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THE BROAD CONSEQUENCES OF NARROW BANKING

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We investigate the macroeconomic consequences of narrow banking in the context of stock-flow consistent models. We begin with an extension of the Goodwin–Keen model incorporating time deposits, government bills, cash, and central bank reserves to the base model with loans and demand deposits, and use it to describe a fractional reserve banking system. We then characterize narrow banking by a full reserve requirement on demand deposits and describe the resulting separation between the payment system and lending functions of the resulting banking sector. By way of numerical examples, we explore the properties of fractional and full reserve versions of the model and compare their asymptotic properties. We find that narrow banking does not lead to any loss in economic growth when both versions of the model converge to a finite equilibrium, while allowing for more direct monitoring and prevention of financial breakdowns in the case of explosive asymptotic behavior.

Keywords: Narrow banking; full-reserve banking; macroeconomic dynamics; stock-flow consistent models; debt-financed investment; financial stability.

1. Introduction

Narrow banking is a recurrent theme in economics, especially after periods of financial turbulence. For example, in the wake of the Great Depression, several prominent economists proposed a set of reforms known as the *Chicago plan*, which included the requirement that banks hold reserves matching the amount of demand deposits [see Phillips (1996)]. The Banking Act of 1935 adopted different measures to promote stability of the banking sector in the United States, such as deposit insurance and the separation of commercial and investment banking, but the idea of narrow banking never went away — and neither did financial crises. After the global financial crisis of 2008, the idea came to the fore again, with outlets as diverse as the International Monetary Fund and the Positive Money movement re-examining the benefits of narrow banking for the current financial system [see Kumhof & Benes (2012) and Dyson *et al.* (2016)], and in at least one case a government explicitly dedicating resources to debate the idea [see Thoroddsen & Sigurjónsson (2016)].

More recently, narrow banking attracted a lot of attention in the context of the cryptocurrency boom, especially in conjunction with the potential introduction of central bank issued digital currencies (Barrdear & Kumhof 2016, Lipton 2016a, Lipton *et al.* 2018). It even made headlines outside academic circles when the Federal Reserve Bank of New York denied an application to open an account by a bank called TNB USA Inc.[a]

The main feature of a narrow bank is its asset mix, which by definition can include only marketable, low-risk, liquid (government) securities, cash, or central bank reserves in the amount exceeding the amount of demand deposits made by its clients. As a result, such a bank is immune to market, credit and liquidity risks, and can only be toppled by operational failures that can be minimized, though not eliminated, by using state-of-the-art technology. Consequently, contrary to a fractional banking system, demand deposits made in a narrow bank can *always* be immediately converted into cash, being therefore equivalent to currency. This provides a maximally safe payment system, which does not require deposit insurance with all its complex and poorly understood effects on the system as a whole, including not so subtle moral hazards.

Because loans and other risky securities are excluded from the asset mix of a narrow bank, lending has to be performed by specially constructed lending facilities, which would need to raise funds from the private sector and the government *before* lending them out, in contrast to fractional reserve banks who *simultaneously* create funds and lend them. In other words, narrow banking separates two functions that are traditionally performed together in conventional banking: loan provision and the payment system.

In practice, a narrow bank and a lending facility as defined above can be combined into a single business, much like the same company can sell both computers and mobile phones. All that is necessary is that any bank accepting demand deposits as liabilities should be required to have an equal or greater amount of central bank reserves as assets. In what follows, we take this full reserve requirement as the operational definition of narrow banking and compare it with the current practice of fractional reserve banking.

We perform the analysis in a stock-flow consistent framework similar to that of Laina (2015) but using the model for debt-financed investment proposed in Keen

[a]See Levine (2018) for details about TNB USA Inc, which stands for The Narrow Bank. In a similar vein, in 2016–17 one of the authors also tried to build a narrow bank in practice — (Burkov *et al.* 2017).

(1995) as a starting point. Our work differs from Laina (2015) in two respects: we use a continuous-time model and consider a growing economy, whereas Laina (2015) uses a discrete-time mode and considers only the zero-growth case.

Our two main findings are: (1) narrow banking does not impede growth and (2) whereas it does not entirely prevent financial crises either, it allows for more direct monitoring and preventive intervention by the government. The first finding is significant because a common objection to narrow banking is that removing the money-creation capacity from private banks would lead to a shortage of available funds to finance investment and promote growth. Our results show that this is not the case, with a combination of private and public funds being sufficient to finance investment and lead to economic growth at the same equilibrium rates in both the full and fractional reserve cases. In our view, this result alone is enough to justify a much wider discussion of narrow banking than has occurred so far, because the clear advantages mentioned earlier, such as the reduced need for deposit insurance, do not need to be necessarily weighed against losses in economic growth.

Our second finding is more subtle. In the context of the model analyzed in this paper, a financial crisis is associated with an equilibrium with exploding ratios of private debt and accompanying ever decreasing employment, wage share, and output. We find that such equilibria are present in both the full and fractional reserve cases and are moreover associated with exploding ratios of government *lending* to the private sector. In the narrow banking case, however, this last variable — namely the ratio of government lending to GDP — exhibits a clearly explosive behaviour much sooner than in the fractional reserve case. Because this is an indicator that is under direct control of the government (as opposed to capital or leverage ratios in the private sector, for example), it is much easier for regulators in the narrow banking case to detect the onset of a crisis and take measures to prevent it.

The rest of the paper is organized as follows. In Sec. 2, we extend the Keen (1995) model by introducing both demand and time deposits as liabilities of the banking sector, as well as reserves and government bills, in addition to loans, as assets of this sector. Accordingly, we introduce a central bank conducting monetary policy in order to achieve a policy rate on government bills and provide the banking sector with the required amount of reserves. As in the Keen (1995) model, the key decision variable of the private sector is the amount of investment by firms, whereas households adjust their consumption accordingly, with the only added feature of a portfolio selection for households along the lines of Tobin (1969). Finally, we assume a simplified fiscal policy in the form of government spending and taxation as constant proportions of output.

In Sec. 3, we modify the model by imposing 100% reserve requirements for banks. This has the effect of limiting bank lending, which now needs to be entirely financed by equity and other borrowing. In the present model, a capital adequacy ratio smaller than one can only be maintained by borrowing from the central bank, which can in effect control the total amount of bank lending.

In Sec. 4, we explore the models introduced in Secs. 2 and 3 through a series of numerical experiments and show how the economy can develop under both beneficial and adverse circumstances by way of examples illustrating the properties described earlier. In Sec. 5, we review our conclusions and outline future research directions.

2. Fractional Reserve Banking

We consider a five-sector closed economy consisting of firms, banks, households, a government sector and a central bank as summarized in Table 1. As it is typical in stock-flow consistent models, the balance sheet, transactions, and flow of funds depicted in this table already encapsulate a lot of the structure in the model, so we start by describing each item in some detail.

