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Creation, dissipation and recycling of resources in non-equilibrium systems. (English summary)

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The paper proposes to extend the stochastic treatment of nonequilibrium statistical mechanical systems to explicitly incorporate the coupling to external reservoirs.

It begins with the general formulation of stochastic dynamics for a system described by a discrete (and finite) sample space X (which the authors confusingly call the "state space", only to define later that a state of the system is really a probability distribution $p(x)$ on X, rather than a point $x \in X$ itself). The dynamics is assumed to occur also in discrete time and is implemented by a stochastic matrix R_{xy}, given the probability of a transition $y \to x$ during the time step δt. The usual concepts of detailed balance, current, entropy and stationary distributions are also reviewed.

The innovative step in the paper comes in the following section, where reservoirs are defined by their own sample spaces Ω_l and general transitions are defined to be maps from $X \times \prod_l \Omega_l$ to itself satisfying certain hypotheses. Apart from being consistent with conservation laws, a transition $\alpha\colon (y,(\eta_l)) \mapsto (x,(\xi_l))$ is assumed to be invertible and to satisfy the central condition of generalized detailed balance, namely

$$R_{xy}^{(\alpha)} \exp(s(y,(\eta_l))) = R_{yx}^{(\alpha^{-1})} \exp(s(x,(\xi_l))),$$

where $s(x,(\xi_l))$ is the entropy of the sample point $(x,(\xi_l))$ (not to be confused with the Shannon entropy $S(p)$, which is a function of the probability distribution). As we soon discover in the paper, this "microscopic entropy" is to be interpreted as "resources" of the system (or the reservoir). For instance, an energy reservoir Ω_l with inverse temperature β_l and discrete levels of energy e_j is assigned an entropy function $s_l(e_j) = \beta_l e_j$.

The essential point is that the coupling with the reservoirs and their different "resources" determines the (linear) stochastic dynamics of the system X itself, through the definition of the transition matrix

$$R_{xy} = \sum_{\alpha} R_{xy}^{(\alpha)},$$

complemented by $R_{xx} = 1 - \sum_{y \neq x} R_{yx}$.

The rest of the paper is dedicated to deducing several notions of dissipation derived from the currents induced by such dynamics, as well as proving lower bounds for them through inequalities relating the different entropy functions with the "information potential" $\Phi(x) = \log p_s(x)$, where p_s is a stationary distribution.

The admittedly general level of the statements and proofs, based on several assumptions which the authors take the care of criticizing along the way, is compensated by a richness of examples in the final section. They include reaction-diffusion systems, the Carnot engine, a rather detailed analysis of three-level systems and a two-spin system at different temperatures.

Reviewed by *M. R. Grasselli*