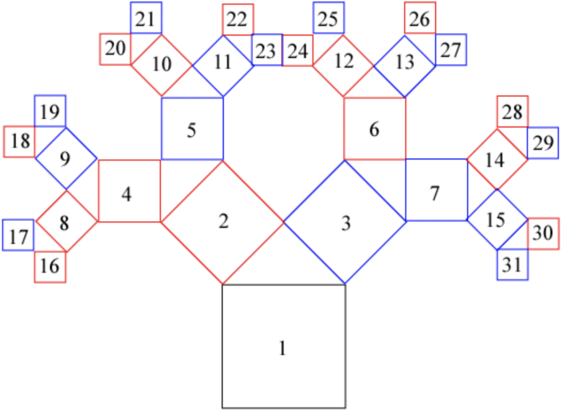
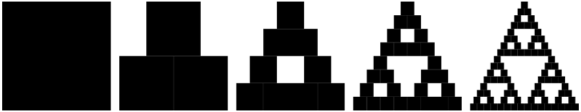
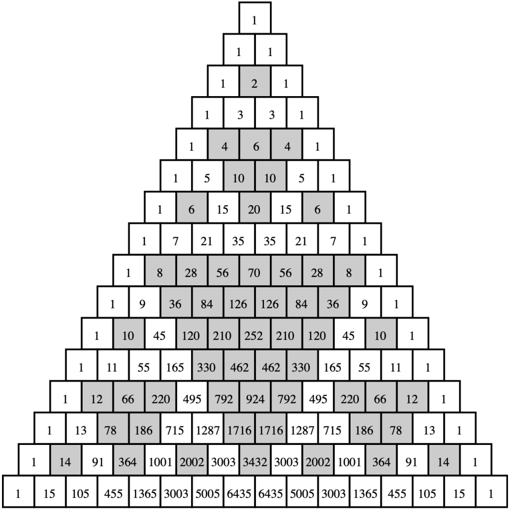


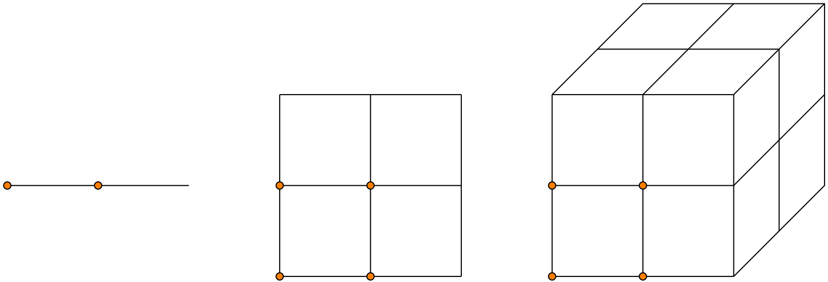
Figure: Pythagoras Tree and Binary Numbers

The evolution of the Sierpinski triangle



Iterating from a square


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 186 715 1287 1716 1716 1287 715 186 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1



Pattern

Magnification by 2 requires 2^d copies for a d-dimensional figure.

Generalization

Scaling by a factor F requires F^d copies for d-dimensional figure.

C copies, scaling by a factor F

$$C = F^d$$

$$\ln C = \ln F^d$$

$$d = \frac{\ln C}{\ln F} \qquad \text{(dimension)}$$

Figure: Middle Third Removed

$$c = \frac{a_1}{3} + \frac{a_2}{3^2} + \frac{a_3}{3^3} + \frac{a_4}{3^4} + \cdots \qquad (a_i = 0, 2)$$

Educated Guess

dimension?

Pattern

2 *copies scale by factor* 3

$$d = \frac{\ln 2}{\ln 3} \approx 0.6309$$

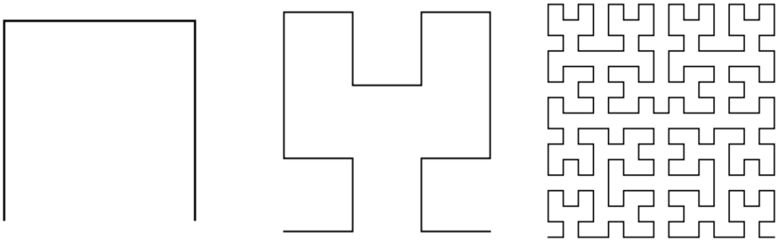
infinitely broken line

Why Why Why?

$$c = \frac{a_1}{3} + \frac{a_2}{3^2} + \frac{a_3}{3^3} + \frac{a_4}{3^4} + \cdots \qquad (a_i = 0, 2)$$

Pattern

$a + b$ with a, b from Cantor set excavated from $[0, 1]$ gives the entire $[0, 2]$.



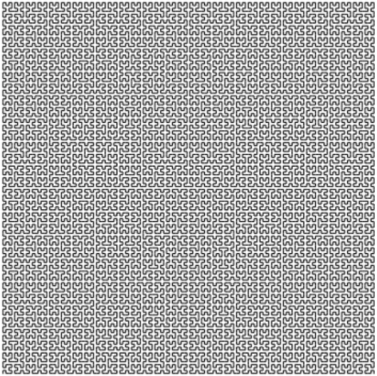


Figure: infinitely iterations later...

