Math 3X03 / Second assignment: due March 14th in class.

Problem # 1: 8th edition: page 171 # 4 / 9th edition: page 170 # 4

Problem # 2: Suppose that D is a bounded domain whose boundary is a simple closed contour $\partial D = C$, and that $f(z)$ is analytic on $D \cup C$.
(a) Show the following "isoperimetric" inequality:

$$\sup_{z\in C} |\bar{z} - f(z)| \geq 2\frac{\text{Area}(D)}{\text{Length}(C)}$$

[Hint: Consider $\int_C (\bar{z} - f(z))\, dz$, and use the estimate on the modulus of a contour integral and exercise #7 page 163 (8th ed)/page 161 (9th ed) (done in the tutorial).]

(b) Show that when D is the unit disk, then there is an analytic function $f(z)$ for which equality holds,

$$\sup_{z\in C} |\bar{z} - f(z)| = 2\frac{\text{Area}(D)}{\text{Length}(C)}$$

Problem # 3: Page 179 (8th ed) #8/ page 178 (9th ed) # 7

Extra problem to work on alongside this assignment. It will be presented in tutorials (but not graded.)

Problem # 4: Suppose that $f(z)$ is entire and is doubly periodic in the sense that $f(z+1) = f(z+i) = f(z), \forall z \in \mathbb{C}$. Show that f is constant.
[Hint: first show that f is bounded in the closed unit square.]