The Fields Institute: thematic program on Quantitative Finance: foundations and applications - January to June, 2010

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The Fields Institute. Credit: Andrea Yeomans.

Organizing Committee: Yacine Aït-Sahalia (Princeton), Matheus R. Grasselli (McMaster), Vicky Henderson (Oxford Man Institute), Tom Hurd (McMaster), Marcel Rindisbacher (Boston), Dan Rosen (R2 Financial Technologies)

We first had the idea to hold a thematic program on mathematical finance at the Fields Institute while attending a similar program at the Isaac Newton Institute in 2005. Back then financial markets were buoyant, all over the world, securitization was in full swing, investors and banks alike had a seemingly infinite appetite for sophisticated financial products, and the future of quantitative finance as a discipline appeared as bright as could be. With

its origins in the work of Louis Bachelier, who modeled prices in the French stock market and correctly derived several option prices in his 1900 thesis, *Théorie de la Spéculation*, quantitative finance became an identifiable subject with the works of Markowitz (1952) and Sharpe (1964) on portfolio allocation, Samuelson (1965) on stock price modeling using geometric Brownian motion, and Black-Scholes-Merton (1973) on option pricing. Its full mathematization occurred in the period between Harrison and Pliska's (1981) introduction of risk-neutral pricing using martingales and stochastic integrals and Delbaen and Schachermayer's (1994) functional analytic proof of the Fundamental Theorem of Asset Pricing. This period saw the banks increasing their use of sophisticated mathematics for utility maximization, risk management, and the modeling of interest rates, stochastic volatility and asset prices with jumps. The decade between two thematic

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programs at the Isaac Newton Institute in Cambridge – *Financial Mathematics* (1995) and *Developments in Quantitative Finance* (2005) – saw the area evolve into a mature academic subject, with the creation of the Bachelier Finance Society, countless masters programs around the world, influential research groups in leading universities and several dedicated journals.

Against this landscape as it appeared in 2005, we envisaged a thematic program that would, in the words of our original proposal submitted in September 2006, 'consolidate the entire field as much as two earlier programs held at the Isaac Newton Institute have defined the area'. We went on to say that 'the proposed design of the program harmonizes two time scales: the ferment and excitement of the latest financial products and techniques used in industry, and the more sedate, but still vigorous, development of rigorous publishable research on mathematical, statistical and computational finance'. As it turned out, the ferment and excitement exploded exactly two years later in the form of the worst financial crisis since the 1930s. With financial mathematics being partially blamed for the debacle, and massive layoffs hitting the quant community in industry, the future of the subject in general, and of our thematic program in particular, looked a bit bleak for a while. What happened next was nothing short of remarkable. In the spirit of not letting a good crisis go to waste, the events that took place in 2008 offered tremendous opportunities for an academic response from financial mathematics in terms of vigorous research in previously untapped areas. These include the limits of arbitrage, the relationship between liquidity and leverage, the importance of systemic risk and stability, the role of incentive structures, not to mention a mathematical theory for the formation and bursting of bubbles and financial crises themselves. These topics represent a substantial broadening of the scope of mathematical finance. None of them were on the horizon when we first imagined hosting a thematic program at Fields, but they figured prominently in the activities that ended up taking place in it.

Over the course of the six months starting in January 2010, we hosted four workshops, two seminar series, three public lectures series, and five two-day industrial-academic forums. Several senior researchers participated in the program as long-term visitors, including Nizar Touzi as the University of Toronto Dean's Distinguished Visitor, who participated in most of the activities and was also the instructor for one of the four graduate courses. To bring the program to a conclusion, the Fields Institute hosted the International Congress on Insurance: Mathematics and Economics and the 6th World Congress of the Bachelier Finance Society, both having to accommodate the heightened security measures put in place for the G8 and G20 meetings, which were certainly not part of our original plan, but nevertheless contributed to the general sense of excitement.

In what follows, we give a personal overview of all the activities. Rather than being exhaustive, we use our somewhat privileged perspective as local organizers to present our own 'view from the trenches', highlighting what in our opinion were the most significant contributions in terms of depth, timeliness and relevance to current issues. With the exception of the public lecture series, which we review individually, we focus on topics and threads rather than particular talks. The full list of speakers, together with the titles, abstracts, slides and audio recording for all the talks, are freely available at the Fields Institute website.

An aspect of the program that is easily overlooked in a catalogue of workshops and seminars is the quiet time between events, when visitors, postdocs and students push their personal research projects further step by painstaking step. It is a time for socializing, eating out, drinking coffee and working together at the blackboard. Perhaps in hindsight we will realize that this time of exploring new ideas with others from around the world was the best part of the program.

