Exercises: Chapter 4

Exercise 4.1. Derive the expression (4.12) for the case $n = 2$ from the Fourier integral, and show that $\omega_2 = 2\pi$. *Hint:* Complex variables techniques would be useful.

Derive the expression (4.12) in the general case for $n \geq 3$ and show that ω_n is the surface area of the unit sphere $\mathbb{S}^{n-1} \subseteq \mathbb{R}^n$.

Exercise 4.2. In the case $n = 2$ the fundamental solution has the property that $\Delta\Gamma(|x - y|) = \delta_y(x)$. Show this by proving that the limit of the expression in (4.17) vanishes, and that the limit in (4.18) holds.

Exercise 4.3. Prove the second version of the Gauss' law of arithmetic mean (4.21).

Exercise 4.4. Prove that a function $u(x) \in C^2(\Omega)$ that satisfies

$$\Delta u \geq 0 \tag{4.22}$$

is subharmonic in Ω. Prove that if $u(x)$ is subharmonic then it satisfies the inequality (4.22).