Pattern

4 *pieces needed to scale by* 2

$$d = \frac{\ln 4}{\ln 2} = 2$$

2*d curve, what?*

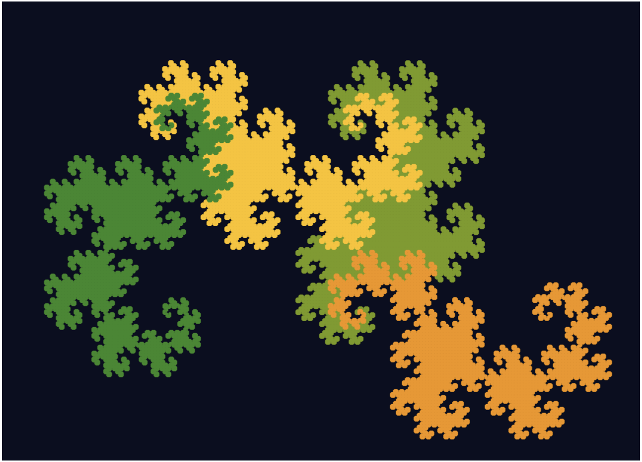
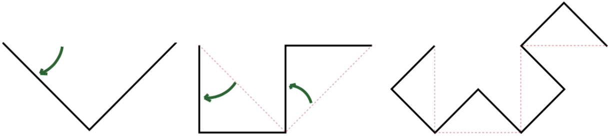


Figure: Dragon Curve



Pattern

2 *pieces needed to scale by* $\sqrt{2}$

$$d = \frac{\ln 2}{\ln \sqrt{2}} = \frac{\ln 2}{\frac{1}{2}\ln 2} = 2$$

Space filling

1. $Z_{n+1} = Z_n^2 + c$
2. starting from origin
3. $z_0, z_1, z_2 \cdots$ does not escape to ∞
4. plot c

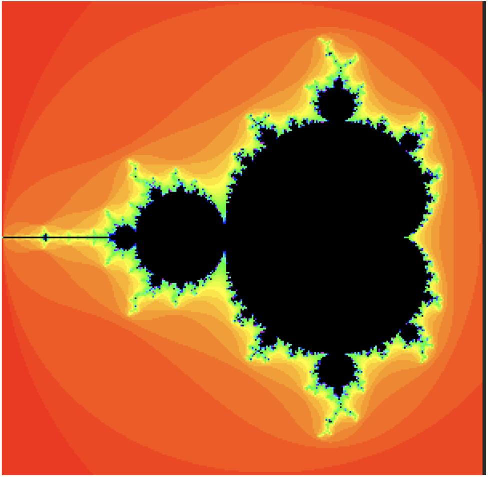


Figure: 2-dimensional boundary!

by Mitsuhiro Shishikura

Get your hands dusty!

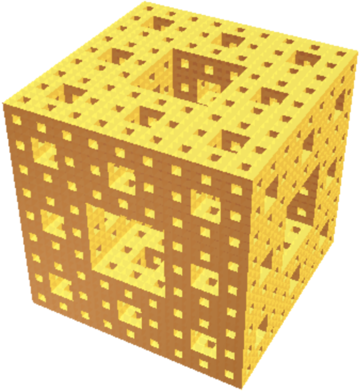
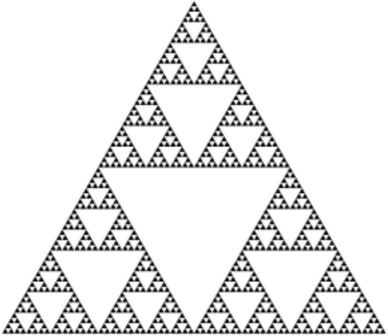


Figure: Sponge ~~Bob~~ Menger



$$d = \frac{\ln C}{\ln F}$$

Beyond "Similarity Dimension"

Say What?

fractal dimension $>$ *topological dimension*

Say What?

The minimum number of sets of diameter at most d need to cover the set S is denoted $N(S, d)$

$$dim(S) = \lim_{d \to 0} \frac{\ln N(S, d)}{\ln 1/d} \qquad \text{(Minkowski Dimension)}$$

Shout out for: "measure theory"

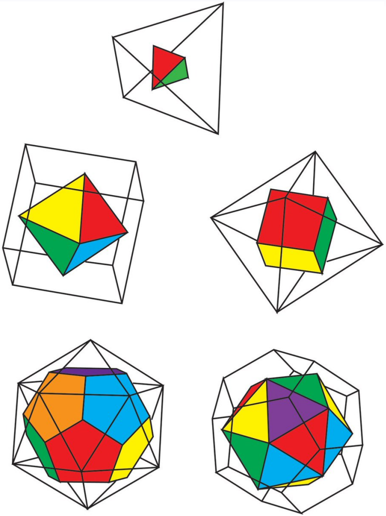


Figure: What do you discover?

