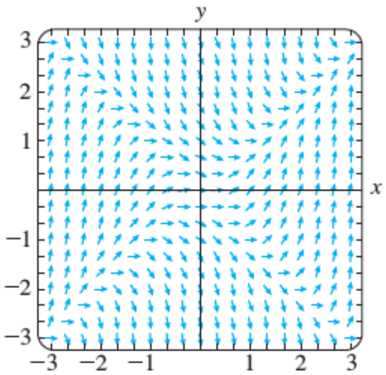
Math 2C03 Practice problem set #2 Jan2021 (18369946)

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

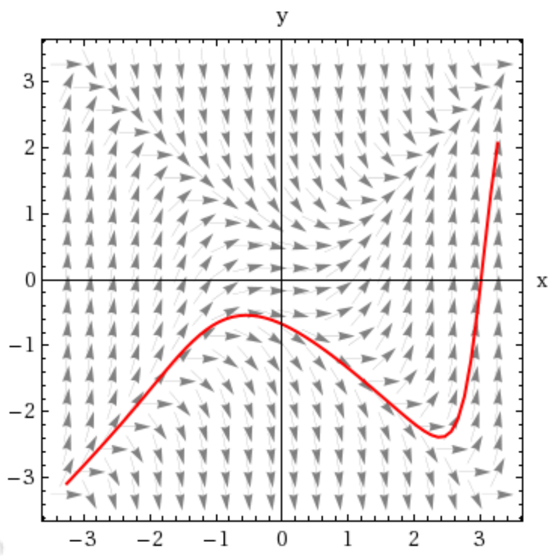
1. Question Details ZillDiffEQ9 2.1.001. [3876604]

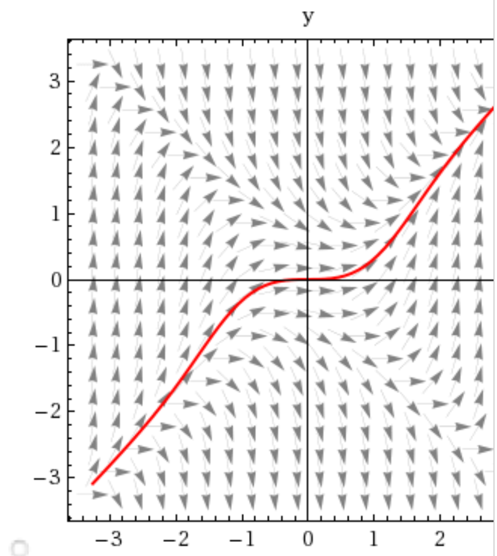
Reproduce the given computer-generated direction field. Then sketch an approximate solution curve that passes through each of the indicated points.