A. Workshops

The four workshops conducted during the program were the periods of greatest intensity of activities, each running at full capacity with large audiences attending the talks by leading international researchers. Their general topics reflect the broad scope of the program, ranging from purely mathematical aspects of the subject to computational and statistical methods, culminating with cutting edge applications to financial derivatives and risk management.

The workshop on *Foundations of Mathematical Finance* (January 11–15), organized by Jaksa Cvitanic, Dmitri Kramkov, Marco Frittelli, Martin Schweizer and Nizar Touzi, opened the Thematic Program with full attendance of more than 130 researchers in mathematical finance, including 25 invited speakers. It set the tone for the semester by presenting the state-of-the-art in rigorous mathematical results pertaining to a broad range of foundational aspects of the subject.

The talks ranged from traditional topics with modern developments, such as dynamic risk measures and bubbles (Hans Föllmer), quasiconvex risk measures (Marco Frittelli) and portfolio optimization with forward utilities (Nicole El Karoui, Thaleia Zariphopoulou), to truly novel subjects, such as incentive structures for hedge funds (Paolo Guasoni), liquidity constraints (Mihai Sirbu, Peter Bank, Dmitry Kramkov), and executives with insider information (Mike Monoyios).

The dominant theme for the workshop ended up being general equilibrium with heterogeneous agents, explored in several different contexts (Kasper Larsen, Jaksa Cvitanic, Gordan Zitkovic, Kostas Kardaras). This shows that, as a community, Mathematical Finance has evolved from arbitrage pricing and risk preferences for individual agents (risk measures, utility, indifference price etc) and is finally arriving at the task of understanding how prices are formed in markets with multiple agents facing different types of constraints, setting our subject on course for exciting and unexplored frontiers.

The workshop on *Computational Methods in Finance* (March 22–24), organized by Mark Broadie, Paul

M. Schweizer, M. Grasselli, T. Hurd, D. Kramkov, M.N. Touzi and M. Frittelli. Credit: The Fields Institute.

Glasserman, David Saunders and Stathis Tompaidis, saw leading experts gather in Toronto to discuss recent advances in computational methods, computer hardware, and their application to financial problems. Solving problems in Finance using computational methods has been a principal reason for the existence of many quantitative finance positions in the financial industry. From algorithms for trading, pricing complicated financial instruments, choosing optimal portfolios, and estimating the exposure of a portfolio to different financial risk factors, the field has proved to be a constant source of innovation.

One stream of talks was devoted to recent advances in well established areas of computational finance, including binomial methods (Ralf Korn), free-boundary problems (Kumar Muthuraman, John Chadam), convex optimization (Lorenzo Garlappi, Chris Rogers) and large scale Monte Carlo simulation (Jeremy Staum, Mike Giles). By contrast, a separate stream focused on algorithmic trading (Ciamac Moallemi, Jim Gatheral, Petter Kolm), which in the words of Jim Gatheral, has employed the talents and energy of quants ever since exotic derivatives were put temporarily out of fashion by the recent market turmoil.

Speaking of turmoil, a panel consisting of Jim Gatheral, Chris Rogers, Ernst Eberlein and Jeremy Staum discussed current challenges in the light of the 2008 crisis. Many interesting suggestions of problems were given, such as optimal liquidation strategies, the effect of latency on trading, dark markets, behavioral finance, liquidity, systemic risk management, interactions between financial institutions, contagion, robust techniques, aggregation of errors at the portfolio level, and modeling appropriate objective functions for institutions

over long time horizons. In a memorable exchange, Ernst Eberlein lamented the use of Gaussian copula models for CDOs, followed by Chris Rogers saying 'I enjoyed Ernst's rant against the Gaussian copula. I think you are kicking a corpse'.

Another highlight of the workshop was the informal presentation by Mike Giles, who gave an overview of the use of GPUs (graphic processing units) in quantitative finance. These have tremendous potential in speeding up computer code that can be executed in parallel, but imply additional effort in software development. It was highly entertaining to hear that banks in Canary Wharf have exhausted the electricity capacity available, with any new power generation being committed to the 2012 Olympic Games, so that they face the choice of either migrating to less power-hungry GPUs – and having to rewrite all their code – or building new data centers on the periphery of London.

