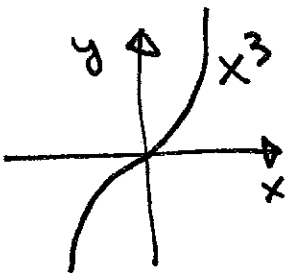
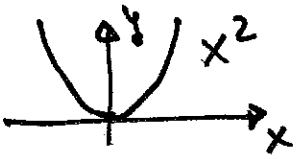
ASSIGNMENT 9

1. (a) $= \lim_{x \to \infty} x^3 = \infty$



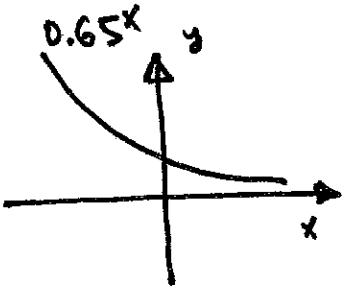
(b) $= \lim_{x \to -\infty} x^3 = -\infty$

(c) $= \lim_{x \to \infty} x^2 = \infty$



(d) $= \lim_{x \to -\infty} x^2 = \infty$

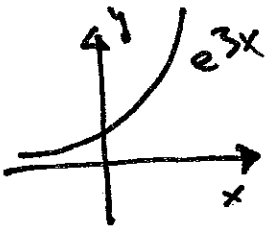
(e) frum the graph:



(f) $\lim_{x \to -\infty} = \infty$

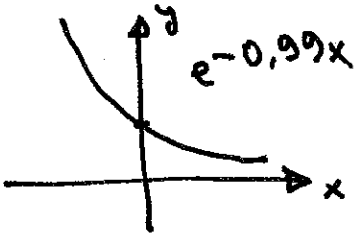
$\lim_{x \to \infty} = 0$

(g)



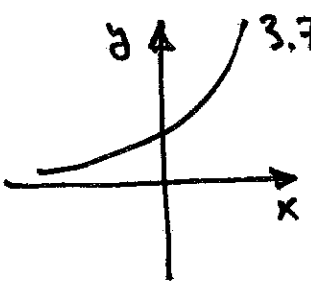
$\lim_{x \to \infty} = \infty$

(h)



$\lim_{x \to -\infty} = \infty$

2. (a)



$\lim_{x \to \infty} 3.76^x = \infty$

(b) $\lim_{x\to\infty} 3.76^{-x} = \lim_{x\to\infty} \frac{1}{3.76^x} = \frac{1}{\infty} = 0$

(c) $\lim_{x\to\infty} e^{-x^2} = e^{-(\infty)^2} = e^{-\infty} = 0$

(d) $\lim_{x\to-\infty} e^{-x^2} = e^{-(-\infty)^2} = e^{-\infty} = 0$

(e) $\lim_{x\to\infty} \ln(x-100) = \ln(\infty) = \infty$

(f) $\lim_{x\to\infty} \ln\left(\frac{x}{10}\right) = \ln(\infty) = \infty$

(g) $\lim_{x\to\infty} \sqrt{x^2+4500} = \sqrt{\infty} = \infty$

(h) $\lim_{x\to-\infty} \sqrt{1-x^3} = \sqrt{1-(-\infty)^3} = \sqrt{\infty} = \infty$

3.(a) $\lim_{x\to\infty} 1^x = \lim_{x\to\infty} 1 = 1$

(b) $\lim_{x\to\infty} \ln(1-x^5) = \ln(1-\infty) = \ln(-\infty) = \text{dne}$

(c) $\lim_{x\to-\infty} \ln(1-x^5) = \ln(1-(-\infty)) = \ln(\infty) = \infty$

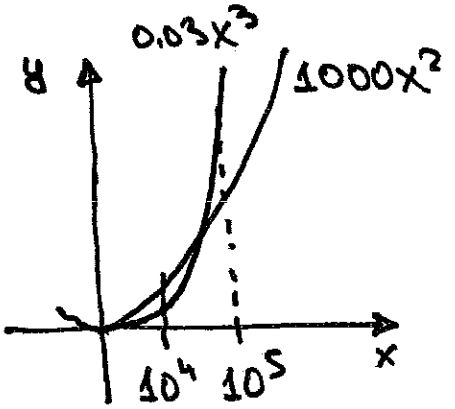
(d) $\lim_{x\to\infty} \sqrt{\frac{x}{10}} = \sqrt{\infty} = \infty$