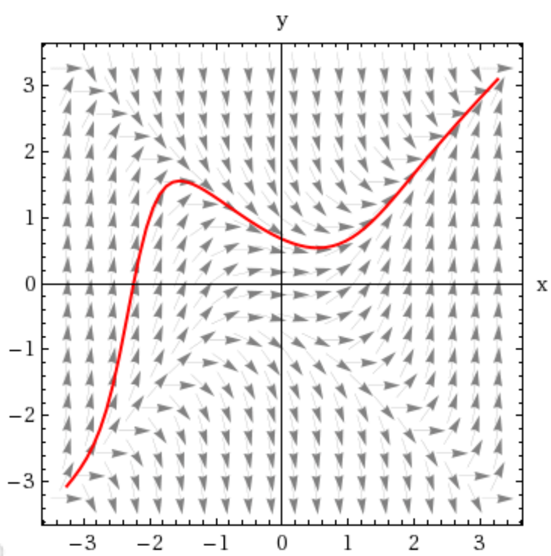
$$\frac{dy}{dx} = x^2 - y^2$$

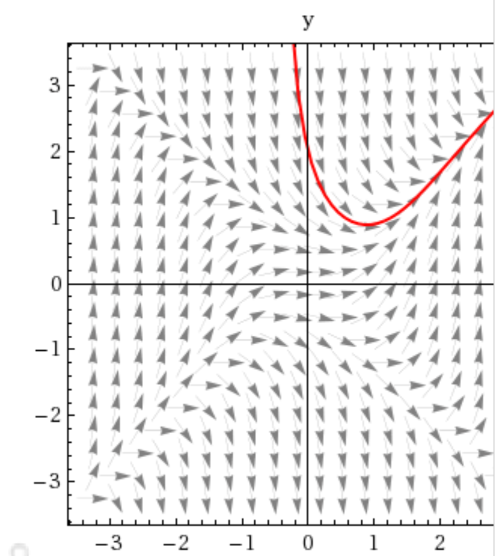


(a) $y(-2) = 1$

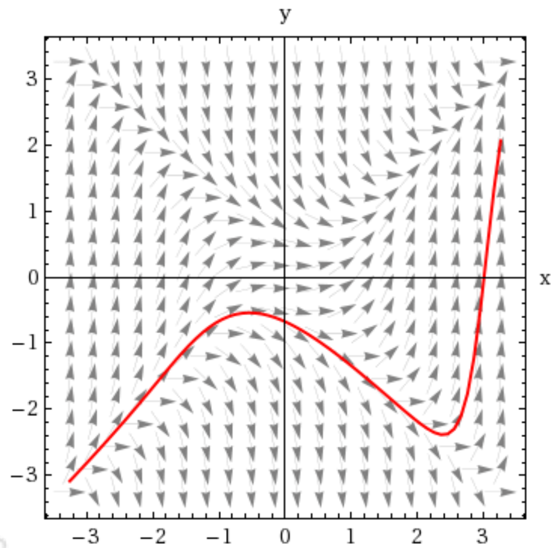


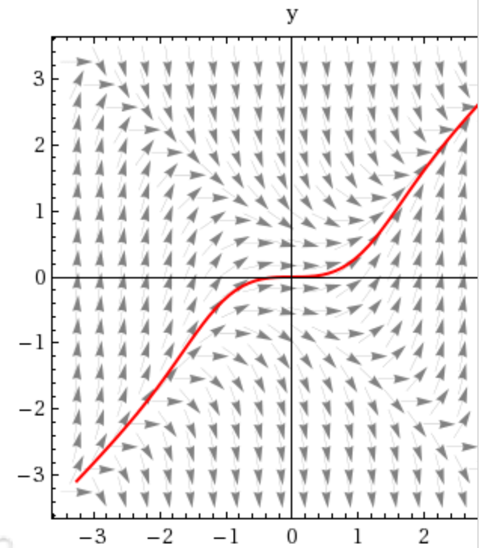


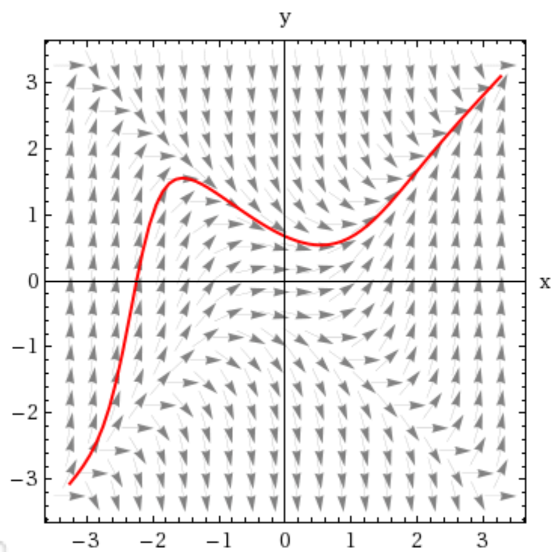


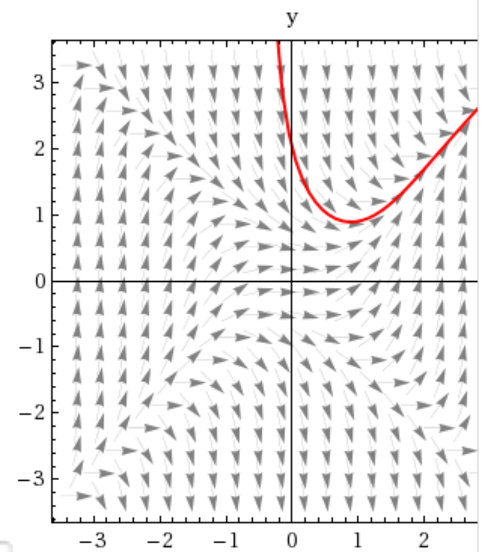


(b) $y(3) = 0$

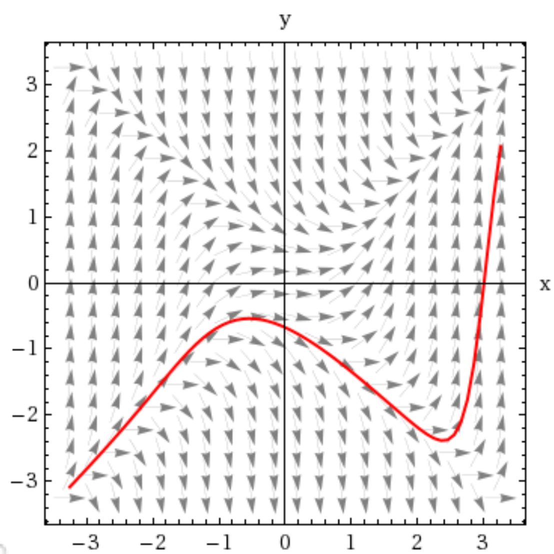


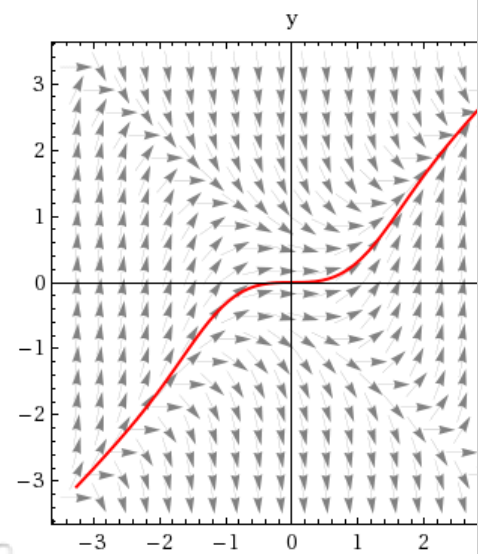


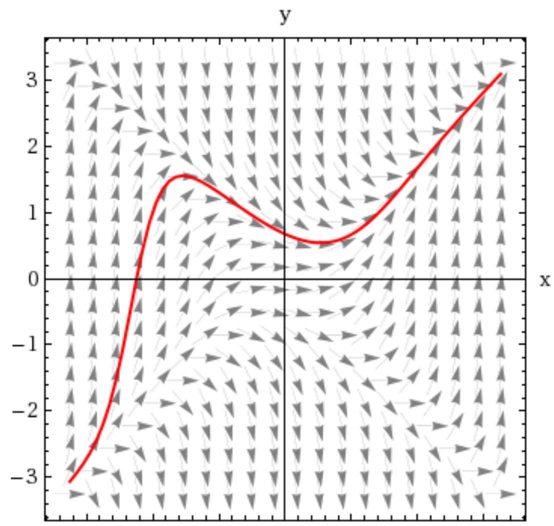


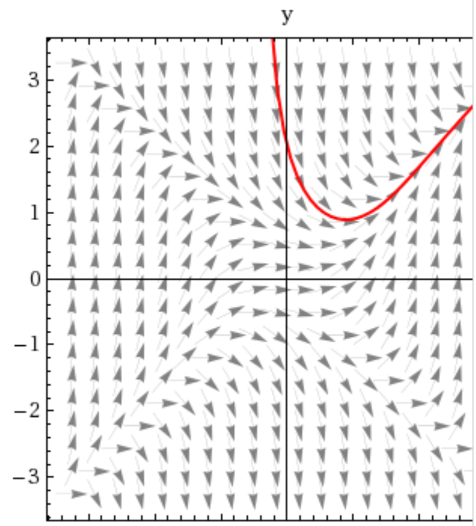


(c) $y(0) = 2$

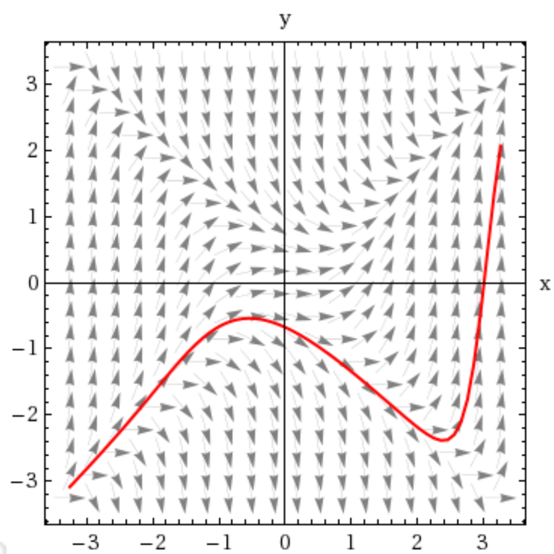


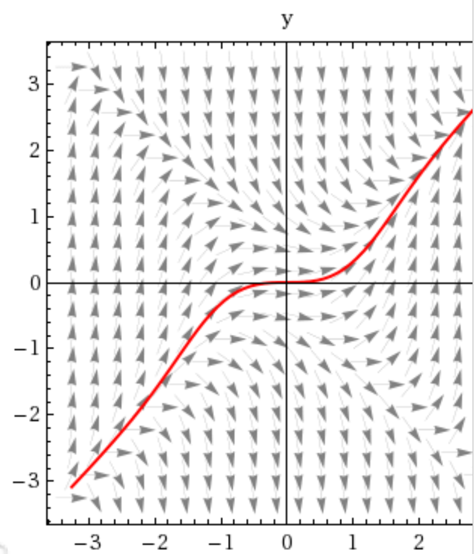


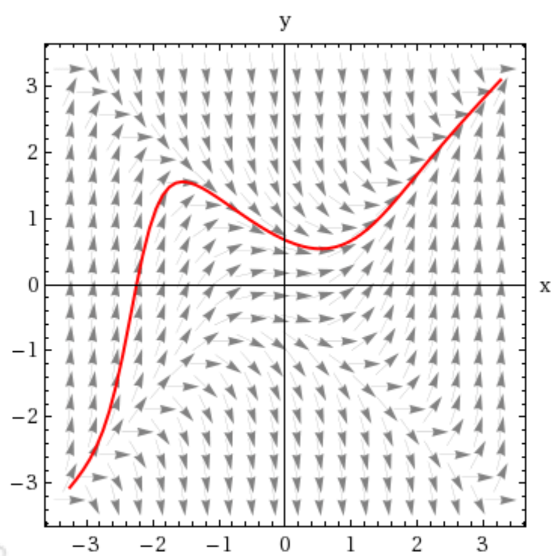


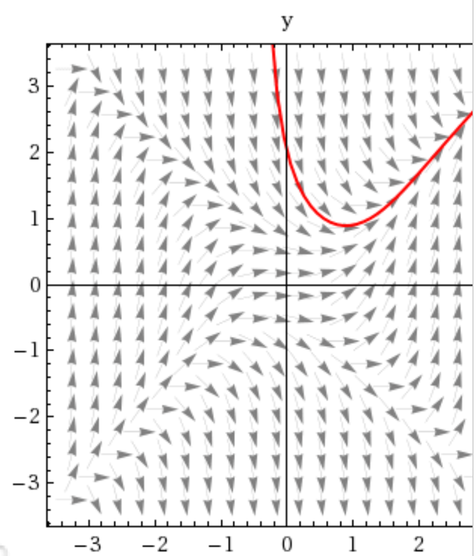


(d) $y(0) = 0$







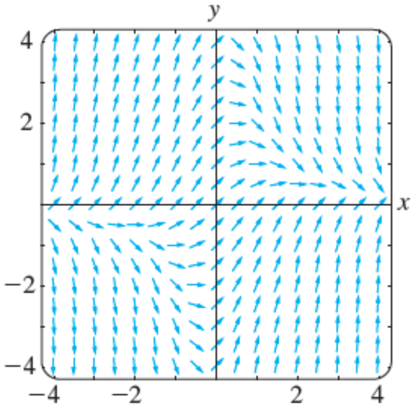


Need Help? Read It

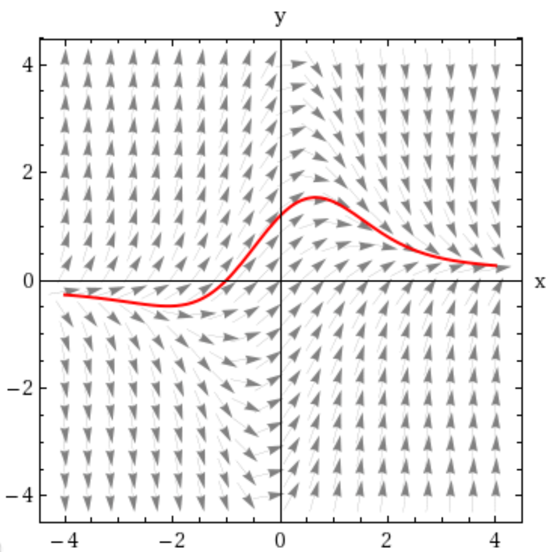
2. Question Details ZillDiffEQ9 2.1.003. [3876592]

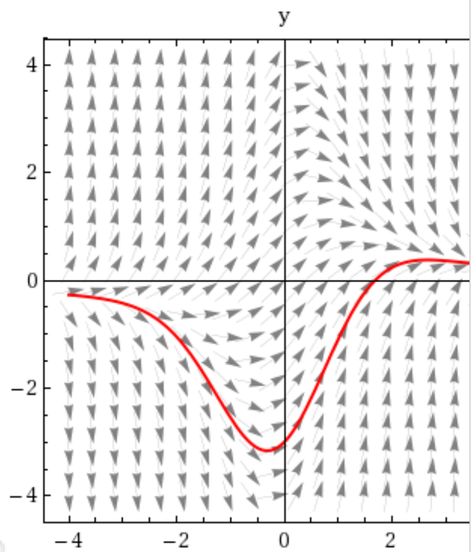
Reproduce the given computer-generated direction field. Then sketch an approximate solution curve that passes through each of the indicated points.

