Course Outline for MATH 4FT
Topics in Differential Equations:
Fourier integral and pseudodifferential methods
Fall term 2018–2019

Course Home Page The course home page is on Avenue to Learn. It can be found at Avenue to Learn.

Instructor Professor Walter Craig, office HH418, ext 23422, craig@math.mcmaster.ca

Office hours: Th 10:30–12:00 and by appointment
Course meeting times: TuThF 2:30–3:20 in HH 305

Course structure There are three lectures per week; you should plan to attend all of them.

Course description and objectives The Fourier transform has many broad applications to wide range of topics, not the least being to partial differential equations. In fact, from one point of view the Fourier transform provides a system of coordinates in Hilbert space in which all constant coefficient differential operators are diagonal. The first part of this course will describe settings in which the Fourier transform and the Fourier inversion theorem apply. This includes the standard Hilbert space $L^2(\mathbb{R}^n)$, Sobolev spaces, Schwartz Class, and the spaces of distributions and tempered distributions. The second part of this course focuses on pseudodifferential operators and their use in analysis. This class of operators includes differential operators with variable coefficients, fractional derivatives, and otherwise operators that allow one to microlocalize analysis; that is, to localize analytic considerations simultaneously in position space ($x \in \mathbb{R}^n_x$) and in Fourier space ($\xi \in \mathbb{R}^n_\xi$).

The course material wll be presented at an advanced undergraduate/masters student level. Despite the apparently advanced and technical nature of the topic, it is actually quite intuitive, a fact that will be emphasized in the course.

The principal goals of the course are: (1) to understand properties of solutions to partial differential equations given through Fourier synthesis, including solutions with classical regularity properties as well as distributional solutions. And (2) to introduce students to the calculus of pseudodifferential operators, opening up the possibilities of a local and microlocal analysis of distributions and their singularities.

Textbooks

1. *A Course on Partial Differential Equations*, by W. Craig, published in McMaster courseware.
2. *Pseudodifferential Operators*, by Michael Taylor, Princeton University Press.

The textbooks are *required*. You will be reading from it regularly, and homework assignments will include selected problems from the two books.

Math 4FT3 Course syllabus

(1) The Fourier transform
- elementary properties
- differential operators

(2) Partial differential equations
- Wave equation - Fourier synthesis
- Schrödinger equation - Fourier synthesis
- Laplace's equation on $\mathbb{R}^n_+$
- Malgrange – Ehrenpreis theorem

(3) Analysis of the Fourier transform
- Hilbert space $L^2(\mathbb{R}^n)$
- Schwartz Class $\mathcal{S}$, C^∞, and C_0^∞
- The Fourier inversion theorem on $L^2(\mathbb{R}^n)$ and on $\mathcal{S}$
- Sobolev spaces and the Sobolev embedding theorem
- distributions and tempered distributions
- support and singular support of a distribution

(4) Pseudodifferential operators
- Fourier integral representation
- symbols and symbol classes
- pseudolocal property
- symbols and their asymptotic expansions
- products and adjoints of pseudodifferential operators
- continuity of pseudodifferential operators on Sobolev spaces
- Gårding inequality

(5) Elliptic pseudodifferential operators
- ellipticity
- preservation of singular support
- Fredholm operators

(6) Hyperbolic equations
- strictly hyperbolic systems
- wave front set
- propagation of singularities
- Egorov's theorem

Assessment

Problem Sets: 25% There will be a homework assignment approximately every two weeks. Homework is due in class on Tuesdays at the beginning of class, and will be returned in a subsequent class meeting.

Reading Assignments: Reading is advised for your preparation for class. You should do the reading before coming to class, as each lecture will contain a substantial amount of material.

Midterm: 25% There will be one midterm that will be held in class.

Final: 50% This will be given on a date scheduled by the registrar's office.

MSAF In the event of an absence for medical or other reasons, students should review and follow the Academic Regulation in the Undergraduate Calendar "Requests for Relief for Missed Academic Term Work." If for some reason you are unable to hand in a problem set, the possibility of an alternative assignment or due date should be discussed with the instructor. If you miss the midterm, there will be a make-up exam scheduled in a following week.

Academic Accommodation of Students with Disabilities Students who require academic accommodation must contact Student Accessibility Services (SAS) to make arrangements with a Program Coordinator. Academic accommodations must be arranged separately for each term of study. Student Accessibility Services can be contacted by phone 905-525-9140 ext. 28652 or email sas@mcmaster.ca. For further information, consult McMaster University's Policy for Academic Accommodation of Students with Disabilities.

Academic Accommodation for Religious, Indigenous or Spiritual observances Students requiring academic accommodation based on religious, indigenous or spiritual observances should follow the procedures set out in the RISO policy. Students requiring a RISO accommodation should submit their request to their Faculty Office normally within 10 working days of the beginning of term in which they anticipate a need for accommodation or to the Registrar's Office prior to their examinations. Students should also contact their instructors as soon as possible to make alternative arrangements for classes, assignments, and tests.

McMaster Policy on Academic Integrity You are expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. The academic credentials that you earn are rooted in principles of honesty and academic integrity. Academic dishonesty is to knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage. This behaviour can result in serious consequences, e.g., the grade of zero on an assignment, loss of credit with a notation on the transcript (notation reads: "Grade of F assigned for academic dishonesty"), and/or suspension or expulsion from the university. It is your responsibility to understand what constitutes academic dishonesty. For information on the various types of academic dishonesty, please refer to the Academic Integrity Policy, located at: Academic Integrity Policy. The following illustrates only three forms of academic dishonesty: 1) plagiarism, e.g., the submission of work that is not one's own or for which other credit has been obtained; 2) improper collaboration in group work; 3) copying or using unauthorized aids in tests and examinations.

Disclaimers Students should be aware that, when they access the electronic components of this course, private information such as first and last names, user names for the McMaster e-mail accounts, and program affiliation may become apparent to all other students in the same course. The available information is dependent on the technology used. Continuation in this course will be deemed consent to this disclosure. If you have any questions or concerns about such disclosure please discuss this with the course instructor.

The instructor and university reserve the right to modify elements of the course during the term. The university may change the dates and deadlines for any or all courses in extreme circumstances. If either type of modification becomes necessary, reasonable notice (if possible) and communication with the students will be given with explanation and the opportunity to comment on changes. It is the responsibility of students to check **their McMaster email** and the **course websites** weekly during the term and to note any changes. Announcements will be made in class and by using the course email distribution list.