(e) $\lim_{x\to\infty} \sqrt{10000-x} = \sqrt{-\infty} = \text{dne}$

(f) $\lim_{x\to\infty} e^{x^2-x-4} = \lim_{x\to\infty} e^{x^2} = e^{\infty} = \infty$

(g) $\lim_{x\to\infty} e^{-x^2-x+44} = \lim_{x\to\infty} e^{-x^2} = e^{-\infty} = 0$

4. (a)

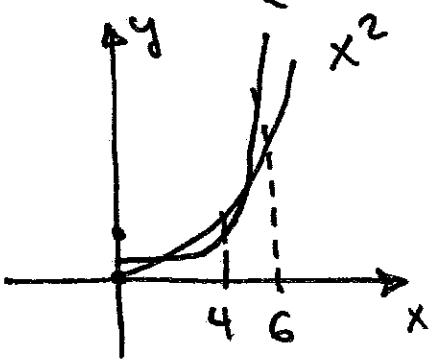


x	$1000x^2$	$0.03x^3$
0	0	0
100	10^7	$0.03\cdot 10^6$
10^3	10^9	$0.03\cdot 10^9$
10^4	10^{11}	$0.03\cdot 10^{12}$
10^5	10^{13}	$0.03\cdot 10^{15} = 3\cdot 10^{13}$

$0.03x^3 > 1000x^2$ if $x = 10^5$

answer: $\frac{0.03x^3}{e^x}$

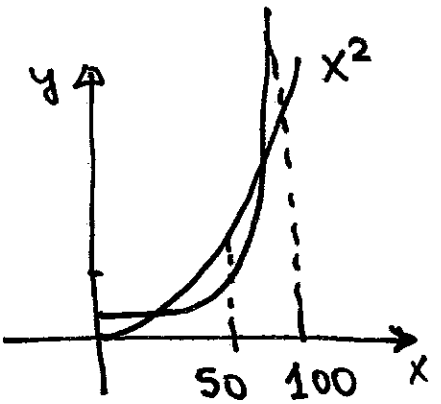
(b)



x	x^2	$0.1e^x$
0	0	0.1
1	1	0.27
2	4	0.73
3	9	2.01
4	16	5.46
6	36	40.34

answer: $0.1e^x$

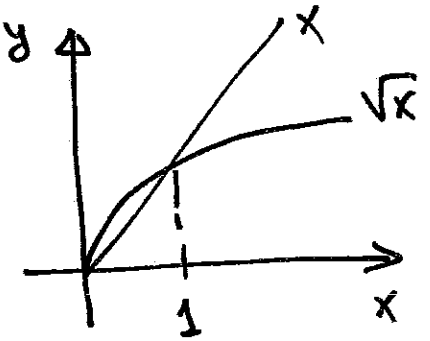
(c)



x	x^2	$e^{0.1x}$
0	0	1
1	1	1.1
2	4	1.22
10	100	e = 2.718
50	2500	148.41
100	10000	22,026.47

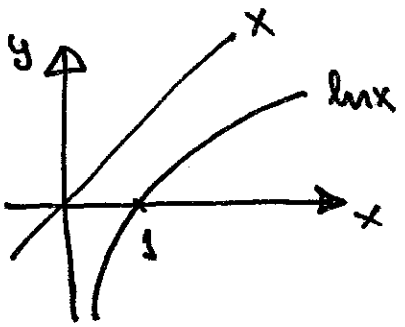
answer : $e^{0.1x}$

(d)