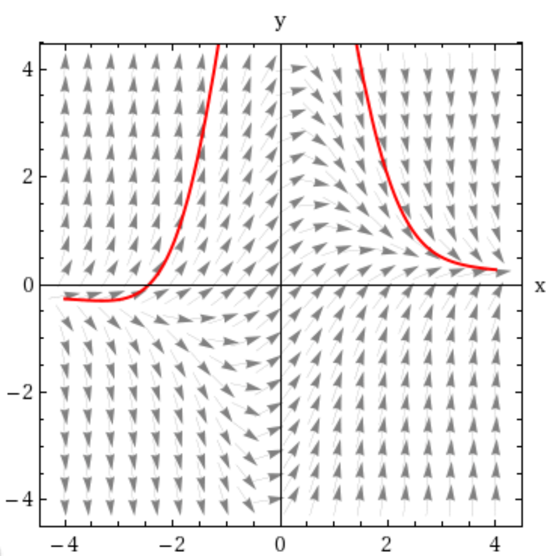
$$\frac{dy}{dx} = 1 - xy$$

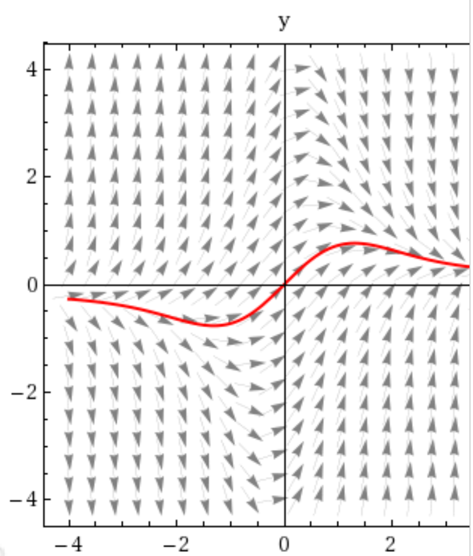


(a) $y(0) = 0$

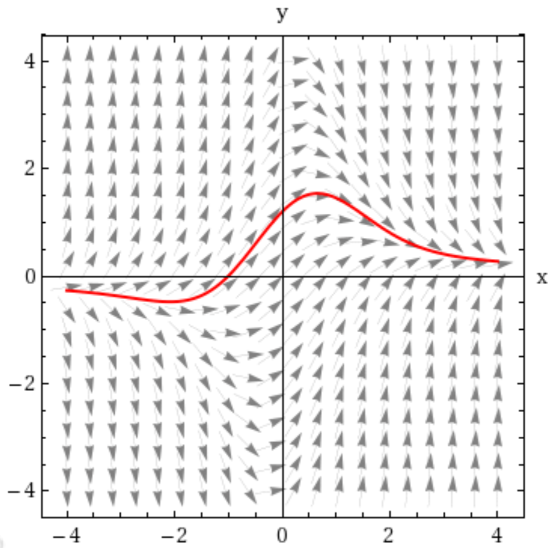








(b) $y(-1) = 0$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x
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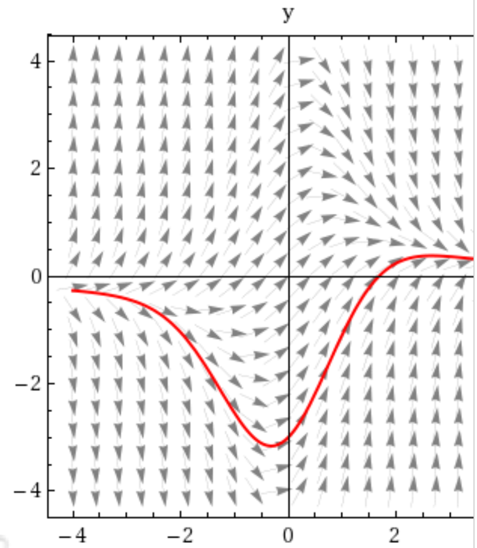
−2
−4
−4
−2
0
2
4


y
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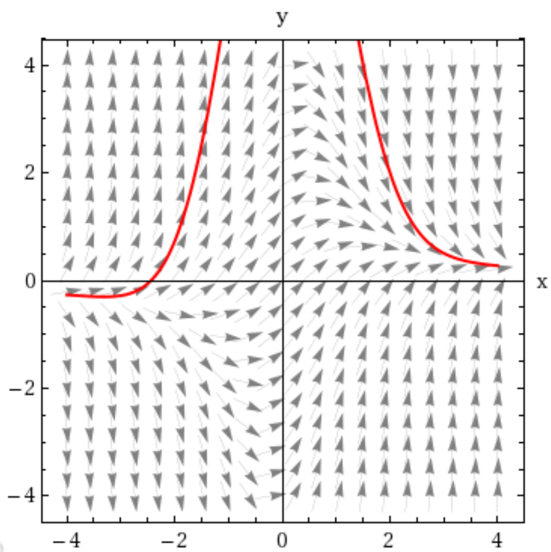
−2
−4
−4
−2
0
2


y
x
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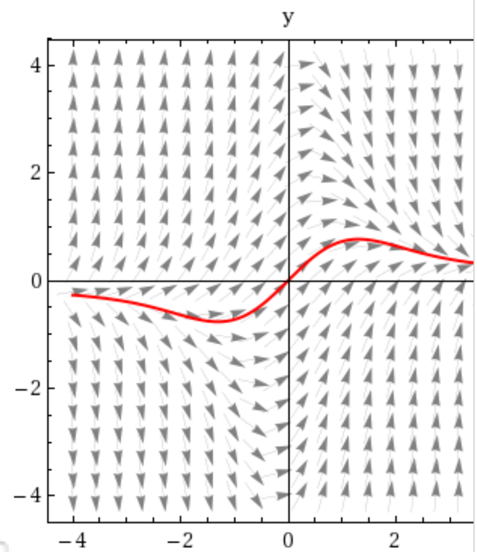
2
0
−2
−4
−4
−2
0
2
4


y
4
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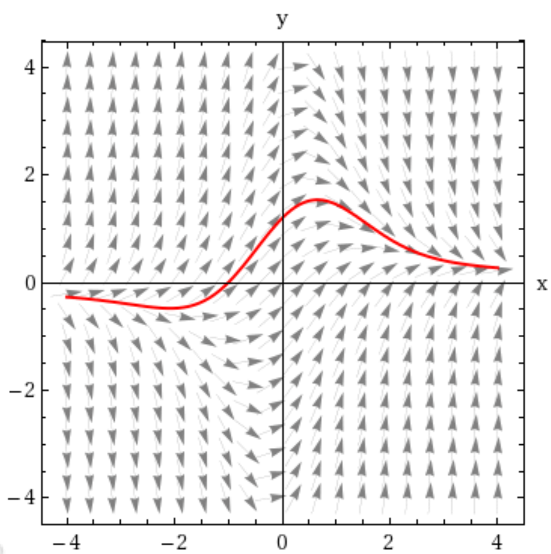
−2
−4
−4
−2
0
2

(c) $y(2) = 2$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x
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0
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−4
−4
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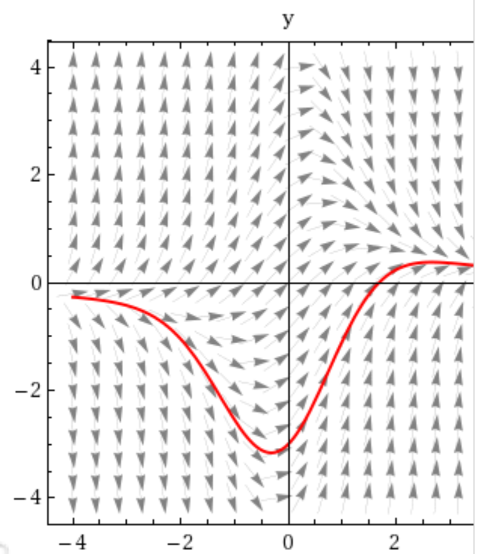
0
2
4


y
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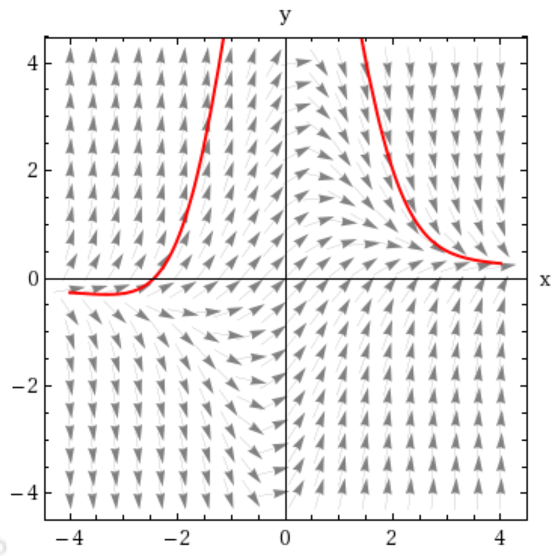
0
2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x
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2
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−4
−4
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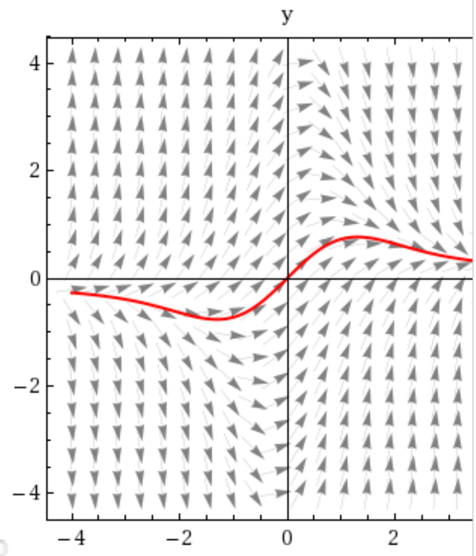
0
2
4


y
4
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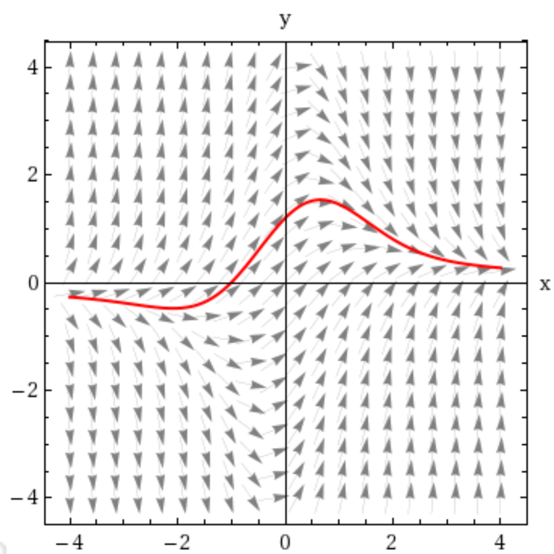
0
−2
−4
−4
−2
0
2