The third workshop, entitled *Financial Econometrics* (April 23–24) and organized by Yacine Aït-Sahalia, Jianqing Fan, and Per Mykland, saw a full house of participants who listened to a very busy program of 31 20-minute talks, plus the added bonus of Darrell Duffie's masterful third and final lecture on dark markets. Overall, it was somehow satisfying to learn that after all these centuries of trading, there is a huge amount still to learn about the basic statistical properties of asset returns. Jumps need to be detected, identified and their distribution understood (Suzanne Lee, Jia Li, Viktor Todorov, Giovanni Urga). Noise must be identified and removed from the observations (Cecilia Mancini). The clustering of volatility, skewness, and the leverage effect are still difficult to measure in the data, and capture in a model (Jean Jacod, Yingying Li, Nour Meddahi, Roberto Reno,

George Tauchen, Allan Timmerman, Roman Valkanov, Dacheng Xiu). Correlations are as difficult to capture as ever (Jianqing Fan, Kaddour Hadri, Eckhard Platen).

The hydra of systemic risk raised its heads in this workshop too: Nobel Laureate Robert Engle asked the question of how data from normal periods in the market can be used to predict the behaviour of assets under stressed market conditions, while Yacine Aït-Sahalia proposed a jump model for the contagion events we have been observing. The foundations of statistical inference are being strengthened by new applications in econometrics: this was illustrated in the talks by Lars Hansen, Stan Hurn, Gael Martin, Joon Park, Paul Schneider, Osnat Straumer, and Zhibiao Zhao. Finally Andrew Lo suggested that understanding of the levels of uncertainty underlying finance can even change our view of physics. All in all, a lot to absorb in two days!

The last of four workshops of the program, on **Financial Derivatives and Risk Management** (May 24–28) and organized by Peter Carr, Rama Cont, Darrell Duffie, Lane Hughston, and Roger Lee, was intended to give a very broad perspective on the current state of quantitative finance research. It showcased an exciting slate of 28 speakers each giving 50 minute presentations, and true to its mandate, the topics they covered gave a snapshot of the diversity of the field. One popular topic, volatility, was seen from a variety of viewpoints: volatility options (Roger Lee, Giuseppe Di Graziano, Thomas Kokholm), the relation between spot and implied volatility (Lorenzo Bergomi, Jim Gatheral), no arbitrage constraints (David Hobson, Alexsandar Mijatovic, Jan Obloj), and models with bubbles (Johan Tysk). Several speakers proposed mathematical frameworks for asset pricing: Lane Hughston, whose new framework highlighted the dynamics of conditional probability densities; Andrea Macrina who modeled the pricing kernel under partial information about economic factors; and Dorje Brody who produced positive interest rate models driven by Lévy processes. Credit risk proved again to be an important topic, with a focus on counterparty risk by Tomas Bielecki, and Monique Jeanblanc's talk on generalizing the reduced form framework. In the same area, Marek Rutkowski talked about forward CDS spreads and Thorsten Schmidt described a top-down approach to CDO valuation. Talks in other areas included Rama Cont's account of Dupire's remarkable functional Ito calculus, whereas Andreea Minca modeled real financial networks and their susceptibility to default cascades. Gustavo Manso studied a Boltzmann equation describing information percolation in a large market. Stuart Turnbull gave a hardhitting view of how to avoid past mistakes in managing the risk of new complex financial instruments. Some talks explored more mathematical questions. Steve Shreve showed how diffusions can be chosen to mimic general Ito processes and this idea can be used to price a variety of exotic derivatives. Nick Bingham described the properties and uses of multivariate elliptic processes. Mark Davis gave some general results on stochastic DEs with state-dependent jump measures while Martijn Pistorius presented results

on pricing barrier options in Markov models. Finally a special mention must be made to Goran Peskir's beautifully mathematical tour of some new results on the Legendre transform, and how these apply to derivative pricing.

B. Seminar Series

Ongoing throughout the program, the **Visitor Seminar Series** met weekly for either one or two research seminars. The series, chaired by Alexey Kuznetsov, gave long and short term program participants an opportunity both to hear and be heard from their colleagues. Amongst the twenty speakers who gave a total of twenty two talks, there were senior international researchers, plus all the postdoctoral fellows connected with the program. While the breadth of the research was impressive, touching on risk measures (Klaas Schulze, Xianhua Peng), computational methods (Vladimir Surkov, Arash Fahim, Jean-François Renaud), stochastic expansions (Anke Wiese), insurance (Tom Salisbury) and probabilistic analysis (Alexander Kreinin), it is also interesting to highlight certain themes that cropped up regularly. One theme addressed by several speakers (Andreas Kyprianou, Alexey Kuznetsov, Pavel Gapeev) was the fine properties of Levy processes of the type now popular in financial modeling. Amongst the new things we learned about was how to compute first passage and related probabilities in some quite general cases. Another theme highlighted was asset correlation and multidimensional time series (Fouad Marri). This was discussed in the context of credit risk by Peng and Lung Kwan Tsui, and in the context of statistical inference and learning by Eddie Ng, while Luis Seco motivated the need for stochastic correlation models. A third dominant theme, stochastic volatility, arose strongly in talks by Hao Xing, Vladimir Vinogradov, Rudra Jena and Joe Campolieti.