2.1. *Balance sheets*

Households distribute their wealth into cash, treasury bills, and demand deposits. The total amount of cash in circulation is denoted by H and is a liability for the central bank. Treasury bills are short-term liabilities of the government sector and pay an interest rate r_θ, which plays the role of the main policy rate in the model. For the purpose of this model they can be thought of as being instantaneously issued or redeemed by the government to finance its fiscal policy. Because of this feature, their unit value is deemed to be constant. The total amount of treasuries issued by the government is denoted by Θ and is divided into the holdings Θ_h of households, Θ_b of banks and Θ_{cb} of the central bank as specified shortly. Demand deposits are liabilities of the banking sector redeemable by cash and paying an interest rate r_m. Observe that we assume for simplicity that households do not borrow from banks. A more complete model can include consumer credit in addition to the credit for firms treated in this paper, but we defer this to further work.

The firm sector produces a homogeneous good used both for consumption and investment. It utilizes capital with monetary value denoted by pK where p is the unit price of the homogenous good. The capital stock of firms is partially financed by loans with total value L at an interest rate r.

The balance sheet of banks consist of demand deposits M and time deposits D as liabilities and firm loans L, treasury bills Θ_b and central bank reserves R as assets. The key feature of fractional reserve banking is that banks are required to maintain a reserve account with the central bank at the level

$$R = fM, \tag{2.1}$$

for a constant $0 \leq f < 1$. We assume that banks maintain this required level of reserves by selling and buying treasury bills to and from the central bank. Observe that there is no reserve requirement assumed for time deposits.

Table 1. Balance sheets, transactions and flow of funds matrices for a fractional banking model. Balance sheet entries have units of currency (e.g. USD), while entries in transactions and flow of funds rows have units of currency per time (e.g. USD/year).

	Households	Firms		Banks	Gov	CB	Row sum
Balance Sheet							
Capital stock		$+pK$					$+pK$
Loans		$-L$		$+L$			0
Cash	$+H$					$-H$	0
Treasury bills	$+\Theta_h$			$+\Theta_b$	$-\Theta$	$+\Theta_{cb}$	0
Demand deposits	$+M_h$	$+M_f$		$-M$			0
Time deposits	$+D$			$-D$			0
Reserves				$+R$		$-R$	0
Column sum (net worth)	X_h	X_f		X_b	X_g	0	pK
Transactions		current	capital				
Consumption	$-pC$	$+pC$					0
Gov Spending		$+pG$			$-pG$		0
Capital Investment		$+pI$	$-pI$				0
Accounting memo [GDP]		$[pY]$					
Wages	$+W$	$-W$					0
Taxes		$-pT$			$+pT$		0
Interest on loans		$-rL$		$+rL$			0
Depreciation		$-p\delta K$	$+p\delta K$				0
Interest on bills	$+r_\theta\Theta_h$			$+r_\theta\Theta_b$	$-r_\theta\Theta$	$+r_\theta\Theta_{cb}$	0
Interest on demand deposits	$+r_mM_h$	$+r_mM_f$		$-r_mM$			0
Interest on time deposits	$+r_dD$			$-r_dD$			0
Bank dividends	$+\Delta_b$			$-\Delta_b$			0
CB profits					$+\Pi_{cb}$	$-\Pi_{cb}$	0
Column sum (financial balances)	S_h	S_f	$-p(I-\delta K)$	S_b	S_g	0	0
Flow of Funds							
Change in capital stock		$+p(I-\delta K)$					$+p(I-\delta K)$
Change in loans		$-\dot{L}$		$+\dot{L}$			0
Change in cash	$+\dot{H}$					$-\dot{H}$	0
Change in bills	$+\dot{\Theta}_h$			$+\dot{\Theta}_b$	$-\dot{\Theta}$	$+\dot{\Theta}_{cb}$	0
Change in demand deposits	$+\dot{M}_h$	$+\dot{M}_f$		$-\dot{M}$			0
Change in time deposits	$+\dot{D}$			$-\dot{D}$			0
Change in reserves				$+\dot{R}$		$-\dot{R}$	0
Column sum	S_h	S_f		S_b	S_g	0	$p(I-\delta K)$
Change in net worth	S_h	$S_f+\dot{p}K$		S_b	S_g	0	$\dot{p}K+p\dot{K}$

Finally, the public sector is divided into a government that issues bills to finance its fiscal deficit (essentially the difference between spending and taxation) and a central bank that issues cash and reserves as liabilities and purchases bills as part of its monetary policy.

The column sums along the balance sheet matrix indicate the net worth of each sector, whereas the row sums are all equal to zero with the exception of the capital stock, as each financial asset for one sector correspond to a liability of another. Observe that we assume that the central bank has constant zero net worth, which in particular implies that it transfers all profits back to the government.

2.2. *Transactions and flow of funds*

Having defined the balance sheet items, the transactions in Table 1 are self-explanatory and lead to the financial balances, or savings, indicated as the column sum for each sector. We now describe how these financial balances are redistributed among the corresponding balance sheet items for each sector. Starting with the government sector, we have that

$$S_g = -pG + pT - r_\theta(\Theta_h + \Theta_b), \tag{2.2}$$

showing that government savings are the negative of deficit spending and interest paid on bills held by the private sector. Because this is entirely financed by net issuance of new bills we have

$$\dot{\Theta} = pG - pT + r_\theta(\Theta_h + \Theta_b). \tag{2.3}$$

Moving to firms, once depreciation is taken into account, we find from Table 1 that savings for firms, after paying wages, taxes, interest on debt, and depreciation (i.e. consumption of fixed capital), are given by

$$S_f = pY - W - pT - rL + r_m M_f - p\delta K \tag{2.4}$$

and correspond to the internal funds available for investment. In this model, the only source for external financing are loans from the banking sector, so that we have

$$\dot{L} - \dot{M}_f = p(I - MK) - S_f = pI - \Pi_p, \tag{2.5}$$

where

$$\Pi_p = pY - W - pT - rL + r_m M_f \tag{2.6}$$

denotes the after-tax, pre-depreciation profits of the firm sector. The exact distribution of the difference $(pI - \Pi_p)$ into net new loans and new deposits depends on portfolio decisions by firms, including a desired rate of repayment of existing debt. For simplicity, we adopt the specification in equations (53)–(54) of Grasselli & Nguyen Huu (2015) with the repayment rate set to zero; namely, we assume that

$$\dot{L} = pI + rL, \tag{2.7}$$

$$\dot{M}_f = pY - W + r_m M_f = \Pi_p + rL. \tag{2.8}$$

The flow of funds for households is slightly more involved, as it requires a choice among different assets. As we can see from Table 1, the savings of households are given by

$$\dot{X}_h = S_h = pY_h - pC, \tag{2.9}$$

where

$$pY_h = W + r_m M_h + r_d D + r_\theta \Theta_h + \Delta_b \tag{2.10}$$

is the nominal disposable income of households. These savings are then redistributed among the different balance sheet items held by households so that