(d) $y(0) = -3$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x
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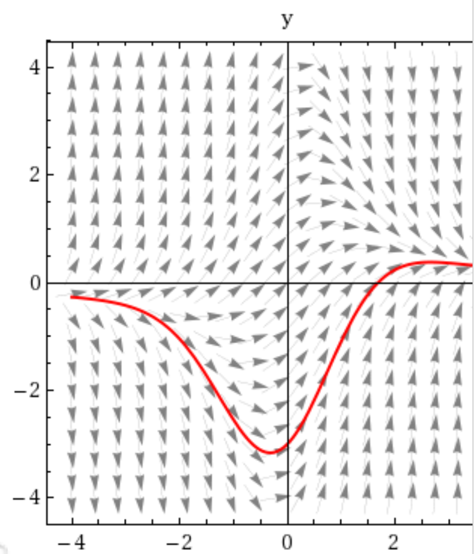
0
−2
−4
−4
−2
0
2
4


y
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−2
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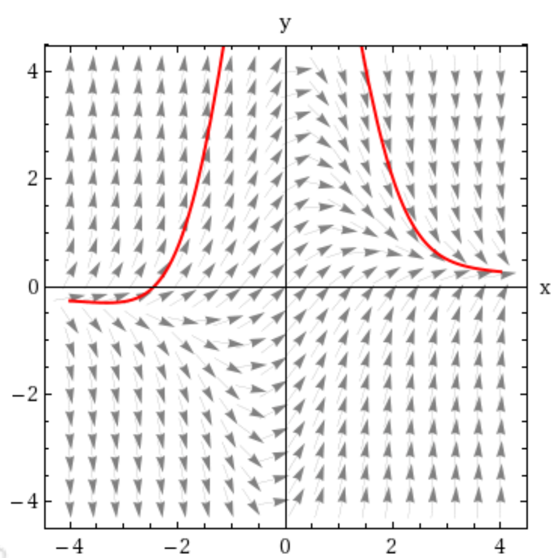
−4
−2
0
2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x
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0
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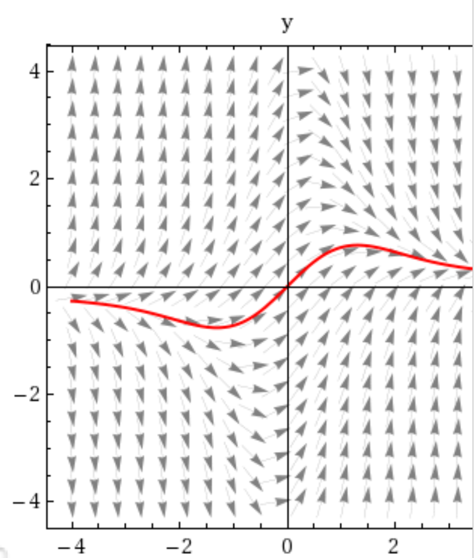
−4
−4
−2
0
2
4


y
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2
0
−2
−4
−4
−2
0
2

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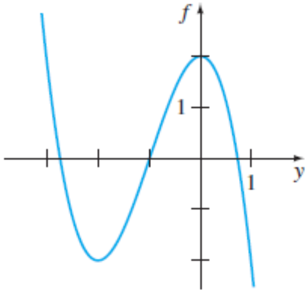
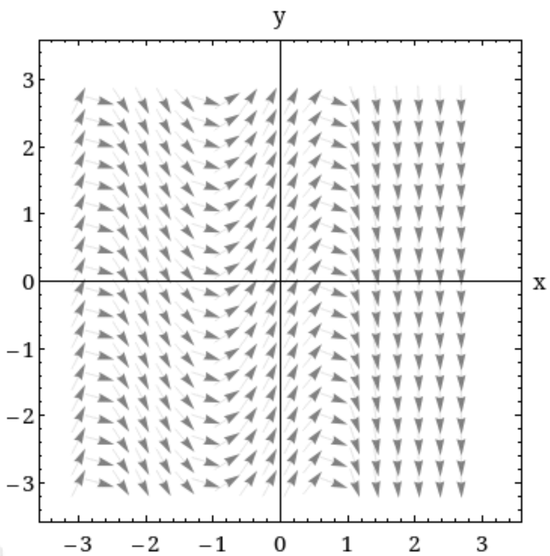
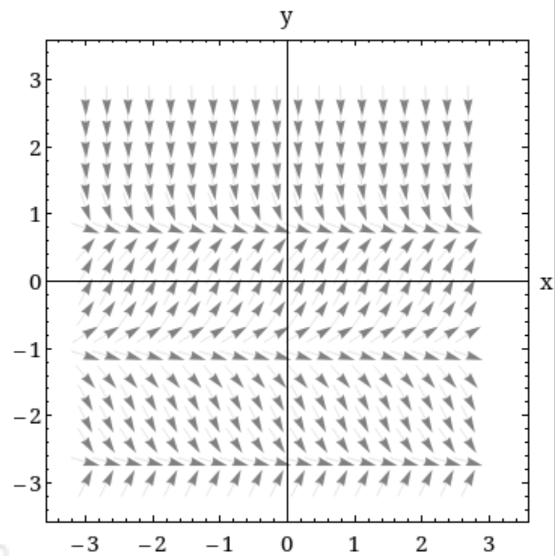
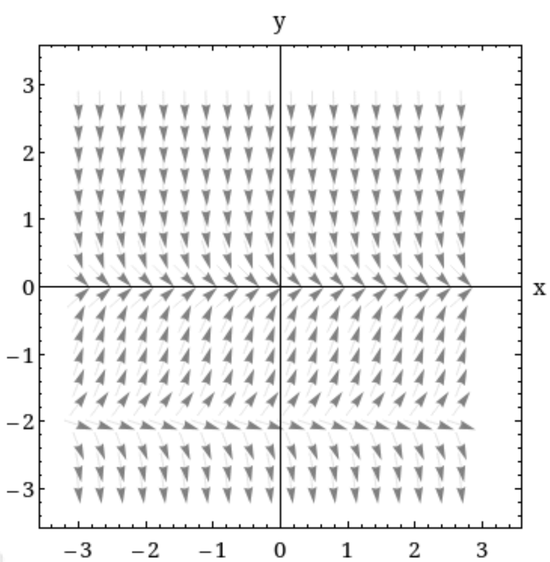
3. Question Details ZillDiffEQ9 2.1.013. [3876540]

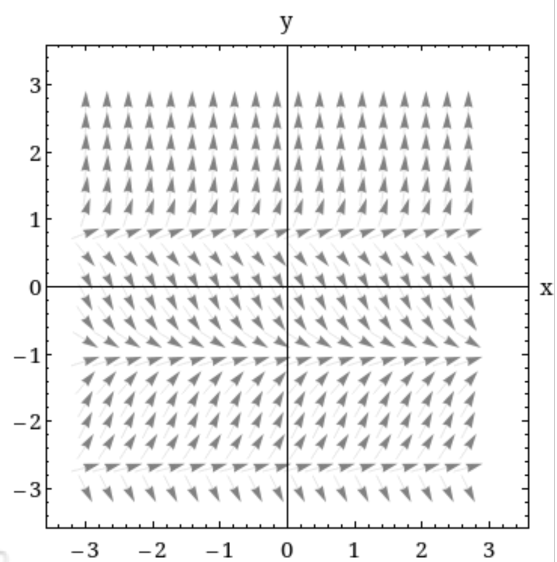
The given figure represents the graph of $f(y)$. Sketch a direction field over an appropriate grid for $dy/dx = f(y)$.









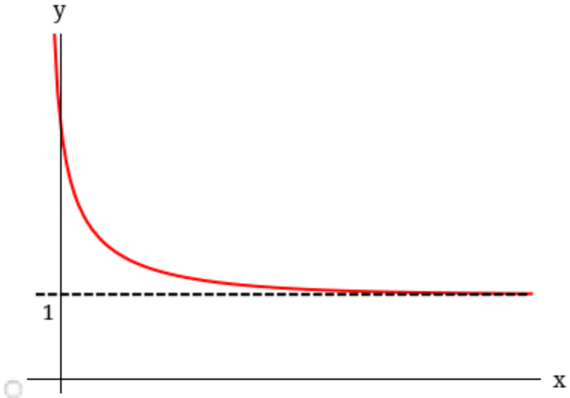


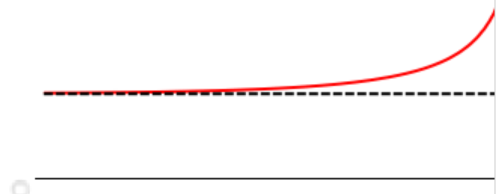
Need Help? Read It

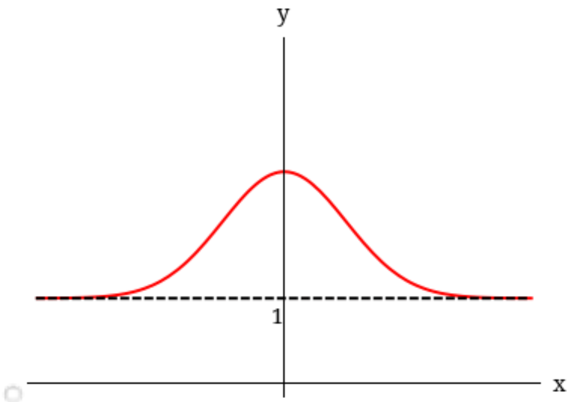
4. Question Details ZillDiffEQ9 2.1.019. [3748710]

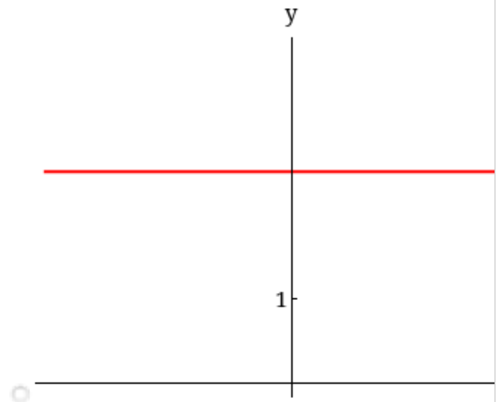
Consider the autonomous first-order differential equation $dy/dx = y - y^3$ and the initial condition $y(0) = y_0$. Sketch the graph of a typical solution $y(x)$ when y_0 has the given values.