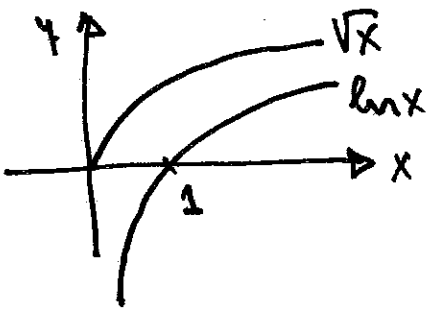
answer : x

(e)



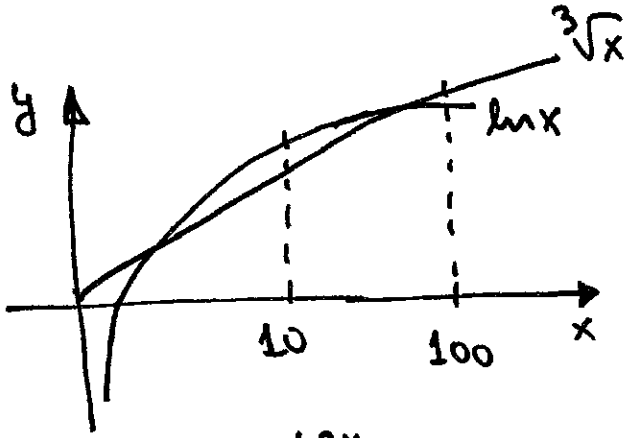
x

(f)



$\sqrt{x}$

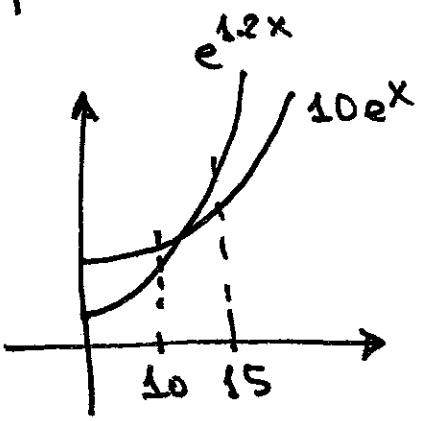
(g)



x	$\sqrt[3]{x}$	ln x
1	1	0
10	2.15	2.30
100	4.64	4.60
200	6.69	5.70

$\sqrt[3]{x}$

(h)



x	$10e^x$	$e^{1.2x}$
0	10	1
1	27.18	3.32
10	$2.2 \cdot 10^5$	$1.62 \cdot 10^5$
15	$3.2 \cdot 10^7$	$6.6 \cdot 10^7$

$e^{1.2x}$

5. (a) $= \lim_{x \to \infty} \frac{-3x^3}{2x^3} = -\frac{3}{2}$

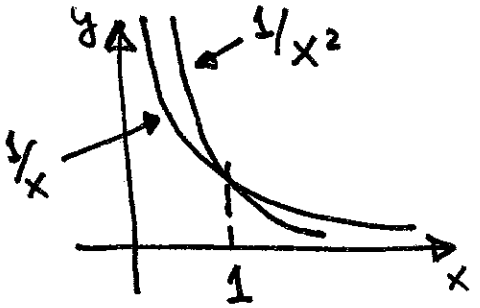
(b) $= \lim_{x \to \infty} \frac{1.4x^2}{24x^3} = \frac{1.4}{24} \cdot \lim_{x \to \infty} \frac{1}{x} = 0$

(c) $= \lim_{x \to \infty} \frac{0.001x^4}{20000x^3} = \lim_{x \to \infty} x \cdot \left(\frac{0.001}{20000}\right) = \infty$

(d) $= \lim_{x \to \infty} \left(\ln \frac{2x^4+1}{x^2-13}\right) = \lim_{x \to \infty} \left(\ln \frac{2x^4}{x^2}\right)$

