CHAPTER 5

The nonlinear economics of debt deflation

Steve Keen

1933 was a pivotal year for economics. Practically, it marked the perigee of the Great Depression – although no end was yet in sight to capitalism's greatest slump. Academically, it saw a bifurcation in economic theory, with two leading economists presenting diametrically opposed interpretations of the cyclical nature of capitalism. In one view, cycles – even, it seems, great depressions – were caused by exogenous shocks to an otherwise stable economic system. In the other, cycles were endemic to capitalism – and indeed, capitalism harbored a tendency toward complete collapse.

The former view was put forth by Frisch in his celebrated and well-known paper "Propagation problems and impulse problems in economics" (Frisch 1933); the latter was put forth by Fisher in his much less well-known paper "Debt deflation theory of great depressions" (Fisher 1933a).[1] The former paper is credited with playing a key role in the development of the then fledgling subdiscipline of econometrics; the latter remained largely ignored[2] until its revival, among the underworld of economics, in the form of Minsky's "Financial instability hypothesis" (Minsky 1977). In this chapter I argue – with the benefit of nonlinear hindsight – that the majority of the profession took the wrong fork back in 1933.

1 Frisch's linear premise

Frisch's initial premise was that "the majority of the economic oscillations which we encounter seem to be . . . produced by the fact that certain exterior

[1] Fisher's views were more fully elaborated in *Booms And Depressions Some First Principles* (Fisher 1933). However, the paper in *Econometrica*, (Fisher 1933a), the journal that Frisch established, is more widely available.

[2] Although Frisch did refer to Fisher's argument concerning the impact of debt (Frisch 1933, pp. 180–81), he did not use it in his modeling.

impulses hit the economic mechanism and thereby initiate more or less regular oscillations" (Frisch 1933, p. 171). This premise was not the product of empirical research into the actual nature of economic cycles, but the byproduct of a linear interpretation of sustained oscillations in a dynamic system: Mathematically stable linear models can generate only irregular cycles if the model is subjected to external shocks. This characteristic of mathematical models of cycles was thus extended by analogy to be seen as a characteristic of the complex real-world system the mathematical model purported to portray.

Today it is well known that Frisch's presumption was incorrect, as nonlinear systems can produce aperiodic cyclical behavior.[3] However, at the time Frisch's argument was convincing, and the profession chose to model the trade cycle by using damped linear models. Although Frisch himself provided a quite elaborate model of the trade cycle, the archetypal specimen of this approach was the Hansen–Samuelson multiplier–accelerator second-order difference equation. The linear weaknesses of this model (before the introduction of ceilings and floors) are well known; before moving on to consider Fisher's more perceptive analysis, I will point out a more crucial weakness: Hansen–Samuelson multiplier–accelerator models are economically invalid.

2 Invalidity of multiplier–accelerator models

Multiplier–accelerator models were purportedly derived by combining the multiplier, which relates consumption to income, with the accelerator, which relates investment to changes in income. When Samuelson's formulation is used (in the absence of a government sector), the derivation starts with the identity that total output is the sum of consumption and investment output:

$$Y_t \equiv C_t + I_t. \tag{5.1}$$

Consumption was defined as a lagged function of income:

$$C_t = \alpha Y_{t-1}. \tag{5.2}$$

Investment was defined as a lagged function of the change in consumption:

$$I_t = \beta(C_t - C_{t-1}). \tag{5.3}$$

[3] Curiously, the final analogy that Frisch used to link the "stable system subject to exogenous shocks" interpretation of the trade cycle with Schumpeter's concept of an innovation-driven cycle – a forced pendulum driven by a continuously replenished water reservoir and a rotating nozzle – describes a forced oscillator that is very similar to the forced dual pendulum and whose behavior may therefore be chaotic (Frisch 1933, pp. 203–205). Had Frisch attempted to model this system, he may well have introduced nonlinear analysis into economics.

Substituting Eqs. (5.2) and (5.3) into Eq. (5.1) yielded the second-order equation:

$$Y_t = \alpha(1+\beta)Y_{t-1} - [\alpha\beta(Y_{t-2})]. \tag{5.4}$$

(Samuelson 1939, p. 76).

The economic fallacy in this model arises from the definition of investment. Equation (5.3) clearly refers to intended investment, yet this was substituted into Eq. (5.1), which is an identity for only actual values of output, consumption, and investment. Actual investment in period t is the increment to capital:

$$I_t \equiv K_t - K_{t-1}. \tag{5.5}$$

This can be related to output by means of the accelerator:

$$I_t = \nu(Y_t - Y_{t-1}). \tag{5.6}$$

When this is substituted into Eq. (5.1), what results is a first-order equation:

$$Y_t = \nu Y_t - (\nu - \alpha)Y_{t-1}. \tag{5.7}$$

This first-order relation generates exponential growth with positive savings, which can easily be seen if α is replaced with $(1-s)$, where s is the propensity to save:

$$Y_t = \nu Y_t - [\nu - (1-s)]Y_{t-1},$$

or

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{s}{\nu - 1}. \tag{5.8}$$

Hansen–Samuelson multiplier–accelerator models were therefore not simply limited by their linearity: They were also badly specified. They effectively equated actual savings to desired investment, two magnitudes that neither pre- nor post-Keynesian economics claim are equal at all times. Because multiplier–accelerator models related both variables to income, the only level of income that guaranteed their equality was zero, and the trade cycles these models generated were simply iterations en route to this trivial solution.

Nevertheless, although linear models in general should be abandoned, it is useful to show that an interesting model of cyclical growth can be derived from a properly specified linear model.

3 A linear model of divergent growth

We start with the Hansen–Samuelson presumption that desired investment is a lagged linear function of changes in consumption (c is therefore a behavioral constant, representing the desired change in capital stock for a given change in consumption[4]):

$$\begin{aligned} I_{d_t} &= c(C_{t-1} - C_{t-2}), \\ C_t &= (1-s)Y_t, \\ I_{d_t} &= c(1-s)(Y_{t-1} - Y_{t-2}). \end{aligned} \tag{5.9}$$

Desired investment then becomes actual investment, so that this amount is added to the capital stock:

$$\begin{aligned} I_t &= I_{d_t}, \\ K_t &= K_{t-1} + I_{t-1}. \end{aligned} \tag{5.10}$$

The new level of capital stock then determines output by means of the accelerator, thus closing the model:

$$\begin{aligned} Y_t &= \frac{1}{\nu}K_t \\ &= \frac{1}{\nu}(K_{t-1} + I_{t-1}) \\ &= \frac{1}{\nu}[\nu \times Y_{t-1} + c(1-s)(Y_{t-2} - Y_{t-3})] \\ &= Y_{t-1} + \frac{c(1-s)}{\nu}(Y_{t-2} - Y_{t-3}). \end{aligned} \tag{5.11}$$

It is obvious by inspection that this model produces both interdependent growth and cycles. Its eigenvalues are 1, $\{[c(1-s)]/\mathrm{v}\}^{1/2}$, and $(-\{[c(1-s)]/\mathrm{v}\}^{1/2})$ in the general case in which $c(1-s) \neq \mathrm{v}$, and (1,1,−1) in the special case that $c(1-s) = \nu$. These indicate that any sustained level of output is a marginally unstable equilibrium[5]: If the model is perturbed, it will generate sustained exponential growth and cycles for $c(1-s) > \nu$, sustained linear cycles for $c(1-s) = \nu$, and diminishing cycles toward a new equilibrium for $c(1-s) < \nu$. The fact that the growth and the cyclical eigenvalues are identical in magnitude means that the cycles generated are always proportional to the level of output, thus making this model the exception to Blatt's rule that linear cyclical models with unstable equilibria are invalid (Blatt 1983, p. 150). The reduced form for the general case of $c(1-s) \neq \nu$ can be decomposed into an

[4] Consumption is unlagged in this model, as the time span for consumption is significantly shorter than that for investment. Consumption was lagged in the Hansen–Samuelson models, not because this made economic sense, but because unlagged consumption gave rise to a first-order difference equation, which of course did not generate the desired cycles. The two-period lag for investment here is because a lag must be presumed between changes in consumption and investment plans based on changes in consumption.