The longstanding **Quantitative Finance Seminar Series** continued to run as usual with pairs of speakers on the last Wednesday of each month, throughout the thematic program from January to June. We were very fortunate to have an elite list of international speakers who told us about the latest theoretical and practical developments in the subject. Eckhard Platen gave a talk on how in the benchmark approach to valuation, the long-term excess return on equity is taken into account, and therefore the pricing of long term contracts is a paradigm shift away from the classic no-arbitrage framework. The meaning of 'long term' is relative to a time scale of about 20 years, and thus his approach is especially relevant to pension planning and insurance problems. Raphael Douady's talk introduced a new risk concept he calls StressVaR, or stress value at risk, which he has developed to evaluate the risk of hedge funds, but appears to be applicable to a wide range of investments. Its aim is to use the fairly short time series normally available in these asset classes, and to identify from it the most relevant risk factors. Dilip Madan spoke about the consequences of what he terms the taxpayer put, which in effect delivers a free put option

in the hands of a firm's debtholders, at the expense of the economy at large. He argued that social costs like this should be valued using a method of probability distortion, and his computations of the taxpayer put value for the six major US banks at the end of 2008 offered a snapshot of the financial system during the crisis. Stan Uryasev's talk revolved around what he called the risk quadrangle, with corners labelled by risk, deviation, regret and error and arrows between them representing mathematical procedures such as maximization. He illustrated how this works with the concept of conditional value-at-risk as an alternative to the industry standard value-at-risk.

Jorge Sobehart talked in April about limitations of rational models of both credit and equity markets and proposed an alternative based on behavior of real investors who are observed to depart significantly from strict rationality. For example, their tendency to follow trends can lead to market overreaction creating positive feedback and potentially unstable dynamics for stock prices such as bubbles and crashes. The final seminar session featured Freddy Delbaen and Mete Soner. Prof. Delbaen gave an insider view of the theory of time consistent dynamic convex risk measures (he is after all a co-inventor of this concept) and their relation to backward stochastic differential equations. He argued that this theory can lead to improved rules for capital requirements for financial institutions that encourage desirable properties such as diversification and the proper accounting for the time aspect of uncertainty. Mete Soner introduced a class of asset price models with uncertain volatility and demonstrated how use of second order BSDEs and the G-expectations introduced by Denis and Martini can solve the difficult problem of hedging and pricing of derivatives.

C. Public Lecture Series

Every thematic program at Fields has the opportunity to host either a Coxeter Lecture Series or a Distinguished Lecture Series, or in some cases, both. Our thematic program was fortunate enough to host both of these prestigious lecture series, plus in addition a special public lecture on the Mathematics and the Social Sciences. These three events, all held in the month of April, provided us with a rare glimpse at the achievements and personalities of three giants of our field.