(a) $y_0 > 1$

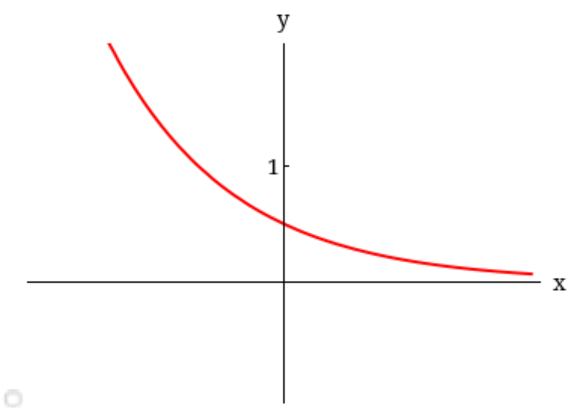


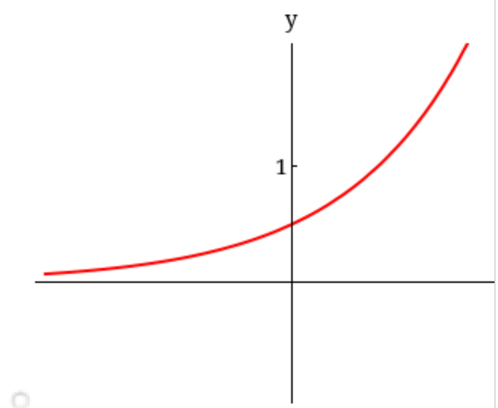


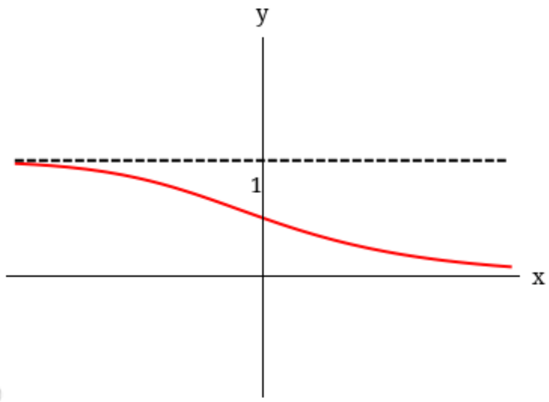


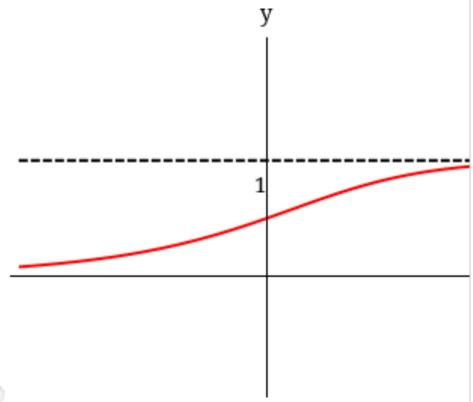


(b) $0 < y_0 < 1$

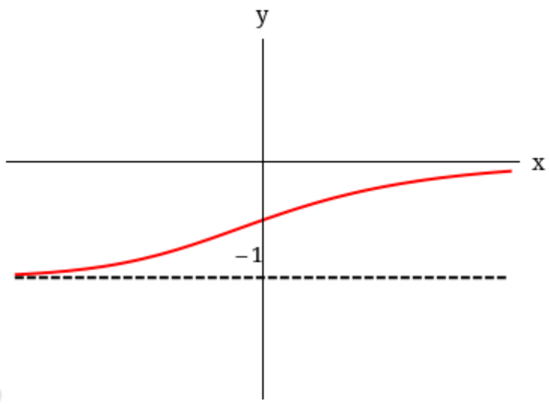


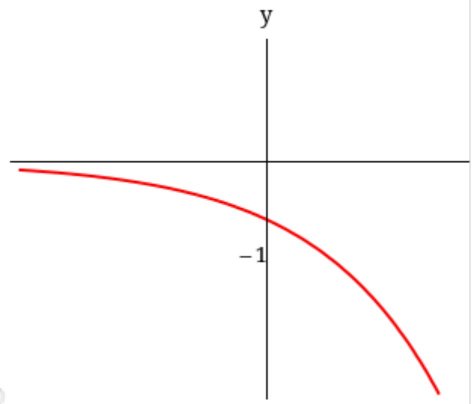


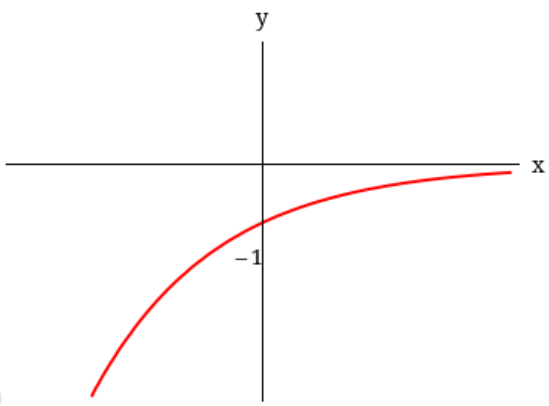


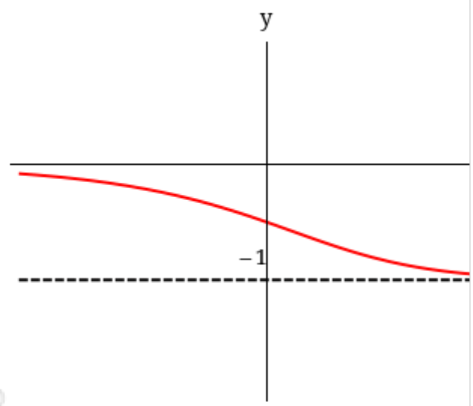


(c) $-1 < y_0 < 0$

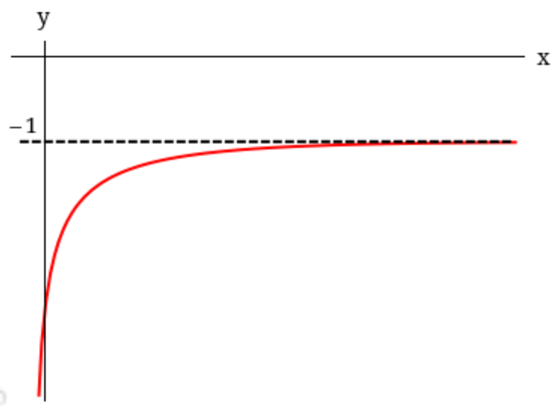


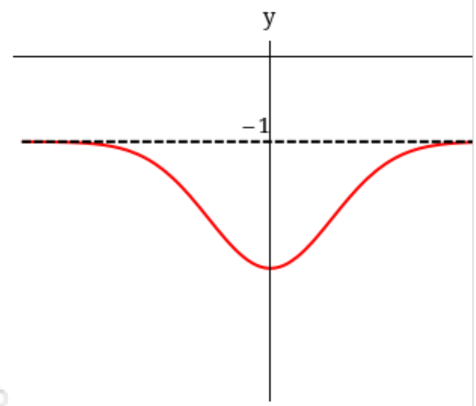


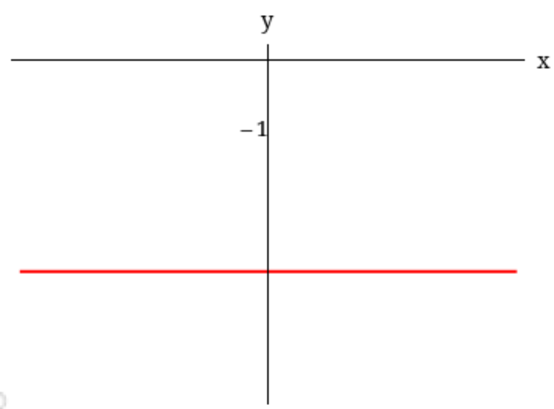


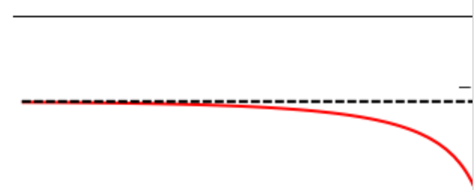


(d) $y_0 < -1$









Need Help? Read It

5. Question Details ZillDiffEQ9 2.1.021. [4568199]

Consider the following autonomous first-order differential equation.

$$\frac{dy}{dx} = y^2 - 2y$$

Find the critical points and phase portrait of the given differential equation.