[5] In the vernacular of econometrics, the system has a unit root.

equilibrium term [which is positive when $\nu > c(1-s)$ and negative otherwise], a growth term, and a cycle term:

$$Y_t = \frac{\nu Y_2 - c(1-s)Y_0}{\nu - c(1-s)} + \frac{1}{2}\left[\frac{\sqrt{\nu c(1-s)}(Y_1 - Y_0) + \nu(Y_2 - Y_1)}{c(1-s) - \sqrt{\nu c(1-s)}}\right]\left[\sqrt{\frac{c(1-s)}{\nu}}\right]^t - \frac{1}{2}\left[\frac{\sqrt{\nu c(1-s)}(Y_1 - Y_0) - \nu(Y_2 - Y_1)}{c(1-s) + \sqrt{\nu c(1-s)}}\right]\left[-\sqrt{\frac{c(1-s)}{\nu}}\right]^t. \tag{5.12}$$

The ratio of the constant in the growth expression to the constant in the cycle expression is

$$\frac{\sqrt{\nu c(1-s)}(Y_1 - Y_0) + \nu(Y_2 - Y_1)}{\sqrt{\nu c(1-s)}(Y_1 - Y_0) - \nu(Y_2 - Y_1)} \frac{c(1-s) + \sqrt{\nu c(1-s)}}{c(1-s) - \sqrt{\nu c(1-s)}}, \tag{5.13}$$

which ensures that the magnitude of cycles will always be smaller than, but in proportion to, the level of output.

The model can also be shown to generate large divergences in growth rates for small differences in c, the desired incremental capital-to-output ratio (ICOR). In the case of two economies that differ only in their ICORs, the ratio of their long-term growth rates is

$$\frac{\sqrt{c_1 c_2 (1-s)^2} + \sqrt{c_1(1-s)\nu} - \sqrt{c_2(1-s)\nu} - \nu}{c_2(1-s) - \nu}, \tag{5.14}$$

which is a quasi-linear but very steep function of the ratio of the preferred ICORs. Figure 5.1 plots this function with $\nu = 3$ and $c(1-s)$ values for the two countries ranging between 3 and 3.3. At one extreme of the function, a 1% difference in ICORs results in a 10% difference in rates of growth for values of the base $c(1-s)$ and ν of 3.3 and 3, respectively. As Fig. 5.1 indicates, the ratio of relative growth rates is more extreme the closer $c(1-s)$ is to ν in the denominator country. For $\nu = 3$, $c(1-s) = 3.1$, a 1% difference between $c(1-s)$ values results in a 30% difference in growth rates. At the other extreme, when $c(1-s)$ is 3.01 for country 2 and 3.3 for country 1, the growth rate of country 1 is 32 times that of country 2.

Finally, this model provides a dynamic equivalent of the "Paradox of Thrift": An increase in the savings rate will cause a decrease in the rate of growth.

All these results – interdependent cycles and growth, divergent growth rates for countries with differing investment propensities, and a dynamic paradox of thrift – stand in strong contrast to the characteristics of multiplier–accelerator

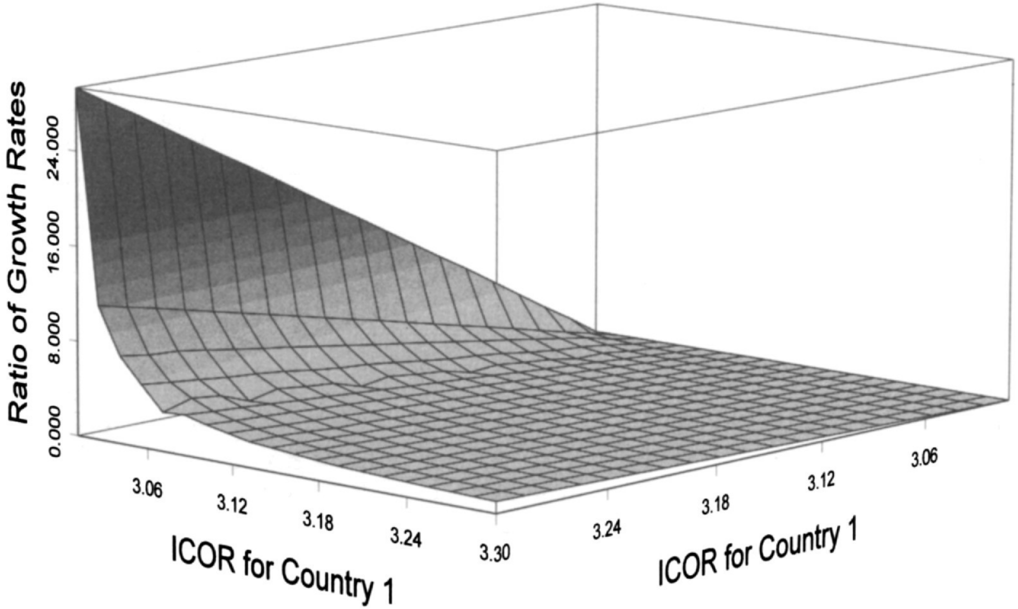

Figure 5.1. Ratio of growth rates for countries with differing ICORs.

models and of neoclassical growth theory. Yet the model is fundamentally the product of a correct specification of the Hansen–Samuelson proposition that investment is a lagged function of changes in consumption. One can only speculate as to how trade and growth theory might have developed had this model been derived at the birth of linear trade-cycle modeling, rather than at its death. Instead, it was Fisher's more enlightened vision that was stillborn.

4 Fisher's vision: the Great Depression as catastrophe

Fisher's appreciation of the dynamics underlying the Great Depression was still constrained by his knowledge of linear models of cycles, and as a result tantalizingly modern perceptions were frequently reduced to embellishments on a linear perspective. However, the gems of nonlinearity in Fisher's thinking stand out strongly against the linear backdrop. Thus, although he concedes that "we may tentatively assume that, ordinarily and within wide limits, all, or almost all, economic variables tend, in a general way, towards a stable equilibrium," the qualifications overwhelm the rule: although equilibrium is stable, it is "so delicately poised that, after departure from it beyond certain limits, instability ensues"; although every variable has an ideal equilibrium, disturbances are so myriad that "any variable is almost always above or below the ideal equilibrium" (Fisher 1933a, p. 339).

These fluctuations can explain mild economic cycles, of course, but Fisher's interest is not in these but in the truly deep declines. Here his thinking strongly departs from the equilibrium norm as he considers the self-reinforcing dynamics that can turn a downturn into a deflation. The process begins with overconfidence, which, although it is crucial to the initiation of a great depression, "seldom does great harm except when, as, and if, it beguiles its victims into debt." The two key factors in the development of a great depression are "*over-indebtedness* to start with and *deflation* following soon after" (Fisher 1933, p. 341). The former results in an exponential growth in the level of nominal debt, as interest on outstanding debts exceeds the repayment ability of some businesses. The latter amplifies this initial disturbance by increasing the real value of debt even as firms attempt to reduce its nominal value: As Fisher evocatively puts it in what can be called Fisher's Paradox, "*the more debtors pay, the more they owe*. The more the economic boat tips, the more it tends to tip. It is not tending to right itself, but is capsizing" (Fisher 1933a, p. 344).

To this point, Fisher's model explains a catastrophe (in the popular sense). To explain cycles, he argues that if one of the two causal factors – overindebtedness and deflation – is absent, then the initial disturbance will correct itself. Thus if a deflation occurs in the absence of overindebtedness or if overindebtedness is not followed by deflation, the situation "is then more analogous to stable equilibrium: the more the boat rocks the more it will tend to right itself. In that case, we have a truer example of a cycle" (Fisher 1933a, pp. 344–45). This analysis leads Fisher to support government reflationary measures as a means to avoid the occurrence of great depressions (Fisher 1933, pp. 346–47). These themes have been elaborated on and combined with a nonstandard interpretation of Keynes by Minsky, resulting in the "Financial Instability Hypothesis" (see Minsky 1977, Keen 1995).

5 The Financial Instability Hypothesis

Fisher's contribution consists of the insights that, in the real world, economic variables will always deviate from equilibrium values and that, in the case of investment behavior, overconfidence can lead to a runaway process of debt accumulation and price deflation. Minsky built on the theories of Fisher (and Keynes) to provide a historicoanalytic explanation for this process.

Minsky's analysis begins at a time when the economy is growing relatively stably, but when firms and banks evaluate investment projects conservatively, because of the memory of a recent economic crisis. The combination of a relatively tranquil economy with conservatively evaluated investment projects means that most projects succeed. Two things gradually become evident to managers and bankers: "Existing debts are easily validated and units that were heavily in debt prospered: it pays to lever" (Minsky 1977, 1982, p. 65). As a result, both managers and bankers come to regard the previously accepted risk premium as excessive. Investment projects are evaluated with less conservative estimates of prospective cash flows, so that with these rising expectations go rising investment and asset prices. The general decline in risk aversion thus sets off the growth in debt-financed investment, which is the foundation both of the boom and its eventual collapse.