$$S_h = \dot{H} + \dot{\theta}_h + \dot{M}_h + \dot{D}. \tag{2.11}$$

To obtain the proportions of savings invested in each type of assets we use the following modified version of the portfolio equations proposed in Chapter 10 of Godley & Lavoie (2007):

$$H = \lambda_0 X_h, \tag{2.12}$$

$$\Theta_h = (\lambda_{10} + \lambda_{11} r_\theta + \lambda_{12} r_m + \lambda_{13} r_d) X_h, \tag{2.13}$$

$$M_h = (\lambda_{20} + \lambda_{21} r_\theta + \lambda_{22} r_m + \lambda_{23} r_d) X_h, \tag{2.14}$$

$$D = (\lambda_{30} + \lambda_{31} r_\theta + \lambda_{32} r_m + \lambda_{33} r_d) X_h, \tag{2.15}$$

subject to the constraints

$$\lambda_0 + \lambda_{10} + \lambda_{20} + \lambda_{30} = 1, \tag{2.16}$$

$$\lambda_{11} + \lambda_{21} + \lambda_{31} = 0, \tag{2.17}$$

$$\lambda_{12} + \lambda_{22} + \lambda_{32} = 0, \tag{2.18}$$

$$\lambda_{13} + \lambda_{23} + \lambda_{33} = 0, \tag{2.19}$$

and the symmetry conditions $\lambda_{ij} = \lambda_{ji}$ for all i, j. These correspond to Tobin's prescription for macroeconomic portfolio selection, whereby the proportion of the total wealth invested in each class of assets depends on the rates of returns, with increased demand for one asset leading to decreased demand for all others.

For our purpose, the important consequence of (2.12)–(2.15) is that

$$\dot{H} = \lambda_0 \dot{X}_h = \lambda_0 (pY_h - pC), \tag{2.20}$$

$$\dot{\Theta}_h = \lambda_1 \dot{X}_h = \lambda_1 (pY_h - pC), \tag{2.21}$$

$$\dot{M}_h = \lambda_2 \dot{X}_h = \lambda_2 (pY_h - pC), \tag{2.22}$$

$$\dot{D} = \lambda_3 \dot{X}_h = \lambda_3 (pY_h - pC), \tag{2.23}$$

where

$$\lambda_0 = 1 - (\lambda_{10} + \lambda_{20} + \lambda_{30}), \tag{2.24}$$

$$\lambda_1 = \lambda_{10} + \lambda_{11} r_\theta + \lambda_{12} r_m - (\lambda_{11} + \lambda_{12}) r_d, \tag{2.25}$$

$$\lambda_2 = \lambda_{20} + \lambda_{12} r_\theta + \lambda_{22} r_m - (\lambda_{12} + \lambda_{22}) r_d, \tag{2.26}$$

$$\lambda_3 = \lambda_{30} - (\lambda_{11} + \lambda_{12}) r_\theta - (\lambda_{12} + \lambda_{22}) r_m + (\lambda_{11} + 2\lambda_{12} + \lambda_{22}) r_d. \tag{2.27}$$

The allocations in the remaining assets is now jointly determined by the interaction between the banking sector and the central bank. To being with, the reserve requirement (2.1) imposes that

$$\dot{R} = f\dot{M} = f[\Pi_p + rL + \lambda_2(pY_h - pC)], \tag{2.28}$$

where we used (2.8) and (2.22). Next, the fact that the central bank transfers all profits to the government sector implies that

$$\dot{\Theta}_{cb} = \dot{H} + \dot{R} = (\lambda_0 + f\lambda_2)(pY_h - pC) + f(\Pi_p + rL), \tag{2.29}$$

where we used (2.8), (2.20) and (2.22).

Finally, denoting bank profits by

$$\Pi_b = rL - r_m M - r_d D + r_\theta \Theta_b = S_b + \Delta_b, \tag{2.30}$$

we see that the holding of treasury bills by banks satisfies

$$\begin{aligned}\dot{\Theta}_b &= S_b + \dot{M} + \dot{D} - \dot{R} - \dot{L} = \Pi_b - \Delta_b + (1-f)\dot{M} + \dot{D} - \dot{L} \\ &= \Pi_b - \Delta_b + (1-f)[\Pi_p + rL + \lambda_2(pY_h - pC)] + \lambda_3(pY_h - pC) - pI - rL,\end{aligned} \tag{2.31}$$

where we used (2.7), (2.22) and (2.28).

At this point it is instructive to observe that it follows from (2.21), (2.29) and (2.31) that

$$\begin{aligned}\dot{\Theta} &= \dot{\Theta}_h + \dot{\Theta}_{cb} + \dot{\Theta}_b = \lambda_1(pY_h - pC) + (\lambda_0 + f\lambda_2)(pY_h - pC) + f(\Pi_p + rL) \\ &\quad + \Pi_b - \Delta_b + (1-f)[\Pi_p + rL + \lambda_2(pY_h - pC)] + \lambda_3(pY_h - pC) - pI - rL \\ &= (\lambda_0 + \lambda_1 + \lambda_2 + \lambda_3)(pY_h - pC) + \Pi_p + rL - r_m M - r_d D + r_\theta \Theta_b - \Delta_b - pI \\ &= (W + r_m M_h + r_d D + r_\theta \Theta_h + \Delta_b - pC) \\ &\quad + (pY - W - pT - rL + r_m M_f) + rL - r_m M - r_d D + r_\theta \Theta_b - \Delta_b - pI \\ &= pG - pT + r_g(\Theta_h + \Theta_b),\end{aligned} \tag{2.32}$$

in accordance with (2.3).

2.3. *Additional behavioral assumptions*

To complete the model, we need to specify several additional behavioral assumptions for each sector. For this, let us first introduce the following intensive variables

$$\omega = \frac{W}{pY}, \quad h = \frac{H}{pY}, \quad \theta_h = \frac{\Theta_h}{pY}, \quad d = \frac{D}{pY}, \tag{2.33}$$

$$m_h = \frac{M_h}{pY}, \quad m_f = \frac{M_f}{pY}, \quad \ell = \frac{L}{pY}, \quad \theta_b = \frac{\Theta_b}{pY}. \tag{2.34}$$

In addition, let the total working age population be denoted by N and the number of employed workers by E. We then define the productivity per worker a, the employment rate e and the nominal wage rate as

$$a = \frac{Y}{E}, \quad e = \frac{E}{N} = \frac{Y}{aN}, \quad \mathrm{w} = \frac{W}{E}, \tag{2.35}$$

whereas the unit cost of production, defined as the wage bill divided by quantity produced, is given by

$$u_c = \frac{W}{Y} = \frac{\mathrm{w}}{a}. \tag{2.36}$$

We assume throughout that productivity and workforce grow exogenously according to the dynamics

$$\frac{\dot{a}}{a} = \alpha, \quad \frac{\dot{N}}{N} = \beta. \tag{2.37}$$

Wage-price dynamics: For the price dynamics we assume that the long-run equilibrium price is given by a constant markup $m \geq 1$ times unit labor cost, whereas observed prices converge to this through a lagged adjustment with speed $\eta_p > 0$. Using the fact that the instantaneous unit labor cost is given by $u_c = \omega p$, we obtain

$$\frac{\dot{p}}{p} = \eta_p \left(m\frac{u_c}{p} - 1 \right) = \eta_p(m\omega - 1) := i(\omega). \tag{2.38}$$

