Math 4A3 / practice pbs on Fourier series

1. Assume that $f \in C^{2\pi} \cap C^1(\mathbb{R})$, with $\displaystyle\int_{-\pi}^{\pi} f(x)\,dx = 0$. Show that

$$\int_{-\pi}^{\pi} [f(x)]^2\,dx \le \int_{-\pi}^{\pi} [f'(x)]^2\,dx,$$

with equality if and only if $f(x) = a\cos x + b\sin x$ for a, b constant.

[Hint: Find the Fourier Series for f' in terms of the coefficients for f, and use Parseval's identity.]

2. Assume that f is 2π-periodic, and continuous on $[-\pi, 0)\cup(0, \pi]$, with a jump discontinuity at $x = 0$,

$$f(0-) = \lim_{x\to 0^-} f(x) \ne \lim_{x\to 0^+} f(x) = f(0+).$$

With f as above, prove that the Cesaro sums of the Fourier Series of f converge to the half-way point of the jump discontinuity:

$$\lim_{n\to\infty} \sigma_n(f)(0) = \frac{1}{\pi} \lim_{n\to\infty} \int_{-\pi}^{\pi} f(t)K_n(t)\,dt = \frac{1}{2}(f(0-) + f(0+)).$$

[Hint: K_n is even, so $\int_0^{\pi} K_n(t)\,dt = \frac{\pi}{2} = \int_{-\pi}^{0} K_n(t)\,dt$. Go back over the proof of Theorem 15.7 with this in mind.]

From the text:

p. 245 # 1, 2; p. 250, # 7;