The economy enters a phase that Minsky describes as "the euphoric economy" (Minsky 1970, 1982, pp. 120–24), in which both lenders and borrowers believe that the future is ensured and that therefore most investments will succeed. Asset prices are revalued upward as previous valuations are perceived to be based on mistakenly conservative grounds. Highly liquid, low-yielding financial instruments are devalued, leading to a rise in the interest rates offered by them as their purveyors fight to retain market share. Financial institutions now accept liability structures both for themselves and their customers "that, in a more sober expectational climate, they would have rejected" (Minsky 1970, 1982, p. 123.).

Asset price inflation in the euphoric economy phase makes it possible to profit by trading assets on a rising market, giving rise to a class of speculators Minsky calls "Ponzi financiers," after the American real estate and bank swindler of the 1920s. These capitalists are willing to incur debts whose servicing costs exceed the cash flows of the assets they buy, because they expect to be able to on-sell these assets at a profit. However, the rising interest rates that also occur in this period eventually force some nonspeculative investors to sell capital assets to meet their debt commitments, and the entry of these new sellers into the asset market pricks the exponential rise in prices on which Ponzi financiers depend. The leading Ponzis go bankrupt, bringing the euphoric economy to an abrupt end and ushering in another debt-induced systemic crisis.

In Minsky's model, the commodity inflation conditions at the time of the crisis determine whether the economy experiences a depression. If commodity price inflation is high, then, although real economic activity collapses, inflation eventually brings corporate cash flows into line with the debts that were accumulated during the boom; the economy limps along with low growth and high inflation, but a true calamity is avoided. If commodity price inflation is low, however, then the level of corporate debt remains beyond that which can be financed out of the depressed cash flows of a recession, and debt continues to accumulate, setting off a chain reaction of bankruptcies – Fisher's Paradox strikes.

Minsky argues that the one means by which a market economy can avoid Fisher's Paradox is by means of the institution of government. With a developed social security system, the collapse in cash flows that occurs when a boom becomes a panic will be at least partly ameliorated by a fall in tax revenues and a rise in government spending – the classic "automatic stabilizers," although this time seen in a more monetary light. Similarly, progressive taxation rates can restrain the ability of capitalists to indulge in speculative investment during the boom phase of the cycle. By proposing that the main purpose of economic policy was not to avoid cycles, but to prevent a debt deflation, Minsky thus puts a Fisherian slant on the Keynesian practice of countercyclical economic policy.

6 Modeling debt deflation

There are elements in Fisher's analysis that are consonant with modern nonlinear analysis, although these are interspersed between comments that are consonant with the generally linear understanding of cycles of the time. However, Fisher had only his creed[6] to offer in competition with Frisch's detailed linear research project, and it had little influence on the development of economics until Minsky.

[6] Fisher set out his analysis as a creed in the sense of an analysis "expressed dogmatically and without proof" (Fisher 1933a, p. 337).

In this section I show that the catastrophic[7] aspect of Fisher's and Minsky's analysis can be modeled by using an extended version of Goodwin's predator–prey model of cyclical growth. The basic Goodwin model reduces to the coupled equations

$$\begin{aligned}\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \omega[w(\lambda) - \alpha],\\ \frac{\mathrm{d}\lambda}{\mathrm{d}t} &= \lambda\left(\frac{1-\omega}{\nu} - \alpha - \beta\right),\end{aligned} \tag{5.15}$$

where λ is the rate of employment, ω is the wages share of output, $w(\lambda)$ is a Phillips curve, α is the output-to-labor ratio, and β is the rate of population growth.

The first step in extending this model is to replace the linear assumption that capitalists invest all their profits [$1-\omega$ in the previous model is the profit to output ratio $\pi = (\Pi/Y)$] with the more realistic assumption that investment is a nonlinear function $k(\,)$ of the rate of profit [$(\Pi_n/K) = (\pi_n/\nu)$], where π_n is the profit share net of interest payments.[8] This does not disturb the underlying nature of the model, which still results in a stable limit cycle, but it sets the scene for the introduction of a finance sector.

We introduce finance into the model by assuming the existence of a banking sector that exists solely to finance capitalist investment. The rate of change of debt in this system is thus simply interest on outstanding debt, plus new investment, minus gross profits:

$$\frac{\mathrm{d}D}{\mathrm{d}t} = rD + I_g - \Pi, \tag{5.16}$$

where $I_g = k(\pi_n/\nu)Y$ represents gross investment (in what follows, depreciation is introduced at the constant rate of γ per annum). This produces the following three-dimensional system:

$$\begin{aligned}\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \omega[w(\lambda) - \alpha],\\ \frac{\mathrm{d}\lambda}{\mathrm{d}t} &= \lambda\left\{\left[\frac{k(\pi_n/\nu)}{\nu} - \gamma\right] - \alpha - \beta\right\},\\ \frac{\mathrm{d}d}{\mathrm{d}t} &= d\left\{r - \left[\frac{k(\pi_n/\nu)}{\nu} - \gamma\right]\right\} + k\left(\frac{\pi_n}{\nu}\right) - \pi,\end{aligned} \tag{5.17}$$

[7] This is not a catastrophe in the sense of catastrophe theory, but an "inverse tangent" (Pomeau and Manneville 1980) chaotic process.

[8] The term Π will be retained for gross profit or output minus wages throughout. The term Π_n will signify gross profit minus all other outgoings, which in this model means interest on outstanding debt. In the next section, Π_n will signify gross profit minus interest payments and taxation minus subsidies.

where d is the debt-to-output ratio D/Y and π_n is the profit share of output:

$$\pi_n = 1 - \omega - rd. \tag{5.18}$$

As is well known, a three-dimensional system introduces the possibility of chaotic behavior, and this particular model follows the inverse tangent route to chaos first identified by Pomeau and Manneville (1980). Even at this basic level, the model contains some important insights into the role of debt in a market economy and the impact of the rate of interest in a model in which, in contrast to the standard IS-LM model, debt is explicitly accounted for.

Perturbation analysis

As is easily shown, with the functional form chosen for the Phillips curve, the equilibrium value of employment is[9]

$$\lambda_e = \frac{\ln(\alpha - C) - A}{B} = 97.12\%. \tag{5.19}$$

There is an equilibrium value for profit share:

$$\pi_e = \frac{\ln[\nu(\gamma + \alpha + \beta) - G] - E}{F} = 16.18\%, \tag{5.20}$$

which corresponds to a rate of profit of approximately 5.4% and, given the investment function, an investment share of output of 16.5%. The equilibrium value for the debt-to-output ratio is

$$d_e = \frac{k\left(\frac{\pi_n}{\nu}\right) - \pi_n}{\frac{k\left(\frac{\pi_n}{\nu}\right)}{\nu} - \gamma} = 7.02\%. \tag{5.21}$$

Because the net profit share is a linear combination of ω and d [Eq. (5.18)], this gives the curious result that, at the equilibrium, workers' share of output and bankers' share are in direct opposition to each other, whereas capitalists' share is constant. This is, unremarkably, significantly different from standard economic models of income distribution, which argue that remuneration reflects relative factor productivity and that are not equipped to deal with a return to accumulated debt. It is also, however, significantly different from the unconventional Sraffian school of economics, which sees a linear trade-off between capitalist and worker shares in the economic surplus.

This equilibrium vector is locally stable but globally unstable, a significant echo of Fisher's intuition in 1933 that the market system has an equilibrium that "though stable, is so delicately poised that, after departure from it beyond certain limits, instability ensues" (Fisher 1933, p. 339).

[9] With the parameter values used in the following simulations, which were derived by a nonlinear regression of Phillip's original data against the rate of unemployment.

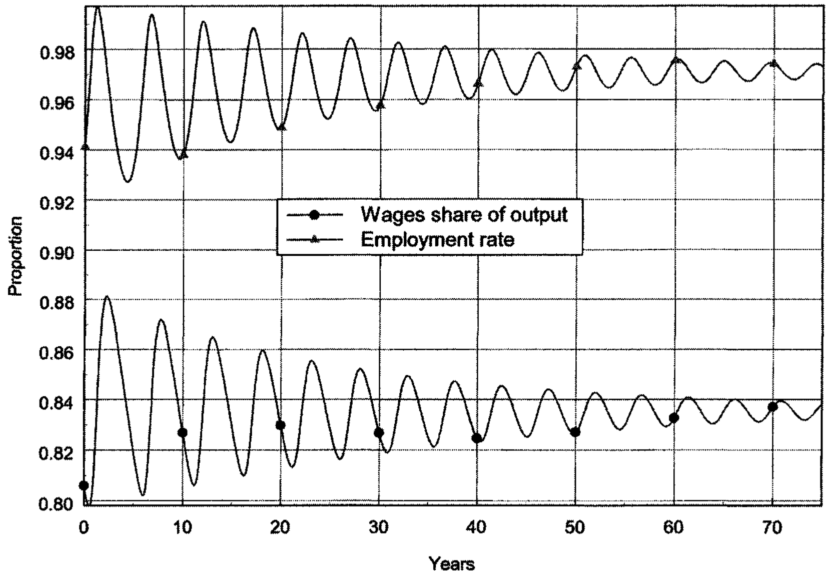

Figure 5.2. Wages share and employment near equilibrium.