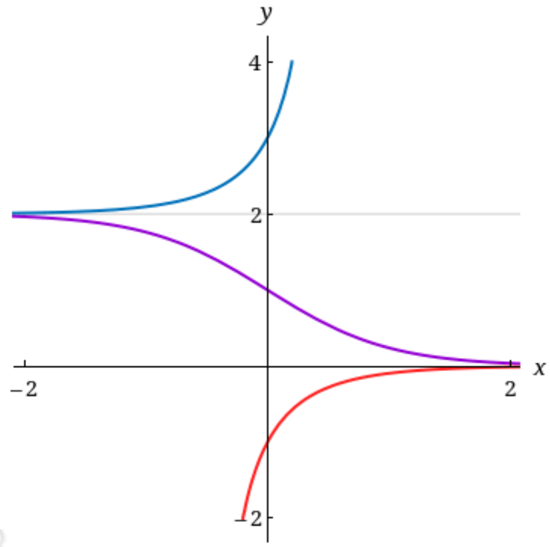
Classify each critical point as asymptotically stable, unstable, or semi-stable. (List the critical points according to their stability. Enter your answers as a comma-separated list. If there are no critical points in a certain category, enter NONE.)

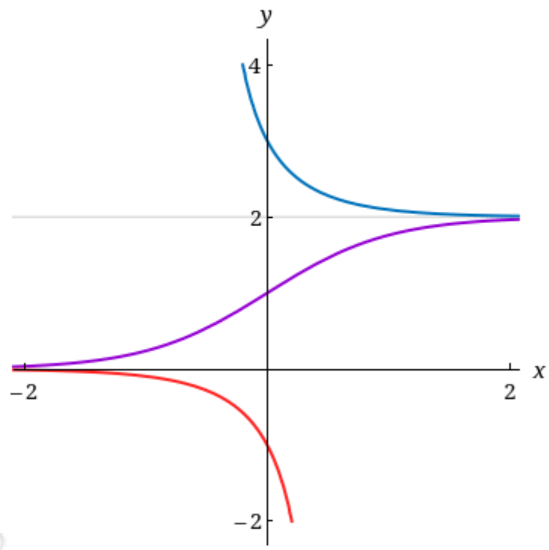
asymptotically stable

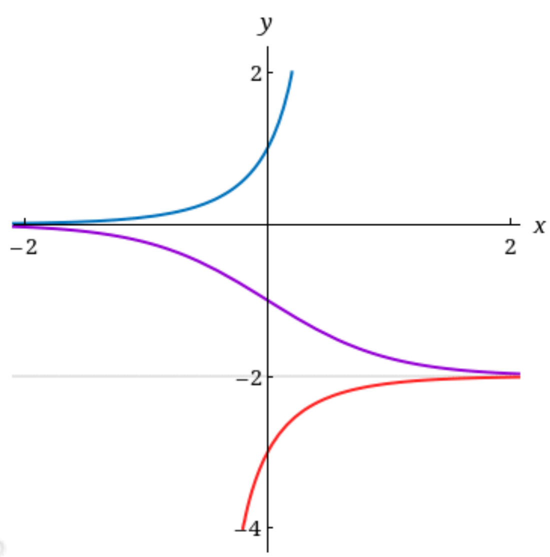
unstable

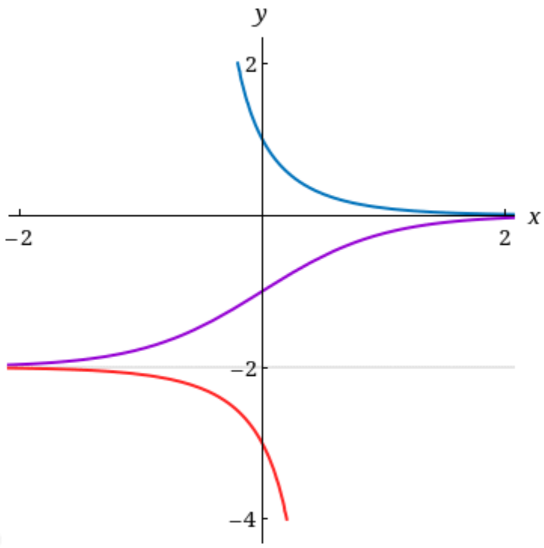
semi-stable

Sketch typical solution curves in the regions in the xy-plane determined by the graphs of the equilibrium solutions.









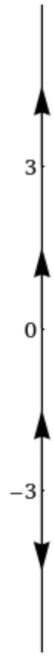
Need Help? Read It Watch It

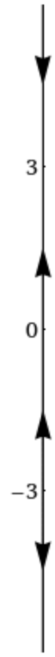
6. Question Details ZillDiffEQ9 2.1.023. [3748722]

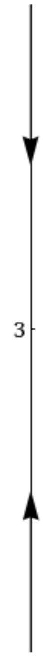
Consider the following autonomous first-order differential equation.

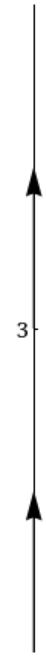
$$\frac{dy}{dx} = (y - 3)^4$$

Find the critical points and phase portrait of the given differential equation.









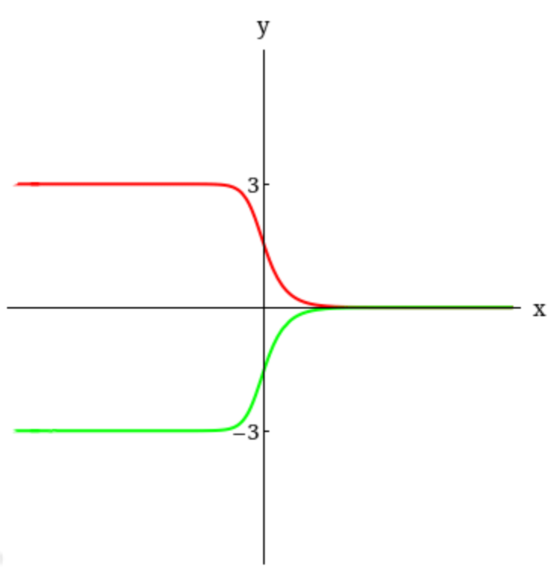
Classify each critical point as asymptotically stable, unstable, or semi-stable. (List the critical points according to their stability. Enter your answers as a comma-separated list. If there are no critical points in a certain category, enter NONE.)

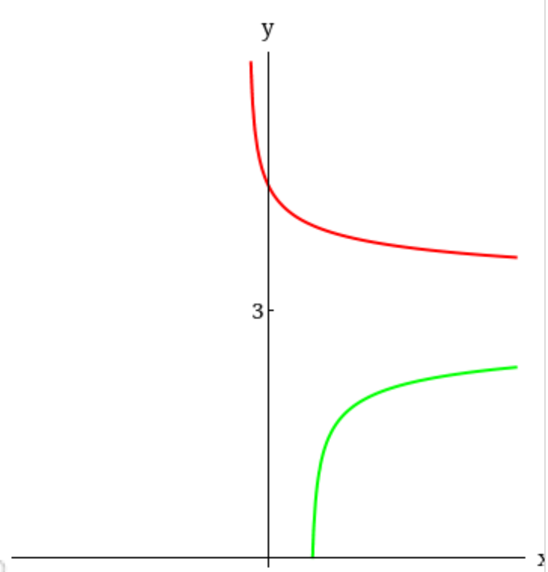
asymptotically stable

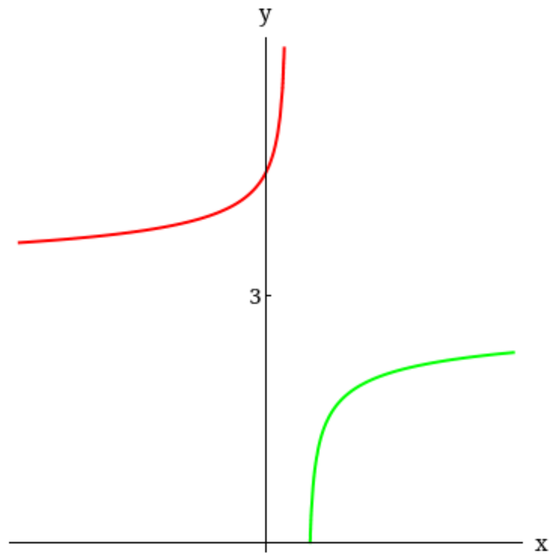
unstable

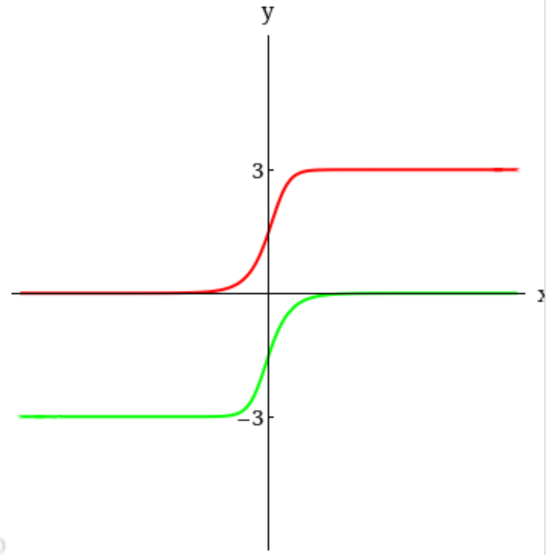
semi-stable

Sketch typical solution curves in the regions in the *xy*-plane determined by the graphs of the equilibrium solutions.









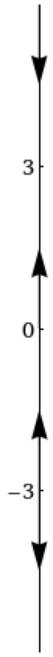
Need Help? Read It

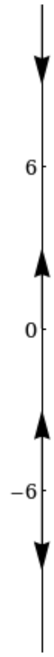
7. Question Details ZillDiffEQ9 2.1.025. [3876574]

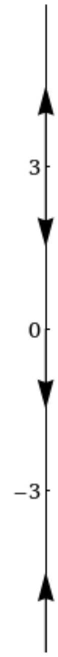
Consider the following autonomous first-order differential equation.