We assume that the wage rate w follows the dynamics

$$\frac{\dot{w}}{w} = \Phi(e) + \gamma\frac{\dot{p}}{p}, \tag{2.39}$$

for a constant $0 \leq \gamma \leq 1$. This assumption states that workers bargain for wages based on the current state of the labor market through the Philips curve Φ, but also take into account the observed inflation rates. The constant γ represents the degree of money illusion, with $\gamma = 1$ corresponding to the case where workers fully incorporate inflation in their bargaining. For the Philips curve, we assume that $\Phi(e) \to +\infty$ as $e \to 1$ in order to prevent the employment rate from going above one.

Fiscal policy: We consider the simplest case of real government spending and taxation given by

$$G = gY, \tag{2.40}$$

$$T = tY, \tag{2.41}$$

for constants g and t.

Investment, production and consumption: As in the Keen (1995) model, we assume that the relationship between capital and output is given by $Y = K/\nu$ for a constant capital-to-output ratio ν. There are many ways to relax this condition, for example by introducing a variable utilization rate as in Grasselli & Nguyen-Huu (2018), but we shall not pursue them here. Capital itself is assumed to change according to

$$\dot{K} = I - \delta K, \tag{2.42}$$

where δ is a depreciation rate. Moreover, we assume that real investment is given by

$$I = \kappa(\pi)Y, \tag{2.43}$$

for a function κ of the profit share

$$\pi = \frac{\Pi_p}{pY} = 1 - t - \omega - r\ell + r_m m_f. \tag{2.44}$$

Using (2.43), (2.42) and $Y = K/\nu$, we find that the growth rate of real output is given by

$$g_Y(\pi) := \frac{\dot{Y}}{Y} = \frac{\dot{Y}}{Y} = \frac{\kappa(\pi)}{\nu} - \delta. \tag{2.45}$$

Furthermore, still in line with the original Keen model, we assume that all output is sold, so that there are no inventories or any difference between supply and demand. Accordingly, real consumption of households is given by

$$C = Y - G - I = (1 - g - \kappa(\pi))Y. \tag{2.46}$$

Bank dividends: There are many alternative definitions of bank behavior that are compatible with the accounting structure described in Table 1. For example, in Godley & Lavoie (2007), with the exception of Chapter 11, it is assume throughout the book that all bank profits are immediately distributed to households, so that the financial balances of banks is always identically zero and, consequently, the net worth of banks is kept constant. The problem with this approach is that, in a growing economy, it leads to vanishing capital ratios, as loans and deposits continue to grow while the equity of the bank remains constant. This was addressed in Chapter 11 of Godley & Lavoie (2007), where banks are assumed to target a desired capital ratio and distribute profits accordingly. We shall adopt the analogous mechanism in continuous-time proposed in Grasselli & Lipton (2019). For the present model, the assumption of targeting a capital ratio k_r translates into

$$X_b = k_r(\rho_L L + \rho_g \Theta_b + \rho_r R), \tag{2.47}$$

that is, we assume that banks distribute enough dividends to keep equity equal to a multiple k_r of risk-weighted assets. For simplicity, we take $\rho_L = 1$ and $\rho_g = \rho_r = 0$, but the same general argument applies to arbitrary risk weights. Because bank savings need to equal the change in bank equity (i.e. net worth), we have that

$$S_b = \dot{X}_b = k_r \dot{L} = k_r(pI + rL), \tag{2.48}$$

which in turn implies that bank dividends are

$$\Delta_b = \Pi_b - k_r(pI + rL). \tag{2.49}$$

Looking back at the expressions involving bank dividends, we see from (2.10) that nominal disposable income for households is equal to

$$pY_h = W + (1 - k_r)rL - r_m M_f + r_\theta(\Theta_h + \Theta_b) - k_r pI \tag{2.50}$$

and from (2.31) that the holding of bills by banks satisfies

$$\dot{\Theta}_b = (1 - f)[\Pi_p + \lambda_2(pY_h - pC)] + \lambda_3(pY_h - pC) - (1 - k_r)pI + (k_r - f)rL. \tag{2.51}$$

2.4. *The main dynamical system*

The dynamics for the wage share $\omega = w/(pa)$ obtained from (2.37), (2.38) and (2.39) is

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} - \frac{\dot{p}}{p} - \frac{\dot{a}}{a} = \Phi(e) - \alpha - (1 - \gamma)i(\omega), \tag{2.52}$$

For the employment rate $e = Y/(aN)$, we use (2.37), (2.45) to obtain

$$\frac{\dot{e}}{e} = \frac{\dot{Y}}{Y} - \frac{\dot{a}}{a} - \frac{\dot{N}}{N} = \frac{\kappa(\pi)}{\nu} - \delta - \alpha - \beta. \tag{2.53}$$

For the household variables $h = H/(pY)$, $\theta_h = \Theta_h/(pY)$, $m_h = M_h/(pY)$ and $d = D/(pY)$, we use (2.20)–(2.23) to obtain

$$\frac{\dot{h}}{h} = \frac{\dot{H}}{H} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \frac{\lambda_0(pY_h - pC)}{H} - \Gamma(\omega, \ell, m_f), \tag{2.54}$$

$$\frac{\dot{\theta}_h}{\theta_h} = \frac{\dot{\Theta}_h}{\Theta_h} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \frac{\lambda_1(pY_h - pC)}{\Theta_h} - \Gamma(\omega, \ell, m_f), \tag{2.55}$$

$$\frac{\dot{m}_h}{m_h} = \frac{\dot{M}_h}{M_h} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \frac{\lambda_2(pY_h - pC)}{M_h} - \Gamma(\omega, \ell, m_f), \tag{2.56}$$

$$\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \frac{\lambda_3(pY_h - pC)}{D} - \Gamma(\omega, \ell, m_f), \tag{2.57}$$

where

$$\Gamma(\omega, \ell, m_f) = g_Y(\pi) + i(\omega) = \frac{\kappa(\pi)}{\nu} - \delta + i(\omega). \tag{2.58}$$

Similarly, for the firm variables $\ell = L/(pY)$ and $m_f = M_f/(pY)$, we use (2.7)–(2.8) to obtain

$$\frac{\dot{\ell}}{\ell} = \frac{\dot{L}}{L} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \frac{pI}{L} + r - \Gamma(\omega, \ell, m_f), \tag{2.59}$$

$$\frac{\dot{m}_f}{m_f} = \frac{\dot{M}_f}{M_f} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \frac{\Pi_p}{M_f} + \frac{r\ell}{m_f} - \Gamma(\omega, \ell, m_f). \tag{2.60}$$