The *Coxeter Lecture Series* (April 6–8) comprised three one-hour lectures by Nicole El Karoui, Professor of Applied Mathematics from Ecole Polytechnique in Paris. Prof. El Karoui is one of the founders of modern mathematical finance, both theoretical and the way it is practiced in industry, and her talks revolved around the theory and practice of backward stochastic differential equations. Prof El Karoui's talks started by showing how these structures turn out to arise at the heart of many applications to finance, stochastic control and PDEs. Her second lecture gave a detailed understanding of the main theoretical developments of BSDEs. In the third and final lecture, after completing the review of theory and numerical methods, she returned to applications, in particular the role of BSDEs in the concept of time consistent dynamic risk measures.
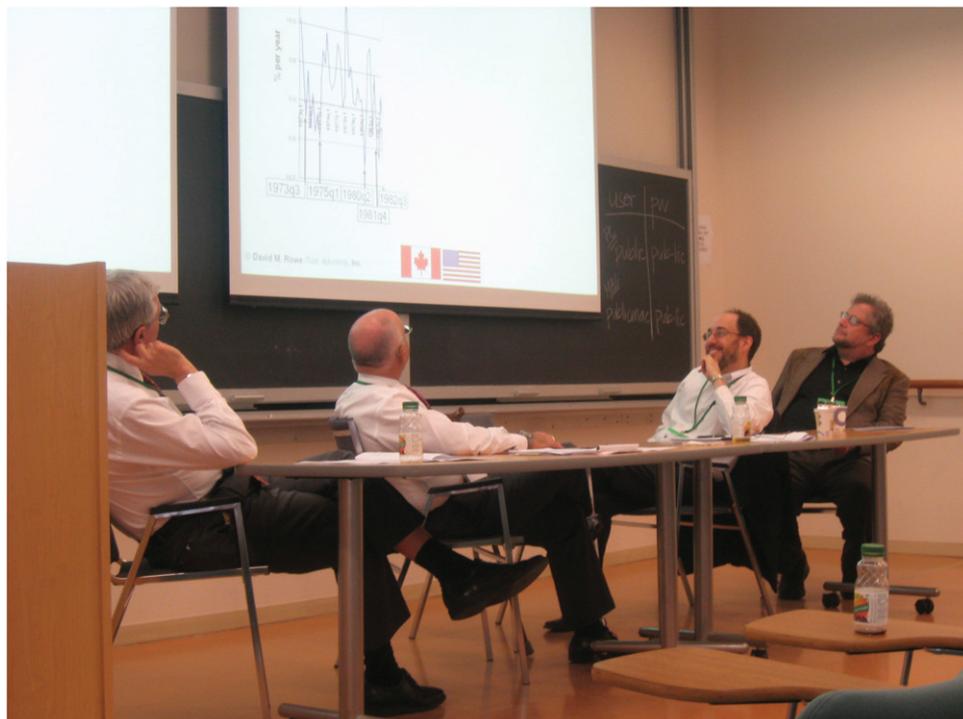
F. Milne, J. Langsam, M. Gordy and A. Brown. Credit: Richard Cerezo.

Robert Merton, the John and Natty McArthur University Professor at Harvard Business School, delivered the Nathan and Beatrice Keyfitz Lecture in Mathematics and the Social Sciences on 'Observations on the Science of Finance in the Practice of Finance: Past, Present, and Future' to a delighted audience of about 300 people who packed the large auditorium at the Bahen Center of the University of Toronto. He touched on many important aspects of the financial crisis, including the structural challenges posed by the intrinsic 'put option' embedded in any risky loan, the role of composition when seemingly prudent actions taken by individuals in isolation lead to large systemic risks, and the limitations of mathematical models, which in his view cannot be judged separately from their users and applications. Contrary to populist clamors for 'common sense', he said that the crisis accentuates the need for *more* quantitative research in finance, since none of these problems will go away by a magical return to 'simplicity' in the financial world.

Apart from the lecture itself, we had the privilege to host him at Fields during the afternoon and to take him out for dinner afterward together with other distinguished guests associated with the Institute. We were unanimously impressed by how engaging he was, not only by being ready to share personal experiences, some very difficult, from the past 40 years, but also being genuinely interested in the research ideas that we timidly put forward for discussion.

Distinguished Lecture Series
April 21–23, 2010

Darrell Duffie, the Dean Witter Distinguished Professor of Finance at The Graduate School of Business, Stanford University, dedicated his Distinguished Lecture Series to the subject of *Dark Markets*.

In the first lecture he described how over-the-counter (OTC) markets differ from centralized ones, in particular with respect to the transfer of capital, which tend to be slower in the former, resulting in asset prices which can show a persistent deviation from 'fundamentals'. He also observed that prices for the same asset at the same time can show a large dispersion, since agents trade bilaterally, with no access to information that can reveal a unique 'fair' price at the time of trade. By way of examples, he showed intriguing evidence from the time signature of prices for treasury bonds (that is, how they vary in time near the moment of issuance), as well as cross sectional dispersion in prices. Towards the end of the lecture, he commented on the benefits of clearing houses for derivative contracts.

Having laid the intuition for OTC markets, Prof. Duffie used his second lecture to explain an idealized mathematical model for a continuum of agents meeting for bilateral trades at random times according to a given intensity rate. Through a heavy use of infinite population, the law of large numbers, and independence, he was able to derive an evolution equation (a version of the Boltzman equation) for the 'types' of agents in the population. Since at equilibrium bids and types are in a one-to-one correspondence, this evolution equation describes how information 'percolates' in the population through an infinite series of double auctions.