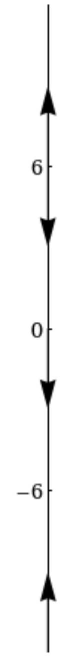
$$\frac{dy}{dx} = y^2(9 - y^2)$$

Find the critical points and phase portrait of the given differential equation.









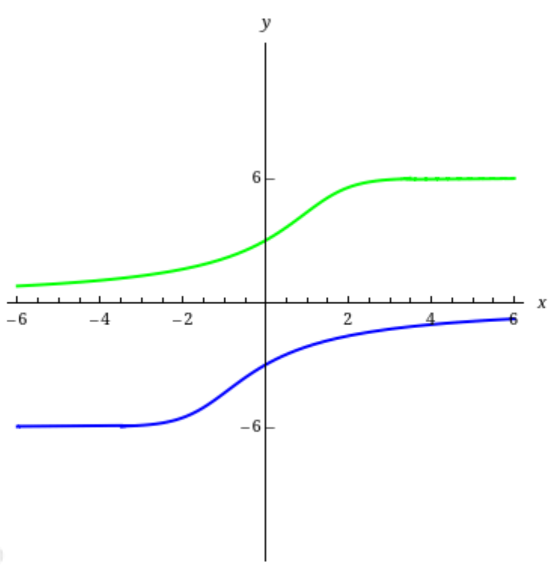
Classify each critical point as asymptotically stable, unstable, or semi-stable. (List the critical points according to their stability. Enter your answers as a comma-separated list. If there are no critical points in a certain category, enter NONE.)

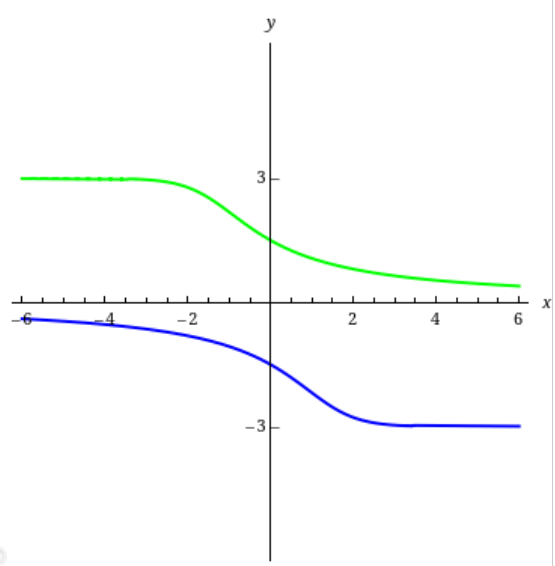
asymptotically stable

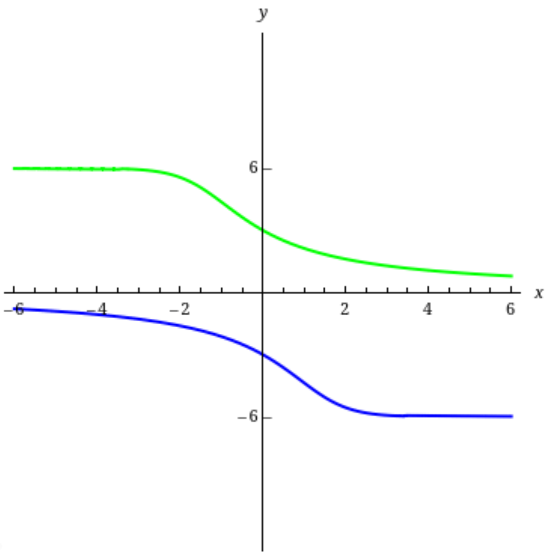
unstable

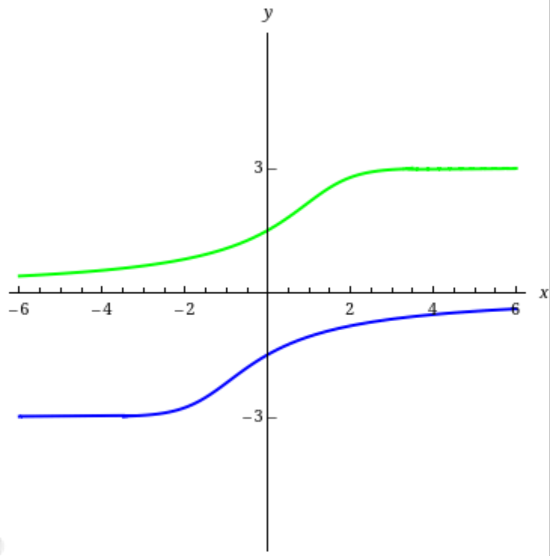
semi-stable

Sketch typical solution curves in the regions in the *xy*-plane determined by the graphs of the equilibrium solutions.









Need Help? Read It

8. Question Details ZillDiffEQ9 2.1.027. [4805224]

Consider the following autonomous first-order differential equation.

$$\frac{dy}{dx} = y\ln(y + 2)$$

Find the critical points and phase portrait of the given differential equation.

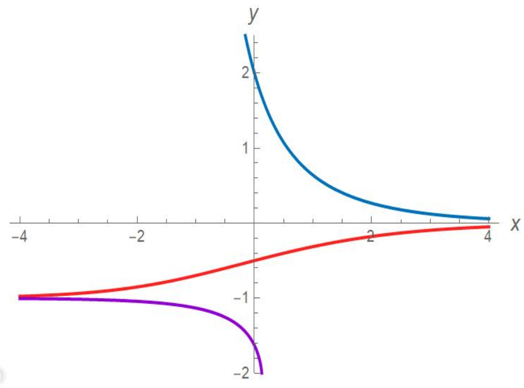
Classify each critical point as asymptotically stable, unstable, or semi-stable. (List the critical points according to their stability. Enter your answers as a comma-separated list. If there are no critical points in a certain category, enter NONE.)

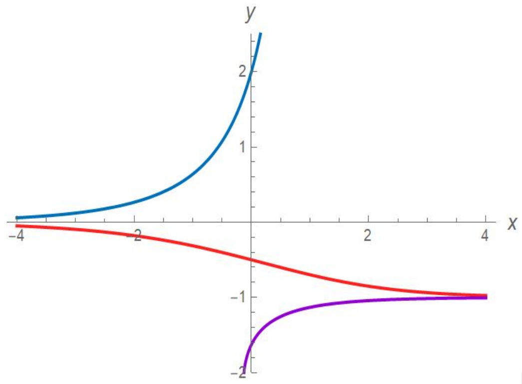
asymptotically stable

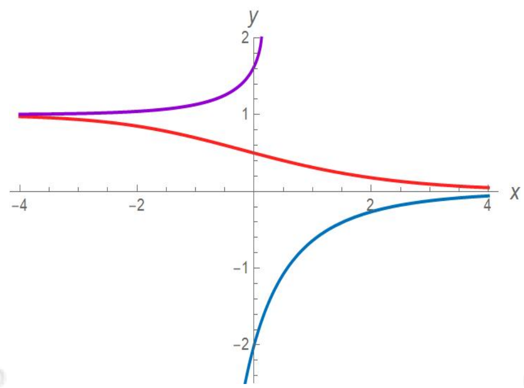
unstable

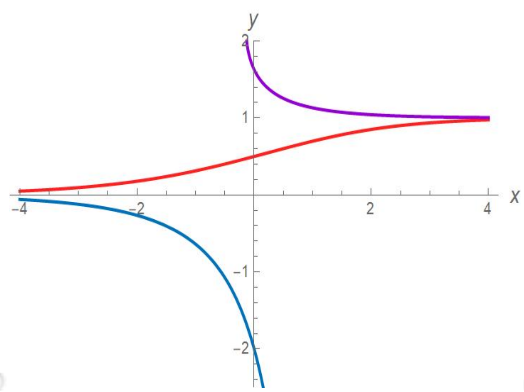
semi-stable

Sketch typical solution curves in the regions in the *xy*-plane determined by the graphs of the equilibrium solutions.









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9. Question Details ZillDiffEQ9 2.R.001. [4568072]

Fill in the blanks.

The linear DE, $y' + ky = A$, where k and A are constants, is autonomous. The critical point ______ of the equation is a(n) ---Select--- for $k > 0$ and a(n) ---Select--- for $k < 0$.

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10. Question Details ZillDiffEQ9 2.R.013. [3876551]

Construct an autonomous first-order differential equation $dy/dx = f(y)$ whose phase portrait is consistent with the given figure.

$\frac{dy}{dx} =$ ______

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11. Question Details ZillDiffEQ9 2.R.035. [3876545]

Solve the given initial-value problem for $y_0 > 0$.

$$\frac{dy}{dx} = \sqrt{y}, \quad y(x_0) = y_0$$

$y(x) =$ ______

Find the largest interval I on which the solution is defined. (Enter your answer using interval notation.)

______

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12. Question Details ZillDiffEQ9 3.2.003.EP. [4903648]

A model for the population $P(t)$ in a suburb of a large city is given by the initial-value problem

$$\frac{dP}{dt} = P(10^{-1} - 10^{-7}P), \qquad P(0) = 4000,$$

where t is measured in months.

Find the population P of the suburb at time t.

$P(t) =$ _____

What is the limiting value of the population?

_____

At what time will the population be equal to one-half of this limiting value? (Round your answer to one decimal place.)