Finally, for the ratio of bank holdings of bills $\theta_b = \Theta_b/(pY)$, we can use (2.51) to obtain

$$\begin{aligned} \frac{\dot{\theta}_b}{\theta_b} = \frac{\dot{\Theta}_b}{\Theta_b} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \frac{(k_r - f)r\ell}{\theta_b} - \Gamma(\omega, \ell, m_f) \\ &+ \frac{(1-f)\Pi_p + [\lambda_3 + (1-f)\lambda_2](pY_h - pC) - (1-k_r)pI}{\Theta_b}. \end{aligned} \tag{2.61}$$

We then find that (2.52)–(2.53) and (2.59)–(2.60) lead to the following system of ordinary differential equations:

$$\begin{cases} \dot{\omega} = [\Phi(e) - (1-\gamma)i(\omega) - \alpha]\omega, \\ \dot{e} = \left[\dfrac{\kappa(\pi)}{\nu} - \alpha - \beta - \delta\right]e, \\ \dot{\ell} = [r - \Gamma(\omega, \ell, m_f)]\ell + \kappa(\pi), \\ \dot{m}_f = [r_m - \Gamma(\omega, \ell, m_f)]m_f - \omega + 1 - t, \end{cases} \tag{2.62}$$

where

$$\pi = 1 - t - \omega - r\ell + r_m m_f, \tag{2.63}$$

$$i(\omega) = \eta_p(m\omega - 1). \tag{2.64}$$

To solve (2.62), it is necessary to specify the behavioural functions $\Phi(\cdot)$ and $\kappa(\cdot)$. For the Philips curve we follow Grasselli & Nguyen Huu (2015) and choose

$$\Phi(e) = \frac{\phi_1}{(1-e)^2} - \phi_0 \tag{2.65}$$

for constants ϕ_0, ϕ_1 specified in Table A.1. For the investment function, we follow the more recent work of Pottier & Nguyen-Huu (2017) and use

$$\kappa(\pi) = \kappa_0 + \frac{\kappa_1}{(\kappa_2 + \kappa_3 e^{-\kappa_4 \pi})^{\xi}} \tag{2.66}$$

that is to say, a generalized logistic function with parameters given in Table A.1.

Once the main system (2.62) is solved for the state variables (ω, e, ℓ, m_f), we can use them to solve the following auxiliary system for the variables (θ_h, θ_b) derived from (2.55) and (2.61):

$$\begin{cases} \dot{\theta}_h = -\Gamma(\omega, \ell, m_f)\theta_h + \lambda_1 \Xi(\omega, \ell, m_f, \theta_p), \\ \dot{\theta}_b = -\Gamma(\omega, \ell, m_f)\theta_b + (\lambda_3 + (1-f)\lambda_2)\Xi(\omega, \ell, m_f, \theta_p) \\ \qquad - (1-k_r)\kappa(\pi) + (1-f)\pi + (k_r - f)r\ell, \end{cases} \tag{2.67}$$

where θ_p is total private holding of government bills:

$$\theta_p = \theta_h + \theta_b, \tag{2.68}$$

and

$$\Xi(\omega, \ell, m_f, \theta_p) = g - t - \pi + r_\theta(\theta_h + \theta_b) + (1-k_r)\kappa(\pi) - k_r r\ell, \tag{2.69}$$

we can then find the following remaining variables separately by solving each of the following auxiliary equations:

$$\dot{h} = -\Gamma(\omega, \ell, m_f)h + \lambda_0 \Xi(\omega, \ell, m_f, \theta_p), \tag{2.70}$$

$$\dot{m}_h = -\Gamma(\omega, \ell, m_f)m_h + \lambda_2 \Xi(\omega, \ell, m_f, \theta_p), \tag{2.71}$$

$$\dot{d} = -\Gamma(\omega, \ell, m_f)d + \lambda_3 \Xi(\omega, \ell, m_f, \theta_p). \tag{2.72}$$

The system (2.62) is very similar to the system analyzed in Sec. 4 of Grasselli & Nguyen Huu (2015) if one sets the speculative flow $F = 0$ in their Eq. (46). We therefore do not repeat the analysis of the equilibrium points of (2.62), except for observing that it admits an interior equilibrium characterized by a profit share defined as

$$\overline{\pi} = \kappa^{-1}(\nu(\alpha + \beta + \delta)) \tag{2.73}$$

and corresponding to nonvanishing wage share and employment rate, finite private debt, and a real growth rate of

$$\kappa(\overline{\pi}) = \alpha + \beta. \tag{2.74}$$

In addition, system (2.62) admits a variety of equilibria characterized by infinite debt ratios and a real growth rate converging to $\kappa_0/\nu - \delta < 0$. In Sec. 4, we explore the properties of these different equilibria in the context of the present model.

3. Narrow Banking

Several alternative definitions for narrow banking have been summarized in Pennacchi (2012), ranging from the familiar full-reserve banking advocated in the Chicago plan to much less recognizable forms of 'banking', such as prime money market mutual funds (PMMMF). Common to all the definitions is a separation between loans and demand deposits. In what follows, we focus on a specific example of such separation, namely by dividing the banking sector into full-reserve bank and lending facilities.

3.1. *Full-reserve bank*

The simplest form of narrow banking corresponds to financial institutions that have only demand deposits as liabilities and are required to hold an equal amount of reserves as assets, which in the context of the model of Sec. 2 this corresponds to setting $f = 1$. These institutions can, in principle, also hold cash or excess reserves as assets in addition to required reserves, with the difference between total assets and demand deposits corresponding to shareholder equity, or net worth in the notation of the previous section. For simplicity, in accordance with assigning a risk weight $\rho_r = 0$ to reserves, we assume that these *full-reserve banks* maintain zero net worth, so that required reserves equal demand deposits at all times. [b] In practice, even a bank holding only reserves as assets should have a small positive net worth to absorb losses due to operational risk, but we shall neglect this effect here. Similarly, because we are assuming zero-interest on reserves, in practice a full reserve bank would need to charge a service fee in order to be able to pay interest on demand deposits and generate a profit. We neglect this effect also and assume that $r_m = 0$ so that our full reserve bank operates with zero profit.[c]

In other words, a full-reserve bank in our model corresponds to the following balance sheet structure:

$$\begin{aligned} \text{Assets:} \quad & R, \\ \text{Liabilities:} \quad & M_h + M_f, \\ \text{Net Worth:} \quad & X_b^1 = R - (M_h + M_f) = 0. \end{aligned}$$

3.2. *Lending facilities*

These correspond to a financial institution that holds treasury bills and loans as assets and time deposits as liabilities. In other words, a lending facility in our model corresponds to the following balance sheet structure:

$$\begin{aligned} \text{Assets:} \quad & L + \Theta_b, \\ \text{Liabilities:} \quad & D, \\ \text{Net Worth:} \quad & X_b^2 = L + \Theta_b - D = k_r L. \end{aligned}$$

At an operational level, in the context of the model of Sec. 2, a lending facility acquires time deposits when a household decides to reallocate part of its wealth away from other assets. For example, a household can transfer funds from its demand deposit account with a full-reserve bank into a time deposit account in a lending

[b] Observe that the assumption of zero net worth is made throughout (with the exception of Chapter 11) in Godley & Lavoie (2007) for the entire banking sector, not only for narrow banks.