Conventional IS-LM analysis argues that an increase in the interest rate will reduce investment (which is portrayed as a monotonically decreasing function of the interest rate, in contrast to this model's argument that the rate of profit determines the level of investment) and thus growth; however, any impact on the accumulation of debt is ignored. The final equation of system (5.17) indicates that, when debt is explicitly accounted for, it is possible for debt to overwhelm the system.

When the initial conditions of the model are in the vicinity of the equilibrium point, the system converges to the equilibrium with cycles of approximately 5 years, a similar period to those of the basic two-dimensional Goodwin model (see Fig. 5.2).

Figure 5.3 shows the time path of the debt-to-output ratio, which rises in a cyclical fashion initially, but then also tapers toward its equilibrium value.

The phase diagram in Fig. 5.4 and the period interactions shown in Fig. 5.5 give a clear picture of the near-equilibrium dynamics of this three-dimensional system. The initial conditions of slightly higher-than-equilibrium debt, workers' share of output, and employment lead to a downturn, as investment stagnates because of the resulting low rate of profit. The excess of profit over investment leads to debt being reduced, but the downturn eventually leads to falling wage demands, and this leads to a boost in investment well before debt is fully repaid. Debt then rises with rising employment as investment boosts output, only to lead eventually to rising wage demands that cut into profits and once again cut

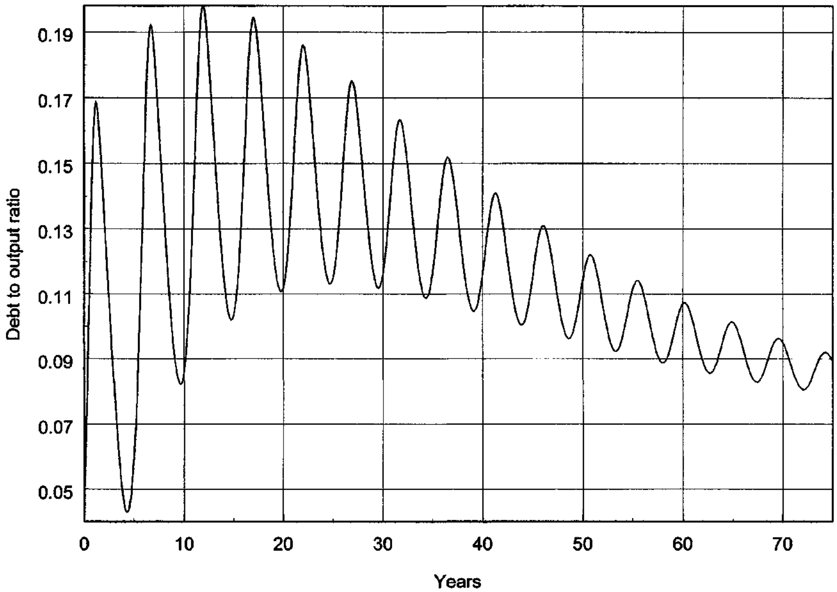

Figure 5.3. Debt-to-output ratio near equilibrium.

off investment. The cycle then continues, with the system tapering toward a stable-equilibrium debt-to-equity ratio, wages share, and rate of employment.

Figure 5.5 shows the interactions among employment, wages share, and debt at the level of a single cycle. During the boom phase, rising investment causes both rising employment and rising debt, as firms borrow to finance investment at above the level of retained earnings. In the early stage of this process, wages share continues to fall because employment, although rising, is still below the level that triggers the demand for wage rises at above the level of productivity growth. However, after roughly half the boom, wage demands lead to a rising wages share that cuts into profit share, adding to the negative effect of the increase in debt repayments. The incentive to invest thus evaporates, growth ceases, profits are devoted to repaying debt, unemployment rises, and eventually wages fall, leading to a renewal of the cycle.

Conversely, as Figs. 5.6–5.8 indicate, at a more extreme distance from the equilibrium vector, the system is unstable, as the nonlinearity of the system results in centripetal forces that drive it toward a debt-induced breakdown. The breakdown can take several forms, given the nature of the initial conditions – the one shown in Figs. 5.6–5.8 is precipitated by an extreme blowout in wages share during a boom, with initial conditions of wages share 0.11 below equilibrium, employment 0.05 below equilibrium, and debt at its equilibrium value.

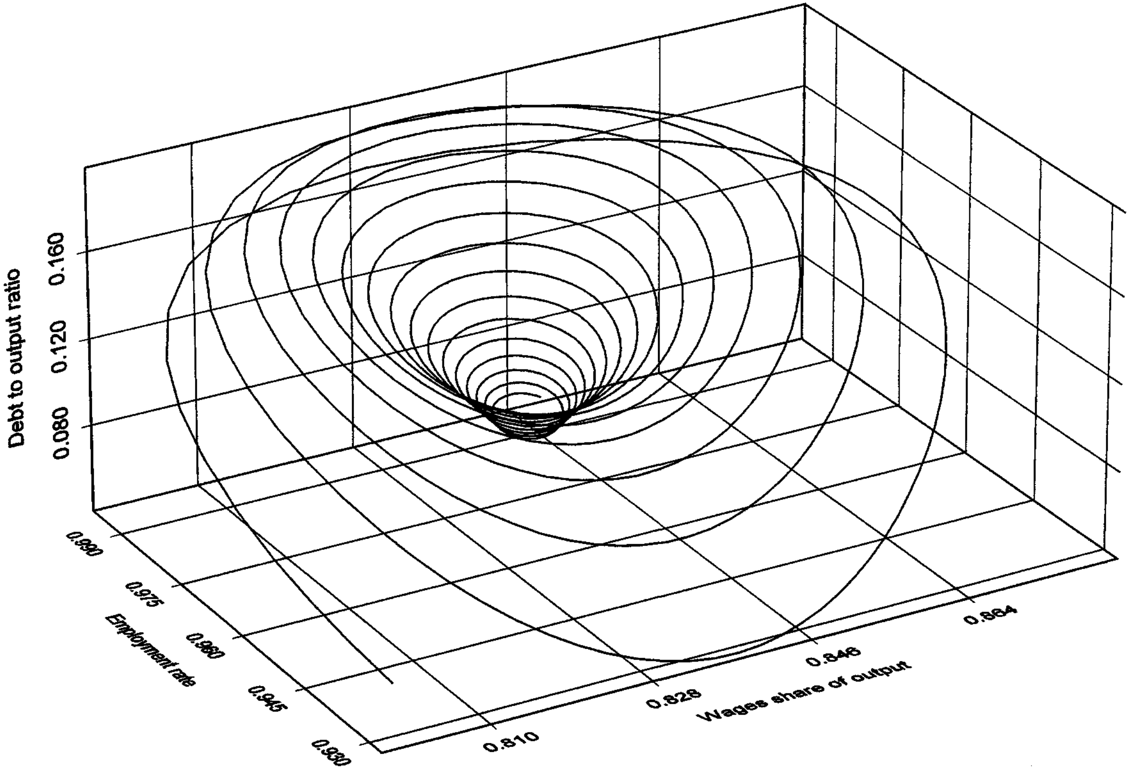

Figure 5.4. Wages share, employment, and debt interactions near equilibrium.