_____ months

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13. Question Details ZillDiffEQ9 3.2.011. [3894163] -

A tank in the form of a right-circular cylinder standing on end is leaking water through a circular hole in its bottom. As we saw in (10) of Section 1.3, when friction and contraction of water at the hole are ignored, the height h of water in the tank in feet after t seconds is described by

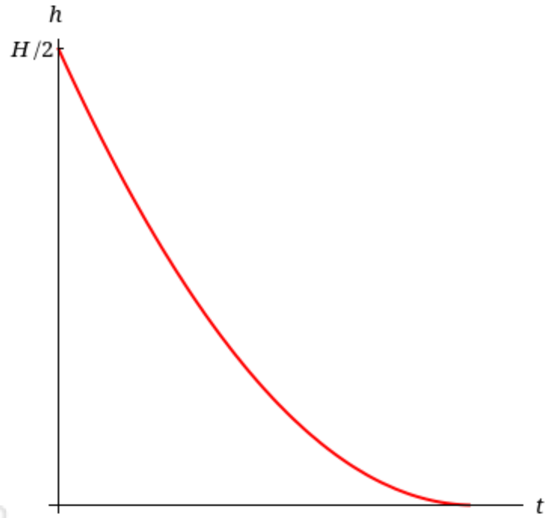
$$\frac{dh}{dt} = -\frac{A_h}{A_w}\sqrt{2gh},$$

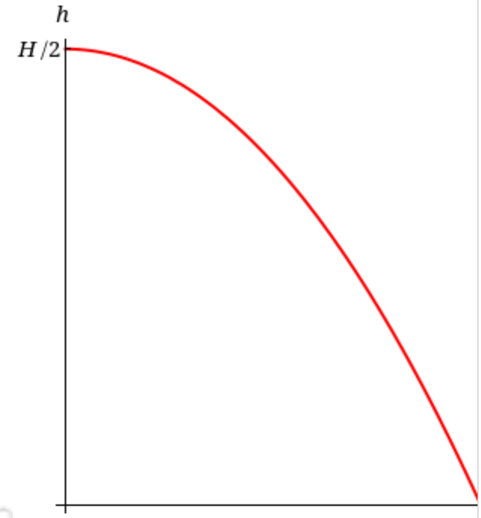
where A_w and A_h are the cross-sectional areas of the water and the hole in square feet, respectively.

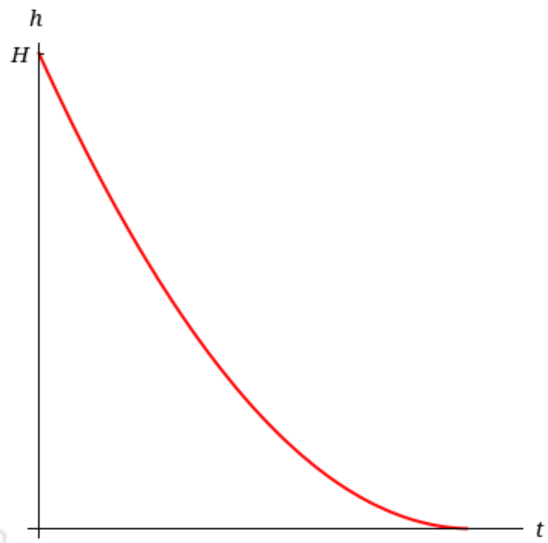
(a) Solve for $h(t)$ if the initial height of the water is H. Give its interval I of definition in terms of the symbols A_w, A_h, and H. Use $g = 32$ ft/s^2.

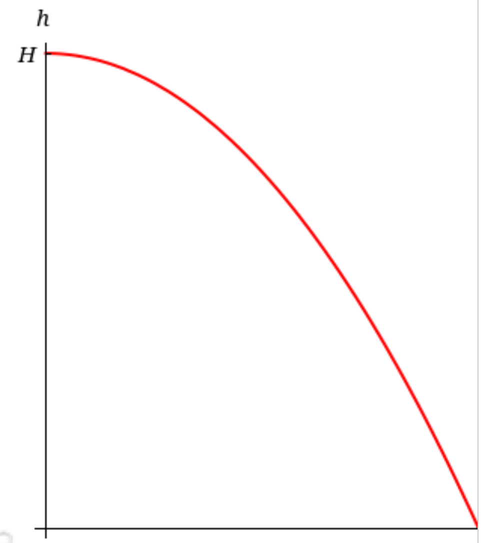
$h(t) =$ ______ , $0 \le t \le$ ______

By hand, sketch the graph of $h(t)$.









(b) Suppose the tank is 13 ft high and has radius 3 ft and the circular hole has radius $\frac{1}{2}$ in. If the tank is initially full, how long will it take to empty? (Round your answer to two decimal places.)

______ sec

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14. Question Details ZillDiffEQ9 3.2.019. [3745191]

(a) A simple model for the shape of a tsunami is given by

$$\frac{dW}{dx} = W\sqrt{4 - 2W},$$

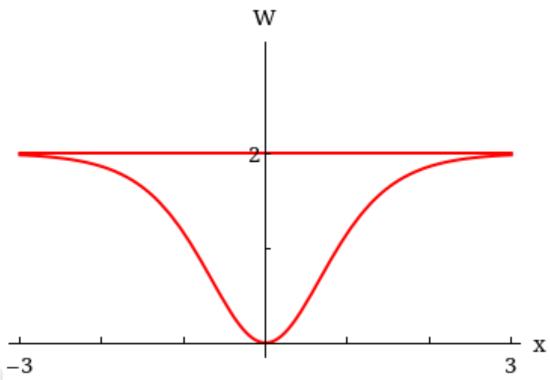
where $W(x) > 0$ is the height of the wave expressed as a function of its position relative to a point offshore. By inspection, find all constant solutions of the DE. (Enter your answers as a comma-separated list.)

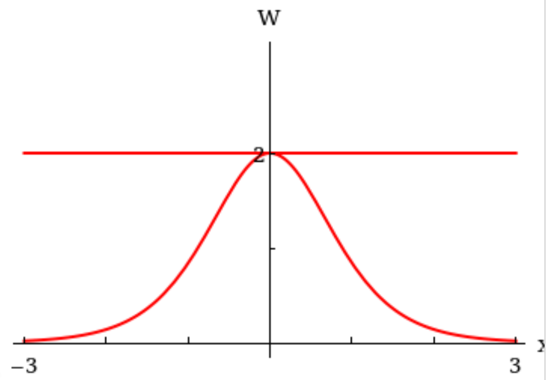
W = [ ]

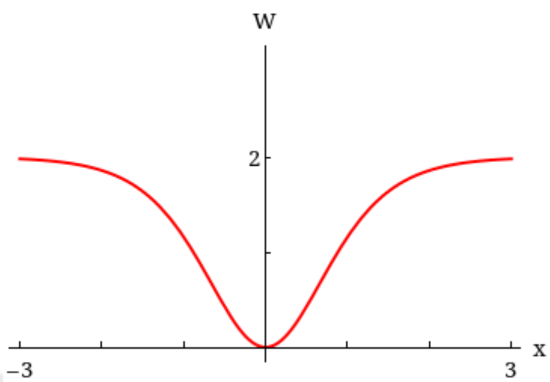
(b) Solve the differential equation in part (a). A CAS may be useful for integration.

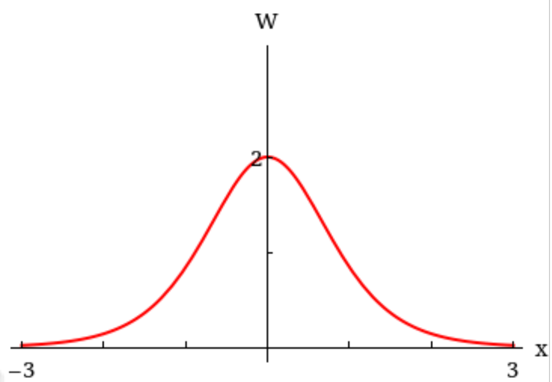
$W(x)$ = [ ]

(c) Use a graphing utility to obtain the graphs of all solutions that satisfy the initial condition $W(0) = 2$.









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15. Question Details ZillDiffEQ9 3.2.021. [4568162] -

Consider the differential equation

$$\frac{dP}{dt} = kP^{1 + c},$$

where $k > 0$ and $c \geq 0$. In Section 3.1 we saw that in the case $c = 0$ the linear differential equation $dP/dt = kP$ is a mathematical model of a population $P(t)$ that exhibits unbounded growth over the infinite time interval $[0, \infty)$, that is, $P(t) \to \infty$ as $t \to \infty$. See Example 1 in that section.

(a) Suppose for $c = 0.01$ that the nonlinear differential equation

$$\frac{dP}{dt} = kP^{1.01}, \; k > 0,$$

is a mathematical model for a population of small animals, where time t is measured in months. Solve the differential equation subject to the initial condition $P(0) = 10$ and the fact that the animal population has doubled in 7 months. (Round the coefficient of t to six decimal places.)

$P(t)$ = [ ]

(b) The differential equation in part (a) is called a **doomsday equation** because the population $P(t)$ exhibits unbounded growth over a finite time interval $(0, T)$, that is, there is some time T such that $P(t) \to \infty$ as $t \to T^{-}$. Find T. (Round your answer to the nearest month.)

T = [ ] months

(c) From part (a), what is $P(60)$? $P(120)$? (Round your answers to the nearest whole number.)

$P(60)$ = [ ]

$P(120)$ = [ ]

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Assignment Details

Name (AID): **Math 2C03 Practice problem set #2 Jan2021 (18369946)**
Submissions Allowed: **20**
Category: **Homework**
Code:
Locked: **Yes**
Author: **Lia Bronsard (bronsard@mcmaster.ca)**
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