6. Verify that the function $u(r, \theta) = \ln r$ is harmonic in the domain $r > 0, 0 < \theta < 2\pi$ by showing that it satisfies the polar form of Laplace's equation, obtained in Exercise 5. Then use the technique in Example 5, Sec. 26, but involving the Cauchy–Riemann equations in polar form (Sec. 23), to derive the harmonic conjugate $v(r, \theta) = \theta$. (Compare with Exercise 6, Sec. 25.)

7. Let the function $f(z) = u(x, y) + iv(x, y)$ be analytic in a domain D, and consider the families of *level curves* $u(x, y) = c_1$ and $v(x, y) = c_2$, where c_1 and c_2 are arbitrary real constants. Prove that these families are orthogonal. More precisely, show that if $z_0 = (x_0, y_0)$ is a point in D which is common to two particular curves $u(x, y) = c_1$ and $v(x, y) = c_2$ and if $f'(z_0) \neq 0$, then the lines tangent to those curves at (x_0, y_0) are perpendicular.

   Suggestion: Note how it follows from the pair of equations $u(x, y) = c_1$ and $v(x, y) = c_2$ that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx} = 0 \quad \text{and} \quad \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{dy}{dx} = 0.$$

8. Show that when $f(z) = z^2$, the level curves $u(x, y) = c_1$ and $v(x, y) = c_2$ of the component functions are the hyperbolas indicated in Fig. 32. Note the orthogonality of the two families, described in Exercise 7. Observe that the curves $u(x, y) = 0$ and $v(x, y) = 0$ intersect at the origin but are not, however, orthogonal to each other. Why is this fact in agreement with the result in Exercise 7?

9. Sketch the families of level curves of the component functions u and v when $f(z) = 1/z$, and note the orthogonality described in Exercise 7.

10. Do Exercise 9 using polar coordinates.

1. Sketch the families of level curves of the component functions u and v when