[c] Notice that we have been ignoring operational costs of the banking sector all along, for example by assuming that they pay no wages to employees, so the assumption of zero profits for a full reserve bank is not much stronger. In reality, under current economic conditions, a narrow bank can be surprisingly profitable.

facility. This is accompanied by a transfer of reserves from the full-reserve bank to the lending facility. Similarly operations take place when households reallocate their wealth from cash and government bills into time deposits, all leading to an increase in reserves temporarily held by the lending facility.

These excess reserves (because there are no required reserves associated with time deposits) can then either be used to purchase bills from the central bank or to create a new loan, which results in the excess reserves being transferred back to the full-reserve bank, but this time as a demand deposit for the borrowing firm.

The key feature of narrow banking is that the lending facility is not able to create new time deposits simply by creating new loans. Instead, the lending facility needs to first obtain excess reserves in the amount of the new loan. This leaves the lending facility with a choice between the following three mechanisms. First, it can obtain excess reserves by attracting time deposits as described above. This leads to an overall expansion of the balance sheet of the lending facility, with the minimal capital requirement being achieved through dividend payments according to (2.49).

Secondly, the lending facility can obtain reserves by increasing equity, for example by paying less dividends than in (2.49). This also leads to an expansion of the balance sheet of the lending facility, but with an equity ratio larger than the minimal capital requirement.

Thirdly, the lending facility can obtain reserves by selling government bills to the central bank, in which lending corresponds to an asset swap, without any expansion of the balance sheet of the lending facility or change in the equity ratio. In all three cases we see that the provision on new loans is ultimately limited by factors outside the direct control of the lending facility, namely: attracting new time deposits, raising equity, and borrowing from the government. This is in contrast to the fractional reserve case, where the creation of new loans is automatically accompanied by the creation of new demand deposits, providing banks with much greater flexibility in lending.

We illustrate in greater detail the third mechanism, namely creating new loans to the private sector by selling government bills, because it is the least intuitive. Assume as before that the lending facility wants to maintain the minimal capital requirement (2.47), which in this case reduces to

$$\Theta_b = D - (1 - k_r)L. \tag{3.1}$$

We therefore see that Θ_b decreases as L increases, corresponding to the sale of government bills by the lending facility in order to expand the amount of loans as mentioned above. Observe that the variable Θ_b becomes negative whenever D drops below $(1 - k_r)L$. This means that in order to offer additional loans to the private sector (that is to say, increase L further), the lending facility needs to borrow from the government sector.[d]

[d]Observe that this is essentially the same mechanism explained in the section "Bank lending under the Sovereign Money system" of Thoroddsen & Sigurjónsson (2016).

As we mentioned in Sec. 1, a full-reserve bank and a lending facility can be owned and managed as a single bank with two distinct business lines. The key point is that demand deposits need to be matched with an equal amount of central bank reserves, regardless of the remaining mix of assets and liabilities of the bank.

For our purpose, a narrow banking regime is therefore characterized by a banking sector with a combined equity $X_b = X_b^1 + X_b^2$ and subjected to $f = 1$ in (2.1). In the next section we compare the properties of fractional and full reserve banking through a series of numerical examples.

4. Numerical Experiments

We perform four experiments to demonstrate the properties of the model under different reserve requirements. In all cases we use the base parameters shown in Table A.1. Details on the parameters used for the wage, employment, and inflation parts of the model, namely $\alpha, \beta, \eta_p, m, \nu$ and δ can be found in Grasselli & Maheshwari (2018), whereas an in-depth discussion of the properties of the investment function and its parameters, namely κ_i, $i = 1, \ldots, 4$ and ξ can be found in Pottier & Nguyen-Huu (2017). The remaining parameters, namely the interest rates r, r_D, r_θ and r_m, the capital adequacy ratio k_r, and the constants g and t related to government spending and taxation are used for illustration only and are based on recent representative values in advanced economies. Initial conditions for each experiment are indicated in the figures showing the results.

Example 4.1 (Fractional Reserve Banking with Finite Debt). We begin with an example of fractional reserve banking where the main system (2.62) reaches an interior equilibrium. We take $f = 0.1$ as the required reserve ratio and choose a moderate level of loan ratio $\ell_0 = 0.6$ as an initial condition. As shown in the left panel of Fig. B.1, the state variables for (2.62) converge to the equilibrium

$$(\hat{\omega}, \hat{\lambda}, \hat{\ell}, \hat{m}_f) = (0.6948, 0.9706, 4.1937, 0.7577). \tag{4.1}$$

The profit share corresponding to this equilibrium according to (2.63) is

$$\hat{\pi} = 0.1249, \tag{4.2}$$

leading to a growth rate of real output of $g(\hat{\pi}) = 0.0451$ according to (2.45). These are very good approximations to the theoretical values $\overline{\pi} = 0.1248$ and $g(\overline{\pi}) = 0.0450$ obtained from (2.73) and (2.74).

The right panel of Fig. B.1 shows convergence of the other variables of the model to finite values. In particular, observe that although the loan ratio ℓ and the deposit ratios d, m_f and m_h all increase before stabilizing at their equilibrium values, the ratio θ_b of bills held by the banking sector remains close to its small initial value $\theta_b = 0.1$, indicating the usual money market interactions between banks and the central bank.

Example 4.2 (Fractional Reserve Banking with Explosive Debt). We consider next an example of fractional reserve banking where the main system (2.62) approaches an equilibrium with infinite private debt and vanishing wage share and employment rate. As before, we take $f = 0.1$ as the required reserve ratio but modify the initial loan ratio to $\ell_0 = 6$, that is to say, ten times larger than in the previous example. Admittedly, this is an extreme initial condition,[e] chosen here for illustrative purposes. The key point is that, as shown in Grasselli & Nguyen Huu (2015), explosive equilibria of this type for (2.62) are locally stable for a wide range of parameters, and therefore cannot be ignored from the outset.

As shown in the left panels of Fig. B.2, both the loan ratio ℓ and deposit ratios m_f for firms eventually explode to infinity in this example, dragging the economy down with a growth rate -0.0522 and causing the wage share and employment rate to converge to zero. The remaining variables of the model are shown in the right panel of Fig. B.2, where we can see the household holdings of cash, demand and time deposits, and bills all exploding to infinity. Characteristically, we see that $\theta_b \to -\infty$, indicating that the banking sector needs to *borrow* from the government in order to increase the loan ratio without bounds.