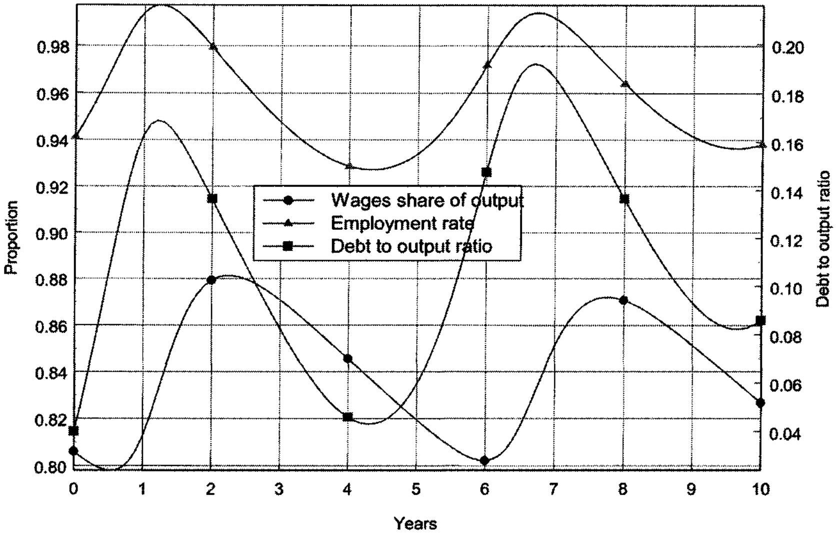

Figure 5.5. Period interactions of wages share, employment, and debt near equilibrium.

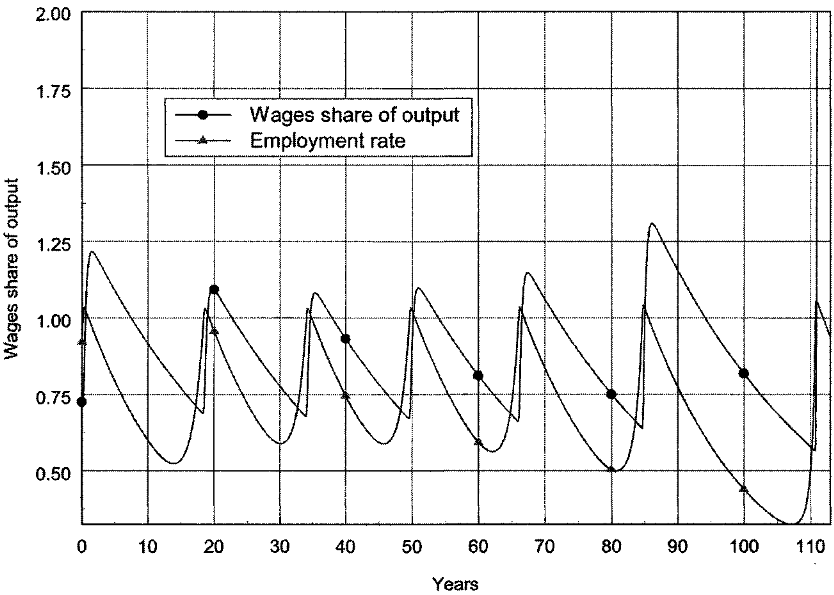

Figure 5.6. Wages share and employment far from equilibrium.

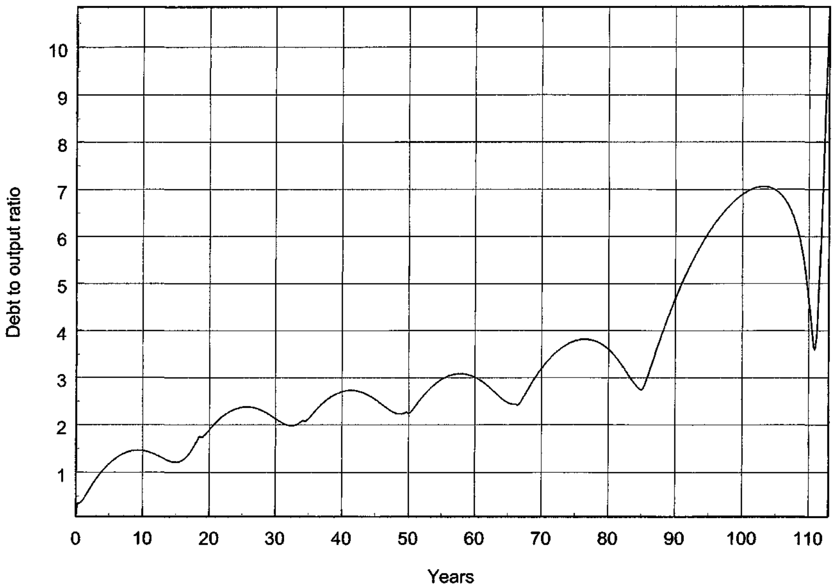

Figure 5.7. Debt-to-output ratio far from equilibrium.

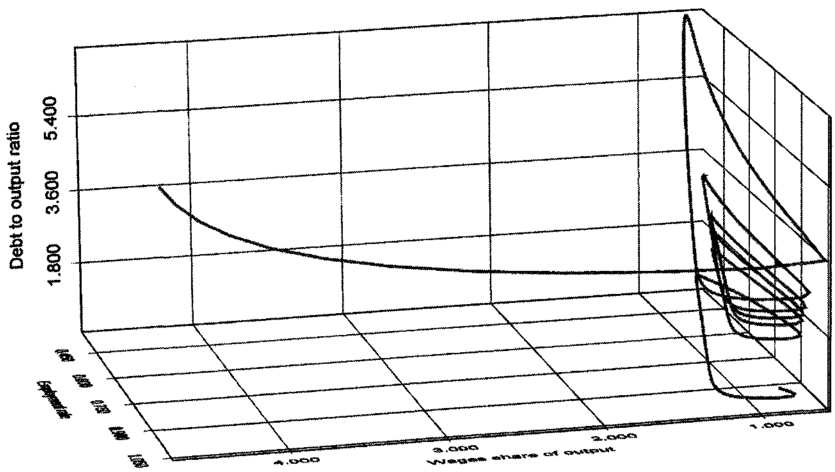

Figure 5.8. Wages share, employment, and debt interactions far from equilibrium.

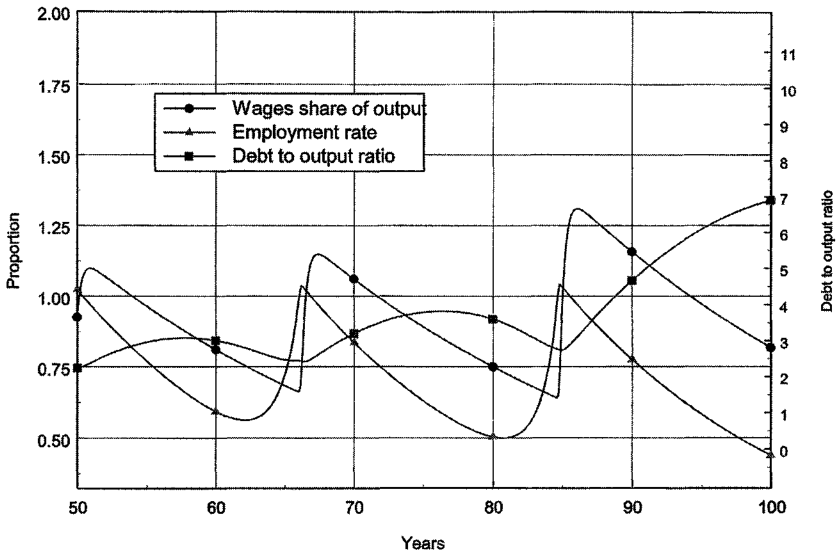

Figure 5.9. Period interactions of wages share, employment, and debt far from equilibrium.

Behind this increasing income distribution and employment instability lies cyclically accelerating debt, which, in contrast to the near-equilibrium simulation, falls primarily during the recovery and the boom phases of the cycle and rises during slumps.

The phase diagram of this simulation in Fig. 5.8 makes the system behavior graphically apparent. What is a stable volcano-shaped phase diagram becomes an unstable vortex in which debt overwhelms the other system variables.

The period interactions shown in Fig. 5.9 give some of the dynamics behind this collapse. Whereas in the near-equilibrium dynamics, the debt-to-output ratio began to fall almost as soon as the boom phase was over, here the debt-to-output ratio continues to rise until well into the slump. With year 63 as our starting point, the boom begins at a point when both wages share and the debt-to-output ratio are falling. This increases profit share and leads to a rapid boom financed by renewed borrowing, although the debt-to-output ratio is actually reduced by the more rapid rise in output. However, this eventually results in an acceleration in wages, sharply reduced profits, and an increased debt burden. These events terminate the boom some 2 years later and lead to a long gradual slump, during which time debt continues to accumulate because profits are so heavily diminished. The cycle recurs, with each successive cycle leading to a more extreme hump in the debt-to-output ratio, until the final boom leads to a wages explosion, the debt financing of which (in addition to the preceding debt financing of investment) leads to an unsustainable level of debt that overwhelms the economy.

