

EXERCISES

1. Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$, when

$$(a) u(x, y) = 2x(1 - y); \quad (b) u(x, y) = 2x - x^3 + 3xy^2;$$

$$(c) u(x, y) = \sinh x \sin y; \quad (d) u(x, y) = y/(x^2 + y^2).$$

$$\text{Ans. } (a) v(x, y) = x^2 - y^2 + 2y; \quad (b) v(x, y) = 2y - 3x^2y + y^3;$$

$$(c) v(x, y) = -\cosh x \cos y; \quad (d) v(x, y) = x/(x^2 + y^2).$$

2. Show that if v and V are harmonic conjugates of $u(x, y)$ in a domain D , then $v(x, y)$ and $V(x, y)$ can differ at most by an additive constant.