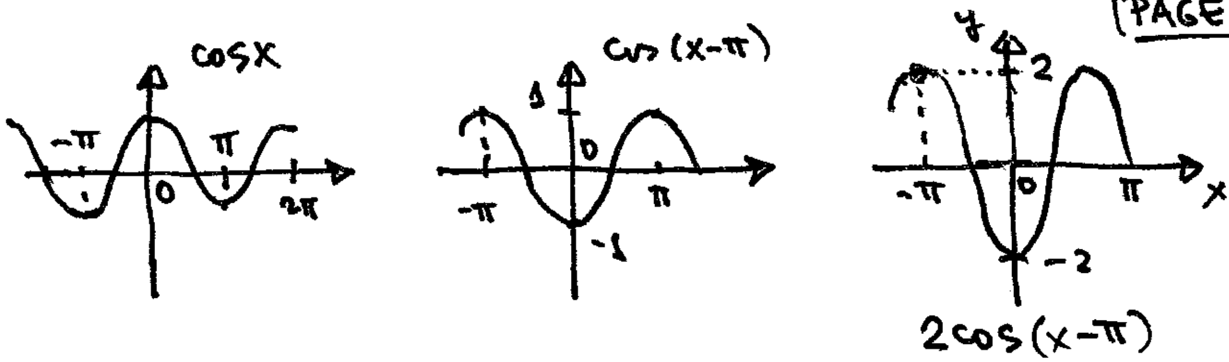
(note: $\lim_{x \to 0^-} \log_{10} x$ dne)

(b)



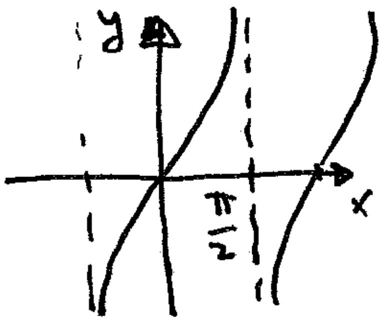
$$\lim_{x \to 0} e^{-1.2x} = 1$$

(c)



$$\lim_{x \to -\pi} 2\cos(x-\pi) = 2$$

(d)



$\lim_{x \to \pi/2} \tan x$ dne because

$$\lim_{x \to \pi/2^+} \tan x = -\infty$$

$$\lim_{x \to \pi/2^-} \tan x = +\infty$$

6. (a) $= \lim_{x \to 0} \frac{(x-2)(x+2)}{x+2} = \lim_{x \to 0} (x-2) = -2$

note: no need to factor!

$$\lim_{x \to 0} \frac{x^2-4}{x+2} = \frac{0^2-4}{0+2} = \frac{-4}{2} = -2$$

(b) $\lim_{x \to -2} \frac{x^2-4}{x+2} = \frac{(-2)^2-4}{(-2)+2} = \frac{0}{0} \rightarrow$ this time we <u>must</u> simplify

$$\lim_{x \to -2} \frac{x^2-4}{x+2} = \lim_{x \to -2} (x-2) = -4$$

(c) $\lim\limits_{t \to 3} \dfrac{t^3-27}{t+3} = \dfrac{3^3-27}{3+3} = \dfrac{0}{6} = 0$

(d) $\lim\limits_{t \to 3} \dfrac{t^3-27}{t-3} = \dfrac{3^3-27}{3-3} = \dfrac{0}{0}$ → need to simplify

$= \lim\limits_{t \to 3} \dfrac{\cancel{(t-3)}(t^2+3t+9)}{\cancel{t-3}} = \lim\limits_{t \to 3} (t^2+3t+9) = 27$

(e) $\lim\limits_{t \to 2} \dfrac{\frac{1}{2}-\frac{1}{t}}{t-2} = \dfrac{0}{0}$, so simplify $= \lim\limits_{t \to 2} \dfrac{\frac{t-2}{2t}}{t-2}$

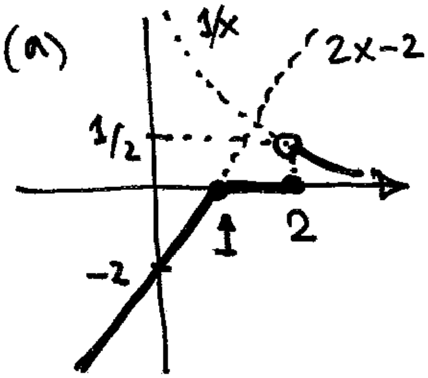
$= \lim\limits_{t \to 2} \dfrac{\cancel{t-2}}{2t} \cdot \dfrac{1}{\cancel{t-2}} = \dfrac{1}{4}$

7. (a) $x-4 \neq 0$ and $x-4 \geq 0$ → $x-4 > 0$

i.e. $x > 4$

(b) $\lim\limits_{x \to 4^+} \sqrt{\dfrac{4.7}{x-4}} = \sqrt{\dfrac{4.7}{+0}} = \sqrt{\infty} = \infty$

8. (a)



(b) $\lim\limits_{x \to 1} f(x) = 0$ because both left and right limits are equal to 0

$\lim_{x\to 2^-} f(x) = 0 \qquad \lim_{x\to 2^+} f(x) = \frac{1}{2} \Rightarrow \lim_{x\to 2} f(x)$ dne

9. (a) $\lim_{x\to 2^+} \frac{1}{x-2} = \frac{1}{0^{\oplus}} = +\infty$

(b) $\lim_{x\to 2^-} \frac{1}{x-2} = \frac{1}{0^{\ominus}} = -\infty$

(c) $\lim_{y\to 2} \frac{\ln y}{y^2-3} = \frac{\ln 2}{2^2-3} = \ln 2$

(d) $\lim_{y\to 0} \sin e^{-y^2} = \sin e^{-0^2} = \sin 1 \approx 0.84147$

(e) $\lim_{t\to -1} \frac{\tan(\pi t)}{t-1} = \frac{\tan(-\pi)}{-1-1} = \frac{0}{-2} = 0$