The behavior of this model thus clearly supports the Fisher–Keynes–Minsky contention that a pure market economy is fundamentally unstable, in that it is prone to fall into a debt-induced depression from which there is no escape, baring "resetting the debt clock" by means of wholesale bankruptcy and debt moratoria. The next extension similarly supports Minsky's claim that the government sector's behavior provides a homeostatic balance that controls and possibly eliminates this tendency to depression.

Adding a government sector

Minsky's contention that countercyclical behavior by government stabilizes the market by constraining its tendency to debt accumulation is explored by the introduction of government spending and taxation as functions of the rate of employment and the profit share of output, respectively. This extension requires a new definition for the net profit share and the rate of change of private debt, and two additional nonlinear functions for the rate of change of government spending with respect to employment and taxation with respect to the gross profit share:

$$\text{net profit share of output } \pi_n = 1 - \omega - t + g - r\,d_k, \tag{5.22}$$

$$\text{rate of change of capitalist debt } \frac{\mathrm{d}}{\mathrm{d}t} D_k = r D_k + I_g - \Pi + T - G, \tag{5.23}$$

$$\text{government spending function } \frac{\mathrm{d}G}{\mathrm{d}t} = g(\lambda) Y, \tag{5.24}$$

$$\text{government taxation function } \frac{\mathrm{d}T}{\mathrm{d}t} = \tau(\pi_n) Y, \tag{5.25}$$

$$\text{rate of change of government debt } \frac{\mathrm{d}}{\mathrm{d}t} D_g = r D_g + G - T \tag{5.26}$$

where $g(\lambda)$ and $\tau(\pi_n)$ are as defined in the glossary of Table 5.1. This extension results in the following six-dimensional model of a mixed-market–state economy:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \omega[w(\lambda) - \alpha],$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = \lambda\left\{\left[\frac{k(\pi_n/\nu)}{\nu} - \gamma\right] - \alpha - \beta\right\},$$

$$\frac{\mathrm{d}d}{\mathrm{d}t} = d\left\{r - \left[\frac{k(\pi_n/\nu)}{\nu} - \gamma\right]\right\} + k(\pi_n/\nu) - \pi + t - g,$$

Table 5.1. *Glossary*

Term	Definition	Formula
Y	Level of output	
π	Profit rate	
L, λ	Employment, employment rate	$L = Y/a, \lambda = L/N$
Y, a	Output, labor productivity	
α	Productivity growth	$\alpha = \alpha_0 e^{\alpha, t}$
N, β	Population, growth rate	$N = N_0\, e^{\beta\, t}$
w	Wage rate	$(1/w)(\mathrm{d}w/\mathrm{d}t) = w(\lambda)$
$P(\lambda)$	Phillips curve	$P(\lambda) = e^{A+B\,\lambda} + C$
π_s	Profit share of output (no finance)	$\pi_s = \Pi/Y$
ω	Wages share of output	$\omega = W/Y = wL/La = w/a$
γ	Depreciation rate	
$k(\pi n)$	Investment function	$k(\pi_n) = e^{D+E\pi_n} + F$
D	Debt	$\frac{\mathrm{d}D}{\mathrm{d}t} = rD + I - \Pi$
d	Debt-to-output ratio	$d = D/Y$
g_y	Output growth rate	$g_y = (1/Y)(\mathrm{d}Y/\mathrm{d}t)$
π_s	Profit share of output (with finance)	$\pi_s = 1 - \omega - r \cdot d$
$g(\lambda)$	Subsidies function	$g(\lambda) = e^{G+H\lambda} + I$
G, g	Subsidies level, subsidies/output	$g = G/Y$
$t(\pi)$	Taxation function	$\tau(\pi) = e^{J+K\pi} + L$
T, t	Taxes, taxes/output	$t = T/Y$
D_k, d_k	Capitalist debt, debt/output	$d_k = D_k/Y$
D_g, d_g	Government debt, debt/output	$d_g = D_g/Y$
π_s	Gross profit share	$\pi_s = 1 - \omega$
π_n	Net profit share	$\pi_n = 1 - \omega - t + g - rd_k$

$$
\begin{aligned}
\frac{\mathrm{d}g}{\mathrm{d}t} &= g(\lambda) - g\left[\frac{k(\pi_n/\nu)}{\nu} - \gamma\right],\\
\frac{\mathrm{d}}{\mathrm{d}t}t &= \tau(\pi_n) - t\left[\frac{k(\pi_n/\nu)}{\nu} - \gamma\right],\\
\frac{\mathrm{d}}{\mathrm{d}t}d_g &= d_g\left\{r - \left[\frac{k(\pi_n/\nu)}{\nu} - \gamma\right]\right\} + g - t.
\end{aligned}
\tag{5.27}
$$

The behavior of this model is consistent with Minsky's hypothesis. The most intriguing aspect, from a complex systems point of view, is that the addition of a government sector transforms a system that was locally stable (about the equilibrium) but globally unstable into a system that is locally unstable but globally stable. At least half the eigenvalues of the linearized version have positive real parts for all values of r, yet, rather than leading to breakdown, the model is constrained by a chaotic limit cycle, as the following simulations indicate.

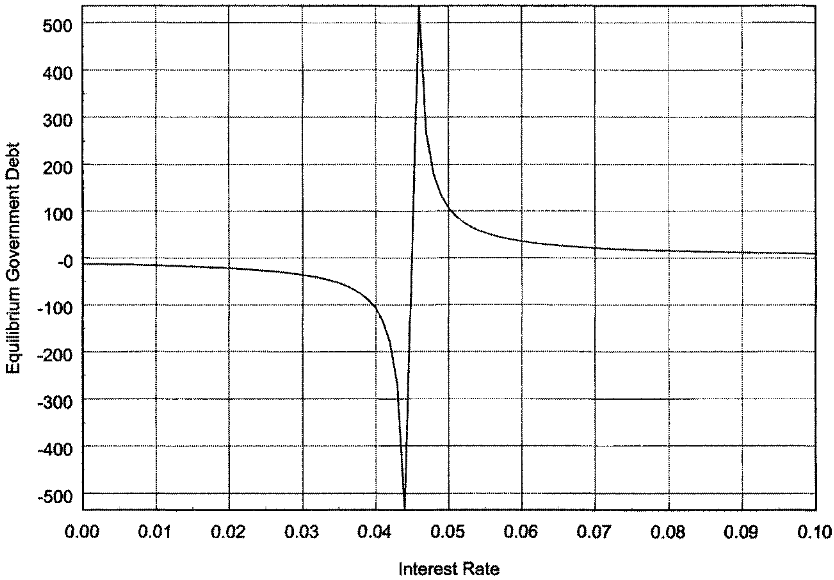

Figure 5.10. Bifurcation in the equilibrium government debt.

The second intriguing feature of this model is the relationship between government debt and the interest rate. As with the preceding model, the equilibrium wages share of output is a negative linear function of the interest rate, but in addition the level of government debt is a rectangular hyperbolic function of the interest rate (see Fig. 5.10):

$$d_g = \frac{t - g}{r - \left[\frac{k(\pi_n/\nu)}{\nu} - \gamma\right]}. \tag{5.28}$$

Thus if the prevailing (real) rate of interest is below the rate of growth of output, then with the equilibrium values for t and g given by the parameter values used in these simulations, the equilibrium value of government debt is negative. Equally, if the rate of interest exceeds the rate of growth, the equilibrium value is positive. Although the actual values differ substantially from equilibrium values because of the system's far-from-equilibrium dynamics, this negative/positive bifurcation remains in any simulation.

Figures 5.11–5.13 show the behavior of the model with an interest rate of 3% and a 0.01 deviation of all system variables from the equilibrium vector.[10]

[10] $(\omega, \lambda, d_k, g, t, d_g) = (0.300604985584, 0.971225057244, 0.070191124862, -0.145020153379, 0.390427727909, -35.696525419245)$.

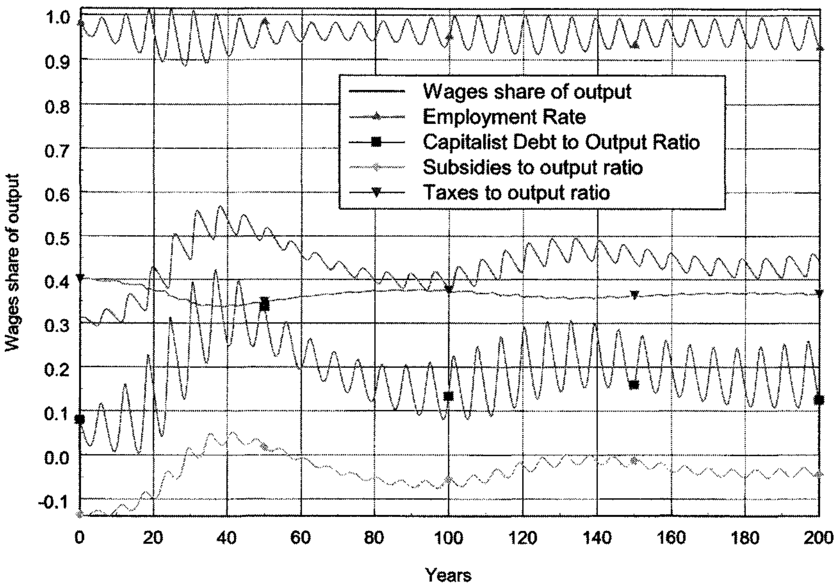

Figure 5.11. Mixed-economy far-from-equilibrium dynamics at low interest.