Example 4.3 (Narrow Banking with Finite Equilibrium). In this example we use the same parameters as in Example 4.1 with the only difference that $f = 1$, namely, we impose a 100% reserve requirement. We also use the same initial conditions as in Example 4.1, except for m_{f_0} and d_0, which need to be calculated differently to achieve the required capital ratio in this case.

The left panel of Fig. B.3 shows the state variables of (2.62) converging to essentially the same equilibrium as before, namely

$$(\hat{\omega}, \hat{\lambda}, \hat{\ell}, \hat{m}_f) = (0.6948, 0.9706, 4.1929, 0.6462), \tag{4.3}$$

corresponding to a profit share and growth rates that are identical up to four decimal places. Notably, narrow banking does *not* lead to any loss in equilibrium growth for the economy.

The only significant departure from the fractional banking case of Example 4.1 is that θ_b drops to negative values almost immediately and continues to become more and more negative as ℓ increases towards its equilibrium value. In other words, in the full reserve case the banking sector needs to borrow more from the government in order to increase its lending to the private sector.

Example 4.4 (Narrow Banking with Explosive Debt). In this example, we use $f = 1$ and the same values for parameters and initial conditions as in Example 4.2

[e]The level of domestic credit to the private sector as a proportion of GDP (which is approximated by ℓ in our model) was approximately 1.3 for the entire world in 2016 (up from 0.5 in 1960) and only larger than 2 for Cyprus. *Source*: IMF, International Financial Statistics (https://data.worldbank.org/indicator/FS.AST.PRVT.GD.ZS).

except for m_{f_0} and d_0, which again need to be calculated differently to achieve the required capital ratio in the full reserve case.

As in Example 4.2, we see in the left panel of Fig. B.4 that the high initial level of debt for the firm sector leads to an explosive behavior for the variables ℓ and m_f in system (2.62) and corresponding collapse of output, wages and employment. The essential difference is that this occurs much earlier in the narrow banking case, namely output begins to decrease shortly after twenty years of debt accumulation, as oppose to after nearly seventy years in the fractional banking case. Moreover, as we can see in the right panel of Fig. B.4, the reliance of the banking sector on *borrowing* from the government is much more pronounced in the narrow banking case, with θ_b surpassing (i.e. becoming more negative than) -1 within a few years.

5. Conclusion

In this paper, we have considered two stock-flow consistent economic models: (A) one with the traditional fractional reserve banking sector and (B) one with the narrow banking sector. We have analyzed their similarities and differences and demonstrated that both can operate in a satisfactory fashion, with a narrow banking system exhibiting features that allow for better monitoring and prevention of crises by regulators. Crucially, the version of the model with a 100% reserve requirement for demand deposits did *not* suffer from any loss of economic growth when compared with the fractional reserve version.

Several improvements can be made to the base model presented here, adding realism at the expense of tractability, as expected. One relates to the usual criticism that the Keen model does not incorporate a realistic consumption function, variable utilization of capital, and inventory management. All of these features can be added to the current model essentially in the same way as in Grasselli & Nguyen-Huu (2018), with the corresponding increase in dimensionality for the system. Similarly, adding stochasticity to some of the underlying economic variables, such as productivity growth, is an important open task that should be carried out along the lines developed in Nguyen Huu & Costa-Lima (2014) and Lipton (2016b). Specifically related to the topic of this paper, a natural extension consists in restricting the supply of credit to firms when the level of government *lending* to the private sector is deemed too high, as a potential stabilization policy. This can be done with the addition of a credit rationing mechanism similar to what is proposed in Dafermos *et al.* (2017). In a similar vein, default by both firms and banks is a very important aspect that needs to be incorporated into the model.

In light of the oversized role that banking and finance play in modern economies, effective regulation of the banking sector remains the number one priority for achieving systemic stability. Narrow banking is a compelling policy tool with a long pedigree but poorly understood properties. While advances in technology make the implementation of narrow banking more feasible than it has ever been, concerns about the macroeconomic consequences of the policy persist, in particular with respect to growth. For example, voters in a recent referendum in Switzerland resoundingly rejected a narrow banking proposal largely because of the uncertainties surrounding the idea.[f] We hope to have contributed to the discussion by showing that the advantages of narrow banking merit serious consideration by regulators and policy makers.

Appendix A. Parameters for Numerical Simulations

The baseline parameters for our simulations are provided in Table A.1. Alternative values for some specific parameters are provided in the legend of each figure.

Table A.1. Baseline parameter values.

Symbol	Value	Description
r	0.04	Interest rate on loans
r_D	0.02	Interest rate on time deposits
r_θ	0.012	Interest rate on bills
r_m	0.01	Interest rate on demand deposits
λ_0	0.1	Proportion of households savings invested in cash
λ_{i0}	0.3	Portfolio parameters for households ($i = 1, 2, 3$)
λ_{11}	4	Portfolio parameter for households
λ_{12}	-1	Portfolio parameter for households
λ_{22}	2	Portfolio parameter for households
α	0.025	Productivity growth rate
β	0.02	Population growth rate
η_p	0.35	Adjustment speed for prices
m	1.6	Markup factor
γ	0.8	Inflation sensitivity in the bargaining equation
g	0.2	Government spending as a proportion of output
t	0.08	Taxes as a proportion of output
ν	3	Capital-to-output ratio
δ	0.05	Depreciation rate
k_r	0.08	Capital adequacy ratio
ϕ_0	0.0401	Philips curve parameter
ϕ_1	6.41×10^{-5}	Philips curve parameter
κ_0	-0.0056	Investment function lower bound
κ_1	0.8	Investment function upper bound
κ_2	1	Investment function parameter
κ_3	2	Investment function parameter
κ_4	10	Investment function parameter
ξ	4	investment function parameter

[f] See https://www.reuters.com/article/us-swiss-vote-sovereign/swiss-voters-reject-campaign-to-radically-alter-banking-system-idUSKBN1J60C0.

Appendix B. Figures

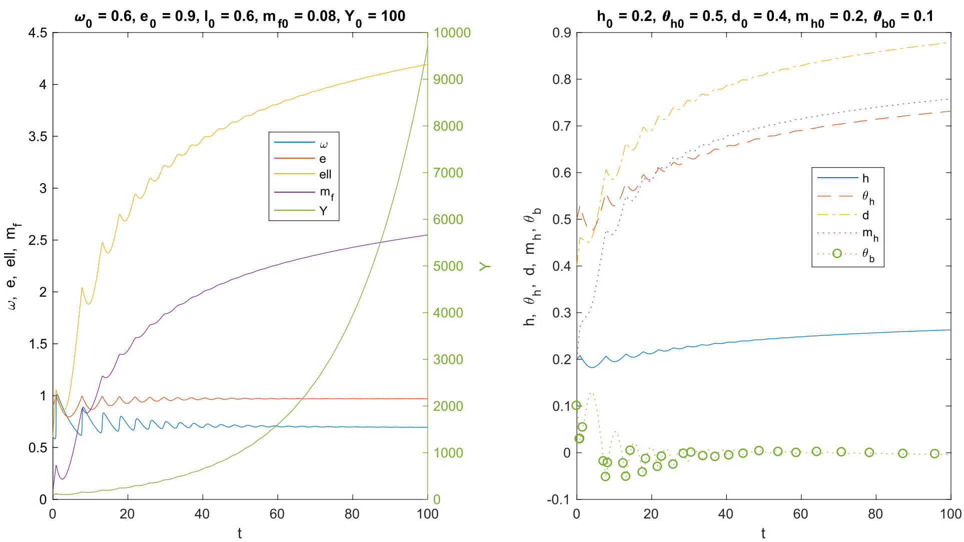

Fig. B.1. Solution of the model (2.62) with fractional reserve ratio $f = 0.1$ and remaining parameters as in Table A.1. With a moderate value for the initial loan ratio $\ell_0 = 0.6$ we observe convergence to an interior equilibrium.