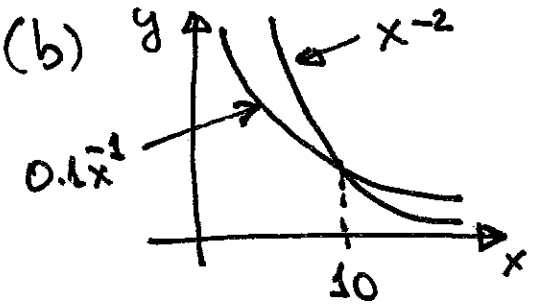
$= \ln \infty = \infty$

6.(a)



answer: x^{-2}

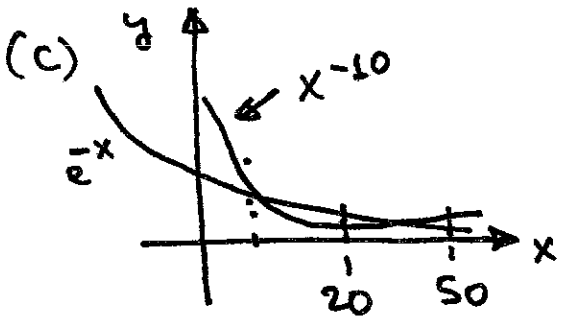
(b)



x	x^{-2}	$0.1\,x^{-1}$
1	1	0.1
2	0.25	0.05
5	0.04	0.02
10	0.01	0.01
20	0.0025	0.005

x^{-2}

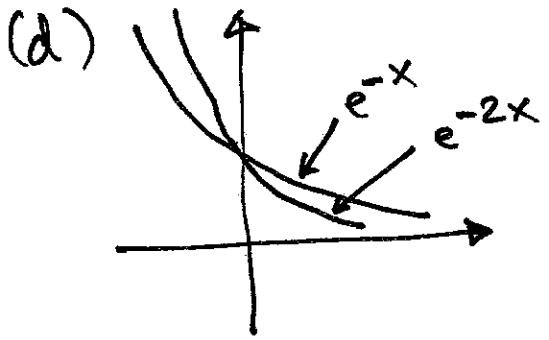
(c)



x	x^{-10}	e^{-x}
1	1	0.37
5	$1.0\cdot10^{-7}$	0.007
20	$9\cdot10^{-14}$	$2\cdot10^{-9}$
50	$1\cdot10^{-17}$	$1.9\cdot10^{-22}$

e^{-x}

(d)



e^{-2x}

good to know: if $x>0$ $\quad e^{-x} < 1 \quad | \cdot e^{-x}$

$e^{-x}\cdot e^{-x} < e^{-x}$

$\Rightarrow e^{-2x} < e^{-x}$!

7.

(a) $$\lim_{x\to\infty} \frac{e^x - e^{-x}}{2e^{-x} + e^x} = \lim_{x\to\infty} \frac{\frac{e^x}{e^x} - \frac{e^{-x}}{e^x}}{\frac{2e^{-x}}{e^x} + \frac{e^x}{e^x}}$$

$$= \lim_{x\to\infty} \frac{1 - e^{-2x}}{2e^{-2x} + 1} = \frac{1 - e^{-\infty}}{2e^{-\infty} + 1} = \frac{1 - 0}{0 + 1} = 1$$

(b) $$\lim_{x\to-\infty} \frac{e^x - e^{-x}}{2e^{-x} + e^x} = \{ \text{divide by } e^{-x} \}$$

$$= \lim_{x\to-\infty} \frac{e^{2x} - 1}{2 + e^{2x}} = \frac{e^{-\infty} - 1}{2 + e^{-\infty}} = -\frac{1}{2}$$

[note: $\frac{e^x}{e^{-x}} = e^{x-(-x)} = e^{2x}$]