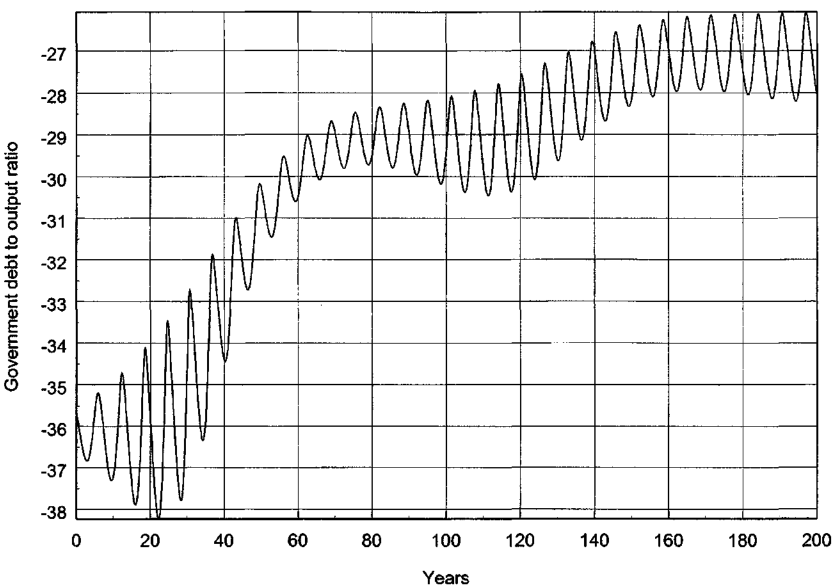

Figure 5.12. Mixed-economy far-from-equilibrium government debt dynamics at low interest.

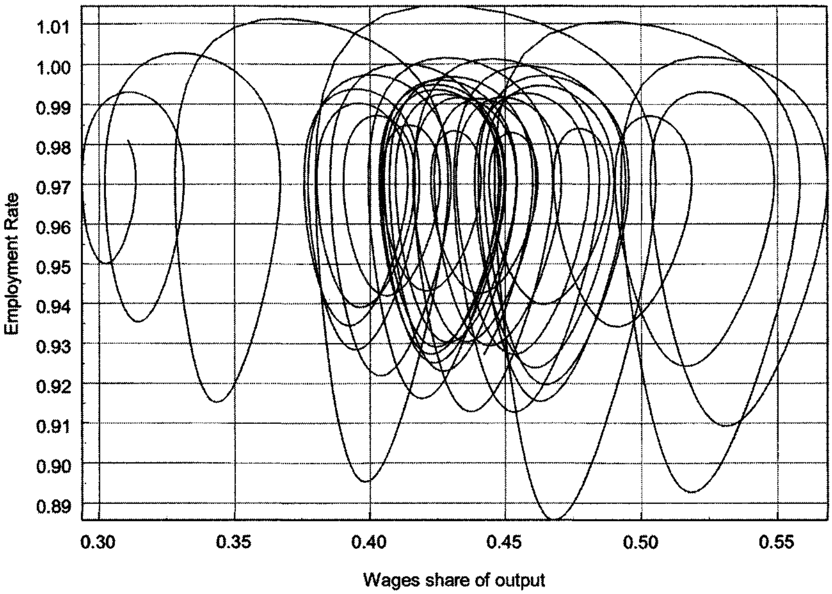

Figure 5.13. Mixed-economy far-from-equilibrium wages share and employment interactions at low interest.

Because the equilibrium vector is a repeller, all system variables move quickly and cyclically away from their equilibrium values.

At this rate of interest, the equilibrium level of government debt is negative (i.e., the equilibrium situation involves a large accumulated government surplus), and although the disequilibrium dynamics reduce this somewhat, the long-term far-from-equilibrium behavior of the system generates a sustained, although cyclical, government surplus. A different initial condition further from the system equilibrium – with an accumulated government deficit for example – could, however, lead to a different long-term outcome for the government sector.

The phase diagram in Fig. 5.13 makes it clear that the dynamics are now governed by a chaotic limit cycle.

The model behavior on the other side of the bifurcation point differs in one highly significant way: Whereas government debt stabilized at a low rate of interest, at a high rate of interest, government debt continues to grow cyclically but exponentially. Rising government deficits have been a feature of post–World War II economies, especially since the adoption of a "fight inflation first" strategy in the mid-1970s in an attempt to control the rate of inflation. The cornerstone of this policy was tight monetary policy – which meant high real

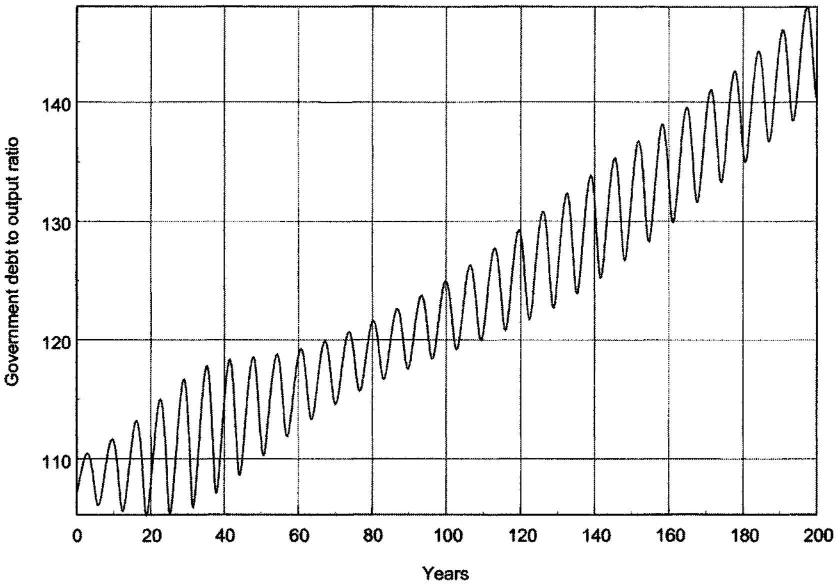

Figure 5.14. Mixed-economy far-from-equilibrium dynamics at high interest.

interest rates. Figures 5.14 and 5.15 demonstrate the behavior of the model with an interest rate of 5% and a 0.01 deviation of all values from the equilibrium vector.

The apparent paradox in Fig. 5.14 of the coincidence of a positive overall government impact on the economy – in that a debt-induced collapse is avoided – and yet a growing accumulated government deficit is explained by the impact of the high rate of interest on the current level of outstanding debt and the already high level of debt implied by starting from the equilibrium position. However, a different initial condition with a low or a negative initial government debt could easily result in a surplus's being accumulated by the government (see Keen 1995), as opposed to the deficit shown here.

As Fig. 5.15 indicates, the qualitative behavior of the model remains the same on either side of the bifurcation in equilibrium government debt.