In the third and final lecture, he focused on the fascinatingly intricate human dynamics of the daily interbank market for Fed Funds. Using a statistical model to describe the probability of a transaction occurring between two banks, he fit a data set comprising 225 million observations of Fed Funds transactions amongst 8000 banks in 2005.

By coincidence, the lectures started on the exact same day that a bipartisan bill on derivatives was introduced to the US Senate floor. As a result, Prof. Duffie divided his time in Toronto between being interviewed by several news outlets, speaking on the phone with government economic advisers and laying down before our eyes the theoretical framework underpinning the type of markets for which exotic derivatives are but one example.

D. Industrial-Academic Forums

These two-day meetings brought together practitioners and academics working in topics of emerging interest in the financial industry. The format we adopted usually consisted of longer plenary talks by established experts, followed by shorter more speculative talks from industry and academia. Each forum hosted a panel discussion where the latest trends and developments in each fields were lively debated.

Kicking off the series, the timely and productive forum on **Operational Risk** (March 26–27), organized by Santiago Carrillo-Menendez, offered an overview of a subject gaining in prominence in this post-crisis period. Among many lessons, we learned that because banks don't make money out of operational risk, as opposed to market and credit risk related to core commercial activities where a lot of money can be made by beating the competition, research in operational risk has been mostly driven by regulation rather than market pressure (Giulio Mignola, Tony Peccia). As a consequence, most effort focuses on how to calculate the exact quantity specified by Basel II, namely VaR 99.9% of operational risk losses using the Loss Distribution Approach. This consists of modeling the losses in each business line as compound Poisson processes, that is modeling the frequency and the severity of the losses separately, and then bringing business lines together under some dependence assumption, say copulas (Elise Gourier). Clever numerical methods (Jörg Fritscher) and analytic approximations (Anupam Sahay) then compete to produce the final shape of the loss distribution and calculate its VaR 99.9%.

Because it is extremely hard to estimate the parameters of heavy-tailed distributions (Beatriz Santa Cruz Blanco, Alberto Suarez), regulators, and just about everyone else, have their work cut out for them in trying to validate the models used by different banks (Emre Balta). Many participants observed that it would be better to have a way to model operational risk from first principles

(Martin Neil), similar to a structural model in credit risk, rather than trying to fit highly unstable statistical models to largely unavailable data.

In general, participants seemed to agree that operational risk research should aim for better risk management rather than better risk measurement. As one participant put it: 'operational risk groups should be seen more like think-tanks instead of data centres'.

The second forum in the series was organized by Rene Carmona and Matt Davison and dedicated to **Commodities, Energy Markets, and Emissions Trading** (April 9–10), one of the most exciting areas for the application of quantitative finance techniques outside the traditional realm of equity and fixed income markets.

The reasons for the excitement were thoroughly discussed in the forum. For some examples, we learned that commodity markets are fertile ground for exotic derivatives because physical constraints strongly influence the design and valuation of even the most basic contracts (Alex Eydeland). In addition, different players in the market, say producers and retailers, have different likes and dislikes, so the story behind the change of measure from physical to risk-neutral probabilities is an elaborate one (Alvaro Cartea). Enforcing consistency constraints while modeling and fitting implied volatility curves and surfaces across time is as difficult a problem for commodities as it is for interest rates, and perhaps might benefit from common techniques (Ehud Ronn).

On the other topics of the forum, we learned that there exist reasonably advanced models for carbon emission markets and they perform well when compared to actual data for the first phase of the European Union Energy Trading System (Gilles Edouard Espinosa, Georg Gruell). On the other hand, a lot needs to be done to model newer features in these markets, such as multiple periods, banking of allowances, and Certificates of Emission Reduction (Marliese Uhrig-Homburg). Because electricity markets are by now quite well developed and understood both in theory and practice, linking them to carbon emission markets in a unified way might shed some light on their structure (Michael Coulon).

The forum concluded with an extremely lively panel, sadly not available in audio recording. Apart from some discussion on the benefits and flaws of carbon emission markets, what really caught the attention of most panelists and people in the audience was the relation between academia and industry. The highlight was Nicole El Karoui standing up at the back of the room and delivering a passionate defense (in French) of the role of independence of academic research. She said that although she learned important lessons from the markets (for example the importance of model robustness, something that academics seldom think about), we need to keep a healthy distance and reject the notion that markets are always right.