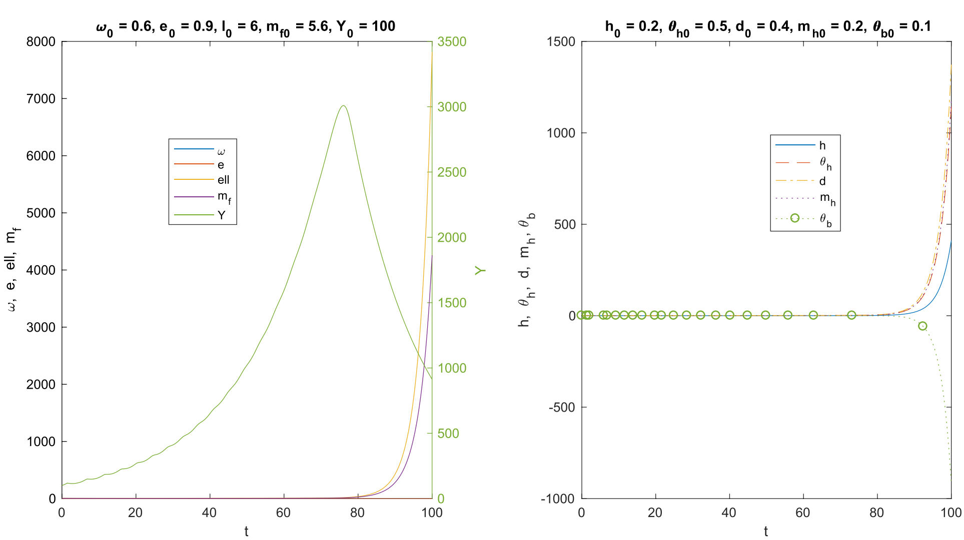

Fig. B.2. Solution of the model (2.62) with fractional reserve ratio $f = 0.1$ and remaining parameters as in Table A.1. With a high value for the initial loan ratio $\ell_0 = 6$ we observe convergence to an equilibrium with infinite private debt.

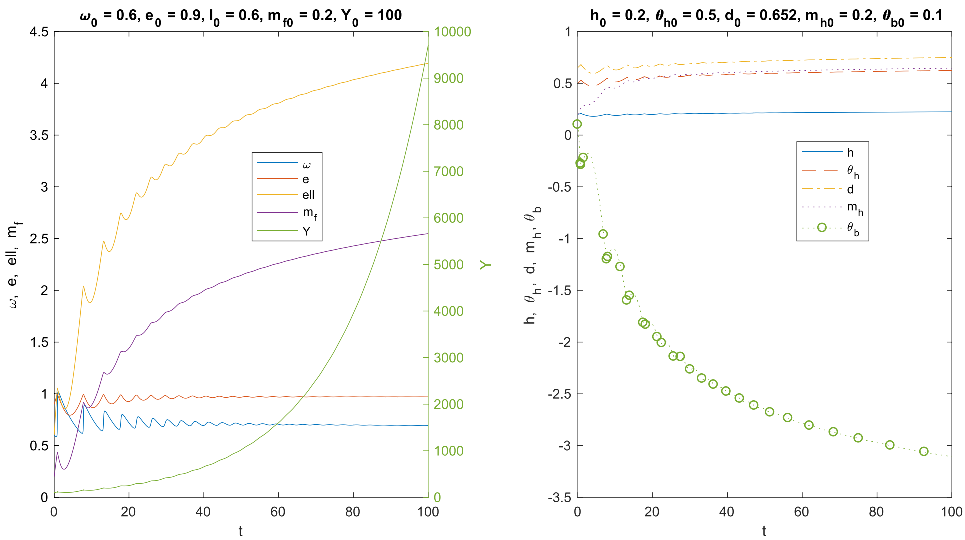

Fig. B.3. Solution of the model (2.62) with full reserve ratio $f = 1$ and remaining parameters as in Table A.1. With a moderate value for the initial loan ratio $\ell_0 = 0.6$ we observe convergence to an interior equilibrium. Observe the negative values for θ_b throughout the period.

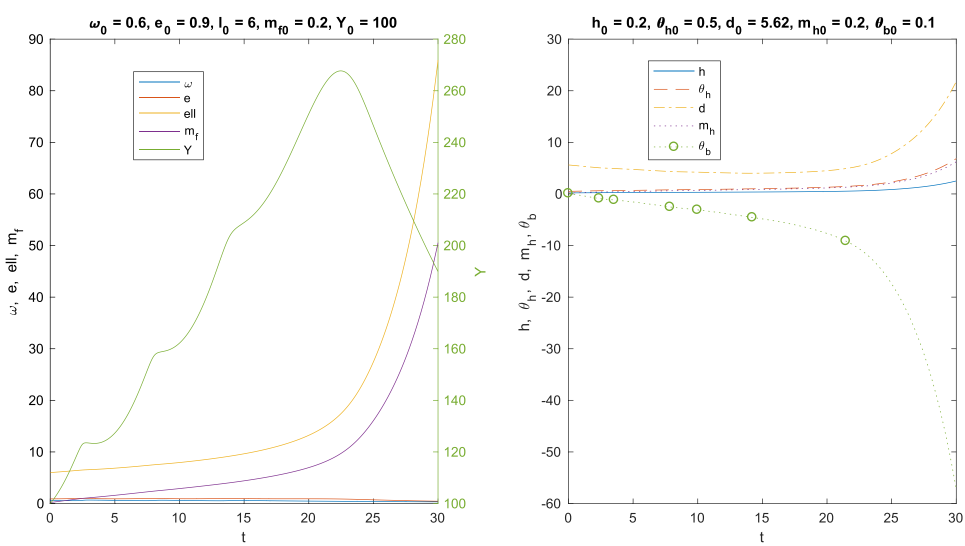

Fig. B.4. Solution of the model (2.62) with full reserve ratio $f = 1$ and remaining parameters as in Table A.1. With a high value for the initial loan ratio $\ell_0 = 6$ we observe convergence to an equilibrium with infinite private debt.

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