7 Prices and Fisher's Paradox

Fisher argued that debt accumulation on its own would not be sufficient to cause a depression, but instead would give rise to cycles. However, the model above indicates the accumulation of debt alone can lead to a depression – as the end product of a series of business cycles – as the fundamental asymmetry

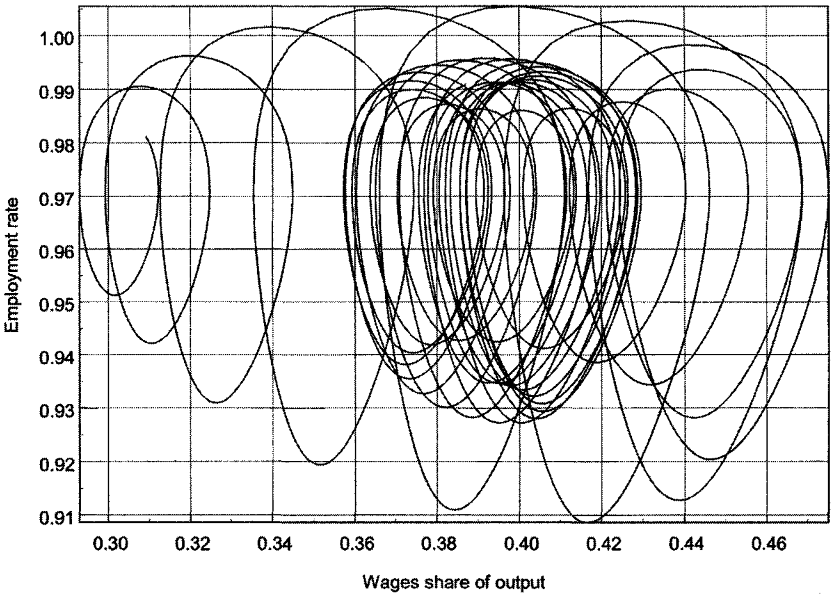

Figure 5.15. Mixed-economy far-from-equilibrium wages share and employment interactions at high interest.

that firms incur debt during booms but have to repay it during slumps asserts itself. Deflation is thus not essential to the occurrence of a depression, but it would accelerate the process and exacerbate its depth by its impact on the rate of bankruptcy. Similarly, Minsky's argument that capital goods prices are expectation driven (Minsky 1982, pp. 64, 80) implies that procyclical movements in capital goods will exacerbate the accumulation of debt, thus hastening the onset of a depression in a market economy.

We can explore these issues by revising the basic system of equations to include consumer prices (P_c) and capital goods prices (P_k). We start with an income shares equation in nominal (money) terms,

$$Y = W + rD + \Pi, \tag{5.29}$$

in which wages can be decomposed into a real wage, a consumer price index, and the level of employment (L):

$$W = wP_cL. \tag{5.30}$$

The wage change relation is now in money terms:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(wP_c) = Ww(\lambda). \tag{5.31}$$

On the other hand, the relations between labor and output and output and capital must now be expressed in real terms:

$$Y_r = \frac{Y}{P_c},\ L = \frac{Y_r}{a} = \frac{Y}{P_c\,a},\ K = \nu Y_r = \nu\frac{Y}{P_c}. \tag{5.32}$$

The introduction of a capital goods price index affects the amount paid by firms for investment goods, but the change in physical productivity continues to depend on the real increment to capital. A distinction is thus required between nominal investment (I_n) that affects bank balances and real gross investment (I_r) that affects the capital stock:

$$I_n = P_k\,k(\pi)Y,\ I_r = k(\pi)Y. \tag{5.33}$$

This results in the following system of equations:

$$\begin{aligned}
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \omega\left[w(\lambda) - \frac{1}{P_c}\frac{\mathrm{d}P_c}{\mathrm{d}t} - \alpha\right],\\
\frac{\mathrm{d}\lambda}{\mathrm{d}t} &= \lambda\left[\frac{k(\pi)}{\nu} - \gamma - \alpha - \beta\right],\\
\frac{\mathrm{d}d}{\mathrm{d}t} &= d\left\{r - \frac{1}{P_c}\frac{\mathrm{d}P_c}{\mathrm{d}t} - \left[\frac{k(\pi)}{\nu} - \gamma\right]\right\} + k(\pi)\frac{P_k}{P_c} - \pi.
\end{aligned} \tag{5.34}$$

Leaving aside the issue of a functional form for the rate of change of the price indices, this set of equations confirms Fisher's and Minsky's insights concerning the impact of commodity price deflation and capital goods prices. As can be seen from the debt relation, a high rate of commodity price inflation reduces the real debt burden, as Minsky emphasizes, whereas conversely price deflation will lead, as Fisher asserts, to an amplification of the real debt burden. The rate of debt accumulation also depends on the ratio of the capital goods price index to the consumer price index, and because the P_k/P_c ratio will rise during a boom, this will accelerate the process of debt accumulation. The price system thus increases the instability of the market economy.

8 Conclusion

The models above and the theories of Fisher and Minsky on which they are based cast a unique light on the economic history of the post-OPEC era. Rather than being a sign of the failure of Keynesian policies, the high inflation of the 1970s may have prevented the 1973 economic crisis from ushering in a depression. Similarly, the eventual success of policies intended to reduce inflation may have unwittingly set the scene for debt deflation to become the most serious economic problem of the late 20th and the early 21st centuries. This prognosis is made all the more likely by the debt crises in East and Northeast Asian developing economies of 1997 (and the subsequent Brazilian crisis of 1998) and Japan's sustained slump since the collapse of its Bubble Economy in the early 1990s.

Indonesia, Malaysia, Thailand, and South Korea have all experienced serious debt-induced crises. None of these countries has developed social security or progressive tax regimes, so that the domestic government sectors cannot significantly temper the deflationary impact of the debt crisis. Although these countries are all likely to suffer significant inflation as a consequence of the currency depreciations, the depreciations may fulfill the same function as price deflation in Fisher and Minsky's theory, as much of the debt involved is unhedged foreign debt. Indeed, the impact of floating exchange rate depreciations may far outweigh anything envisaged by Fisher, with the money market showing a proclivity to heavily devalue a country's currency whenever it believes that the country will not be able to repay its debts. Although it was possible that many of Indonesia's nongovernment borrowers would be unable to repay their debts at an exchange rate of 2500 rupiah to the dollar, it is certain that only the least indebted of them can repay at a rate of over 15,000 rupiah to the dollar. There is no prospect of Indonesia's trading its way out of its private debt crisis at a market-determined exchange rate – and the same quite possibly also applies to South Korea, Thailand, and Malaysia. A debt rescheduling comparable with that of Latin American during the 1970s could, however, be feasible if the precrisis exchange rate were reestablished.

The Japanese economy has been crippled by debts accumulated during the Bubble Economy period of the late 1980s, when real estate speculation resulted in Tokyo's nominal land value exceeding that of Canada. Japan's absence of foreign-denominated debt and substantial financial reserves in the form of foreign bond holdings insulate it from the currency depreciation problems of its Asian nations. Japan's peculiar combination of massive internal debts with its status as the world's leading creditor nation means that any action it takes to avert its domestic crisis will have significant consequences for the global economy.

Although the Japanese government is in a position to give its economy a massive fiscal and monetary boost, the only action that is likely to ameliorate Japan's internal situation and simultaneously ease the problems of its debtors is one that would simultaneously increase the Japanese price level, thus reducing the real debt burden, and cause a devaluation of the yen, thus reducing the burden of debt owed to Japanese nationals by those of other nations. One such action would be an across-the-board increase in Japanese wages, which, unlike failed attempts at monetary and fiscal stimuli during 1997 and 1998, would necessarily lead to inflation by means of the impact of uniformly higher wages on nominal production costs. This action would lead to a devaluation of the yen by the Forex markets, which would thus reduce the burden of yen-denominated debt for Japan's Southeast Asian debtors and lead to an overall approximately neutral impact on Japan's international competitiveness.

It goes without saying that such policies – arbitrated exchange rates, debt rescheduling, and deliberately engineering inflation by means of direct government manipulation of input prices – goes strongly against the dominant grain in both economic theory and policy. There is thus little likelihood of such policies being adopted, at least in the immediate future. There also may be no painless way out of this debt-deflationary process, now that it has begun. The essential policy message of the Financial Instability Hypothesis was that we should avoid such crises in the first place by developing and maintaining institutions and policies that enforce "a 'good financial society' in which the tendency by businesses and bankers to engage in speculative finance is constrained" (Minsky 1977, 1982, p. 69). Because we have manifestly failed to maintain such institutions and policies, we may have to reap the consequences in the form of the second Great Depression of the 20th century. As with the first, it will usher in social upheaval and widespread debt repudiation and conceivably lead to the dismantling of the current international financial system.

In this crisis, as evidenced by the International Monetary Fund's disastrous interventions in Indonesia, conventional economic theory will be one of the most important barriers to understanding what is happening and to working out what can be done to attenuate the damage. Economics has been conditioned by 50 years of moderate to high inflation to regard reflation with suspicion and to be too sanguine about low to negative rates of price change.[11] Yet, as the models in this paper and Fisher and Minsky's theories indicate, reflation and deliberately manufactured inflation may provide the only means by which the debt-deflationary process can be contained. The world may yet pay a high price for the economics profession's choice of the exogenous, linear explanation of cycles some 65 years ago in the depths of the last Great Depression.

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[11] Although Alan Greenspan's observation that "deflation can be detrimental for reasons that go beyond those that are also associated with inflation" (see http://www.bog.frb.fed.us/boarddocs/speeches/19980103.htm) gives cause for hope.

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