The third forum in the series, entitled **Credit-Hybrid Risk** (April 15–16) and organized by Tom Bielecki, Stéphane Crépey and Monique Jeanblanc, focused on very recent topics in credit risk modeling, in particular its relationship with equity risk. A quick glance at the program reveals that counterparty risk in general, and Credit Value Adjustment in particular, was the prominent theme of the forum, followed closely by game options. This made for a very diverse forum, since the former is mostly grounded in practical day-to-day considerations for both banks and regulators, for example how to deal with so called 'wrong-way risk' (Agostino Capponi, Dan Rosen, Giovanni Cesari), whereas the latter is as theoretical as it gets. Game options (Jan Kallsen) include European and American options as special cases, and pricing and hedging them quickly leads one to think deeply about the limitations of arbitrage, replication, and utility-based approaches.

A third pillar for the forum was the joint modeling of equity and credit markets, which sits between the other two pillars in the theory-practice spectrum. A few subtly different frameworks were presented (Rafael Mendoza-Arriaga, Julien Turc, Tom Hurd), ultimately relying on a deeper understanding of the capital structure of a company.

The forum on **Systemic Stability and Liquidity** (May 17–18), organized by Michael Gordy and Mark Reesor, was one of the most anticipated activities in the thematic program, not only because it relates directly to the financial crisis, but because it brought to Fields a diverse group of speakers, including some renowned economists, who are hard to find in typical mathematics conference.

On the economics side, many talks showed how to model funding choices and diverse beliefs (Wei Xiong), liquidity provisions (Jennifer Huang, Frank Milne), self-fulfilling credit freezes (Itay Goldstein) and risk appetite (Jon Danielsson, Erkko Etula). More towards financial engineering, different ways to identify and measure systemic risk were proposed using network models (Rama Cont), hypothetical insurance analogies (Hao Zhou), default probabilities for large fractions of the banking system (Kay Giesecke), and the level of under-capitalization of a financial institution (Viral Acharya).

A panel at the end of the first day provided unique insights on key factors leading up to the crisis and a lively discussion on policy, legislation and general banking practices.

The last forum, called **Financial Engineering and Insurance Mathematics** (June 21) and organized by Huaxiong Huang, Moshe Milevsky and Tom Salisbury, allowed university researchers in the field of finance and insurance mathematics to interact with practitioners in the field. The morning session consisted of three high-profile plenary talks (Arthur Fliegelman, Stan Pliska, Robert R. Reitano) with ample time allocated to questions and answers as well as dialogue with the audience. The afternoon was used for short technical presentations by junior researchers, followed by feedback and discussion about directions for further research.

E. Graduate Courses

Graduate student activities are an important facet of the Fields Institute mandate. During the six months of the

program, we offered four advanced graduate courses that could be taken for credit by students from any of Fields' partner universities. These courses were each directed by a lead instructor, but in addition a number of guest lecturers were invited to give lectures on special topics. Each instructor has posted the lecture notes and exercises on the graduate course website for the program.

The course on *Foundations of Mathematical Finance* was conducted by Matheus Grasselli, with guest lecturers Eckhard Platen, Marco Frittelli and Stanley Pliska. This course focused on proving the fundamental theorems of asset pricing (FTAP) in various settings, starting with the discrete time/finite state case, and working up to the continuous time/continuous state setting, as well as reviewing the theory of utility optimization. Prof. Platen's lectures gave a comprehensive picture of the 'benchmark approach' to asset pricing, which provides a coherent picture of investment over time horizons as long as a lifetime rather than just months. Prof. Frittelli lectured on the problem of portfolio selection framed in the most general semimartingale setting. He developed the theory of convex risk measures and introduced an Orlicz space approach to utility maximization and indifference pricing. Prof. Pliska's lectures looked back over the history of the subject and recalled how the concepts of semimartingale and the equivalent martingale measure arose from a concerted effort to understand how developments in probability could used as the natural setting for arbitrage theory of asset pricing.

Tom Hurd conducted a course on *Interest Rates and Credit Risk*, which as the title suggests addresses the theory of fixed income securities, beginning with default free bonds, and then developing the theory of firm default, and the pricing of defaultable bonds and their associated derivatives. Tomas Björk and Kay Giesecke were guest lecturers. Prof Björk delivered a masterful survey of interest rate theory, including the Heath-Jarrow-Morton framework and Libor Market modelling. His lectures culminated in the classification of finite dimensional realizations of HJM models. Prof. Giesecke lectured on the topic of portfolio credit risk from the top down. He told us about possibilities for analytically tractable models of the portfolio loss process, and demonstrated exact simulation methods for the relevant point processes.

