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**Ay, Nihat** (D-MPI-NS); **Tuschmann, Wilderich** (D-MPI-NS)

**Dually flat manifolds and global information geometry. (English summary)**

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The paper offers a refreshing point of view by addressing global aspects of information manifolds, rather than just local ones, as has been the case in differential geometric approaches to families of probability distributions. Its first main theorem provides a partially negative answer to a long standing question posed by Amari (the creator of information geometry as we know it): does there always exist a dually flat structure $(g, \nabla, \nabla^*)$ on a Riemannian manifold? The answer: for compact manifolds, only if its fundamental group has infinite order.

The second theorem and its corollary present a structural characterization of dually flat Riemannian manifolds $(M, g, \nabla, \nabla^*)$ under the assumption that at least one of the connections is complete. For instance, the authors show that the universal covering of $M$ is diffeomorphic to $\mathbb{R}^n$ and its fundamental group $\pi_1(M)$ is isomorphic to a subgroup of the group of affine motions that acts freely and properly discontinuously on $\mathbb{R}^n$. Given this result, an interesting question is how to construct the dually flat information manifold (with its probabilistic ingredients) whose fundamental group is isomorphic to a given such subgroup of $\mathbb{R}^n \ltimes \mathrm{GL}(\mathbb{R}^n)$.

In their third theorem, the authors prove that any two points on a dually flat manifold $(M, g, \nabla, \nabla^*)$ can be joined by a $\nabla$–geodesic, provided $\nabla$ is complete. The theorem has clear applications in estimation theory, as pointed out by them.

All the proofs given in the paper are elegantly written and should give enough motivation to working information geometers to enlarge their toolboxes with concepts such as universal covering and homotopy groups. Having opened this path into the global analysis of information manifolds, the hard task ahead is to obtain the statistical and probabilistic counterparts of the concepts discussed in the paper.

**Reviewed** by *M. R. Grasselli*