Pragmatism and Conservatism in the Egyptian Mathematics

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1 Introduction

The first ideas that come to mind when we refer to the Ancient Egypt are the grandness and the longevity of their civilization, most remarkably the great architectural works, such as the pyramids, the temples and the monuments, alongside with the hardly imaginable timescales, like the 31 dinasties and the three millenia of continuum history. It is therefore inevitable that we expect to find the same elements of grandness in their scientific activities, especially their Mathematics. The common opinions, almost legends, about the rigor and precision of the Egyptian Mathematics, come from this kind of expectations.

A less superficial look, however, shows two quite distinctive features in the mathematical activities of the ancient Egyptians. The first, of a technical character, is the total absence of generalizations in all aspects of their Mathematics. In all the available sources for its study (which comprise mainly of two papyri, the Rhind Papyrus and the Moscow Papyrus, containing a series of problems and some mathematical tables), we do not find even a single instance of a theorem or a general rule. The rules, as we shall see, are quite specific and instead of proofs what we find are descriptions of particular procedures.

The second distinctive feature, of a more historical character, is that all the mathematical development in Egypt took place in the very first centuries of the formation of their civilization, in the period traditionaly called Ancient Empire, more specifically in its first subdivision, known as Archaic Period.

In the following two millenia of civilization, what we see is the stagnant continuity of what was already done, or even in some cases a retrogression.

To appreciate these mathematical and historical facts, we need to seek relationships between the development and use of science in Egypt in the way of thinking and seeing the world of the ancient Egyptians.

2 Mathematical Characteristics

The starting point for Egyptian Mathematics is its practical inspiration. Herodotus wrote that geometry appeared in Egypt out of the necessity to measure, demark and distribute land. Basic arithmetic evolved from the management and control of public spending, distribution of goods and a whole array of pragmatic applications.

The fundamental characteristic of this arithmetic comes from the Egyptian number system. It is a non-positional system, since positional number systems were only introduced later in Mesopotamia. Thus, there are hieroglyphs for the (positive) powers of 10, which take always the same value, regardless of their position. Any given (positive integer) number can then be formed by the repetition of these hieroglyphs, just like assembling coins of different values.

This system tends to overemphasize the importance of addition, since it only takes the grouping of hieroglyphs of the same kind, replacing them by the one immediately above when one reaches 10 of them, and one has the additon of any two numbers. As we know, our own algorithm for multiplication utilizes largely the positional principle. On the contrary, the Egyptian multiplication is carried on by successive duplications, which is the most additive form of multiplication (consisting of adding a given number to itself). They then use the distributive property of multiplication, in an algorithm later known as 'duplication and mediation' (doubling and halving). Division, as expected, was done as the inverse process, duplicating the divisor.

In the Egyptian algebra we find mostly linear and some quadratic equations, which are solved by a method coherent with their mathematical specificities. It is the method of 'false position', consisting of choosing a covenient initial guess for the unknown (called the *Aha*), put it back in the equation and then adjust it according to the discrepancy between the obtained and the expected result. The validity of the method is proved by explicit verification, which is predictable, since verifications are proofs for particular processes, not for general methods.

One other feature is worth mentioning: their fractions, in order to ease the notation and calculations in the duplication technique, were always expressed with the unit as a numerator. Apart from the 2/3, which has a somewhat mysterious motivation in the Egyptian calculations, all non-unitary fractions had to be decomposed into the sum of unitaries. Here again, the decomposition process varies from fraction to fractio, with no clear explanation of why a particular process is better than any other.

In their geometry we find the least neurotic branch with regard to the specificities of the Egyptian mathematics. Here is where the first relations between different geometric figures appear, in a trend to generalization that would be laudable. However, besides gross mistakes that went unnoticed to the scribes, what we perceive in the sparse general rules for calculating areas and volumes is a preocupation with producing easier practical computations, instead of with the amelioration of the abstract understanding.

3 Philosophical and Cultural Interpretation

As we have seen, pragmatism and specificities were the tone of the developments in all branches of mathematics which florished in the Ancient Egypt.

That corresponded to the necessity to build a society with an agricultural economic basis and a highly centralized State. In fact, we would not expect that its *begining* were any different, since even nowadays a great deal of mathematical discovery has its origin in empirical problems, being later on generalize and investigated on its own right. The central question is this: why, after such starting on practical basis, has the mathematics in Egypt not evolved towards abstraction and generalizations, taking the consistence of the science that we understand by Mathematics today. That is, we are faced again with our second crucial puzzle: the premature and long-lasting stagnation in the Egyptian mathematics.

The answer resides almost entirely in the Egyptian culture, religion and psychology.

In a brief manner we can say that - unlike us who believe that a perfect world will be achieved in the future and that, consequently, the present must be transformed - the Egyptians believed during their entire history that perfection was to be found in the past, in a time where gods lived on Earth. Therefore the present had to be preserved exactly like the past, in order to be the past invoked in the future. The social, economic and political stabilities were gifts left by the gods, and any alteration would mean to move away from the perfect order and bring social chaos. To maintain itself in power, the centralized State always emphasized that its absence would signify hunger, disorder and civil war, as in fact it had occured in two chaotic periods in the history of Egypt.

The historical foundation for this religious belief in the *Golden Age* can be traced back to the first tribes to established themselves around the Nile, who had indeed a tremendous increase in their quality of life, allowing them to be unified in a great nation. Well, it was exactly during this time that the bases for the Egyptian mathematics were formed. Therefore, it was neither desirable nor necessary to alter this mathematical legacy. In this way, the immense Egyptian conservatism, which manifested itself so heavily in their institutions, arts and daily life, also prevented the evolution of their Mathematics, as well as any other science and, particularly, any kind of Philosophy. The burden of thinking, abstracting, pondering and transforming was long taken by the gods, who left to the Egyptian people of all posterior times the task of integrating themselves into the natural order of the Universe and extracting from it their ideal of happiness.

References

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