The University of Toronto Dean of Science Distinguished Visitorship, which provides funds to cover the prolonged stay of an internationally prominent researcher, was awarded to Nizar Touzi from Ecole Polytechnique in Paris. As part of the duties attached to this award, Prof Touzi delivered an advanced graduate course on *Stochastic control, BSDEs, and applications to finance* over a period of six weeks. These challenging lectures covered the foundations of stochastic control and backward stochastic differential equations that lie at the heart of rational decision making in finance. Prof. Touzi's notes for the course will be published as a volume in the Fields Institute Monograph series. Bruno Bouchard gave guest lectures on the theory of target control problems. Mete Soner gave a guest lecture on singular control and

its application to portfolio selection in models with transaction costs. Finally, Agnes Tourin, the Fields Research Immersion Fellow, gave a guest lecture on numerical schemes for nonlinear partial differential equations.

Dan Rosen of R2 Technology Inc, offered an intensive four day graduate course on *Advanced Risk Management Methods*. This covered such topics as economic and regulatory capital, counterparty credit risk, and capital allocation, risk contributions and risk aggregation.

F. IME and Bachelier Congresses

The Fields Institute partnered with two major scientific organizations to host two of the largest international conferences in the area of quantitative finance, both in the second half of June. Running both conferences involved intense logistics that were to a great extent handled by Fields staff members.

The *14th International Congress on Insurance: Mathematics and Economics* took place on the campus of University of Toronto on June 17–19, with logistical support from the Fields Institute. The IME Congress has been an annual event in the quantitative finance calendar since 1997 and is an important forum for researchers to focus on the latest advances, trends and developments and to bring together scientists, academicians and insurance sector professionals from all over the world in the area of actuarial science and insurance and financial modeling. The theory and methods of actuarial science, and its applications in insurance, finance and risk management fields form the core of the presentations at the Congress.

One of the ideas driving the original thematic program proposal way back at its outset in 2005 was that it would include the *6th World Congress of the Bachelier Finance Society*, due to occur in 2010. Indeed this was the original rationale for running the program in this year: the partnership between the Fields Institute and the Bachelier Finance Society was hoped to create a synergy with the whole greater than the sum of its parts. All this indeed came to pass, and the BFS World Congress that took place at the Hilton Hotel in Toronto from Tuesday June 22 to Saturday June 26 was both a major international scientific event and a fitting closure to the Fields Thematic program. The statistics of BFS 2010 are impressive: approximately 500 participants came from all parts of the globe to hear nearly 360 presented papers and posters, including 9 plenary talks and 30 invited theme talks. The bulk of the talks were in the format of submitted papers presented in six parallel sessions plus an additional 60 papers that were selected for poster presentation in a well attended session that occurred over two days.

The presentations at BFS World Congresses give a comprehensive slideshow of the state of the quantitative finance discipline at two year intervals. To have an impression of the current state of the subject in 2010, one cannot do better than to visit the congress website

www.bfs2010.com to see the range of talks presented and to read the abstracts and slides. The scientific content is subdivided into two dozen themes that include the central topics of interest rates, credit risk, options and futures, computational finance, portfolio optimization, plus some newer areas such as liquidity and systemic risk, insurance finance, commodities and real options. Many of the presentations reflected in some way or other on the causes and effects of the great financial crisis of 2007/08.

The venue at the Hilton Hotel in central Toronto worked out extremely well, offering excellent lunches and snacks at regular intervals, a well connected cluster of lecture halls for the six parallel sessions, plus great space for publishers and sponsors to display their wares. Fields staff, a team of motivated undergrad and grad students, media specialists, plus the helpful hotel staff worked together to handle all the logistics of this complex operation. This included some special difficulties, such as an earthquake on the second day, but chiefly the G20 summit that rather inconveniently shut down almost the entire downtown area for much of the week. G20 security measures included thousands of police and military personnel who made a tough stand against the thousands of protesters that arrived from around the globe to air their grievances with world leaders. All this happened in the blocks surrounding the Hilton during the week of the Congress. Indeed the final Saturday of the congress was conducted while the hotel was under 'lockdown' due to the confrontations happening in the streets. Thankfully all attendees were able to make their travel connections at the end of the congress, even if some skirted burning police cars in the street on their way to the airport.

The Bachelier Congress marked the end of the thematic program on Quantitative Finance, and long term participants and organizers didn't tarry long before leaving, many seeking a quieter place to recharge batteries and recall the numerous highlights from an intense half year.

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