Correspondence

1. *of solids*
2. *of edges*
3. *(faces at vertex)* $\longleftrightarrow$ *(edges on face)*

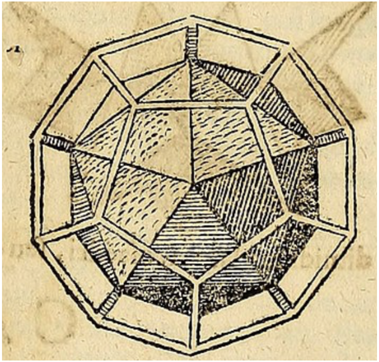


Figure: From Kepler's Book

$$20 - 30 + 12 = 2$$

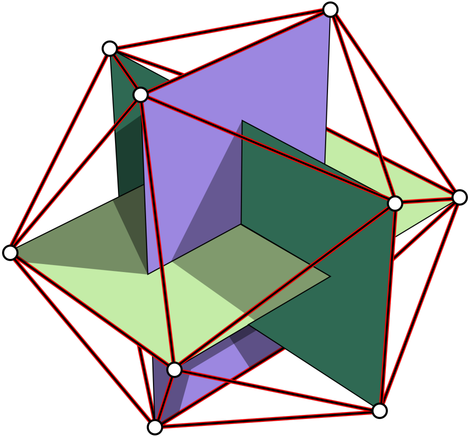


Figure: notice the pentagon diagonal

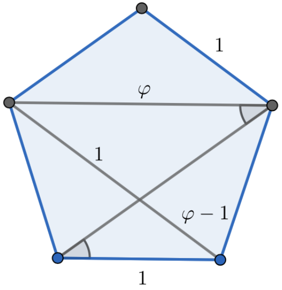


Figure: find diagonal, φ

$$\frac{1}{\varphi} = \frac{\varphi - 1}{1}$$

Definition

Two segments are in golden ratio when the smaller to the larger is in the same ratio as the larger is to the whole.

$$\frac{a}{A} = \frac{A}{a+A}$$

Pattern

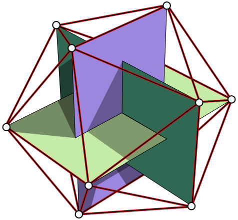
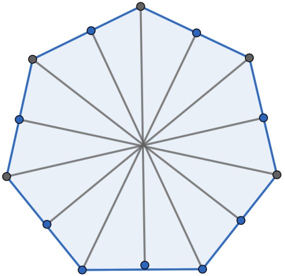
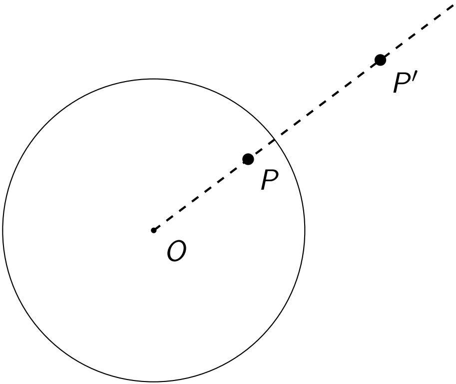


Figure: Icosahedron from golden rectangles

Rotational Groups in $\mathbb{R}^3$ (Finite Ones)

1. *rotational symmetry of dodecahedron (equivalently, its dual)*
2. *cube (octahdron)*
3. *tetrahedron*
4. *regular n-gon*
5. *rotations in multiples of* $\frac{2\pi}{k}$





$$OP \cdot OP' = r^2$$ (inversion about a circle)

Pattern

1. *all points on circle stays fixed*
2. *exchanges center and infinity*
3. *pairs up points*

Where do Things Go?

1. *line through center maps to itself*
2. *circle not through center goes to circle not through center*
3. *circle through center goes to line not through center*
4. *line not through center goes to circle through center*

invariant: angle

Draw a Circle Tangent to Three Given

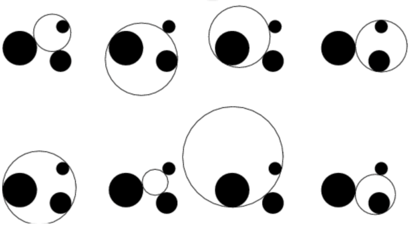
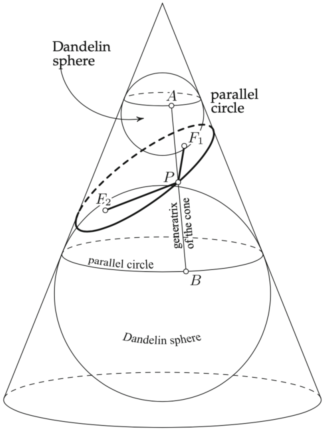


Figure: Apollonius Problem & Inversion

Projective Transformation



$$PF_1 + PF_2 = AB = const. \qquad \text{(ellipse)}$$

Theorem

The "cross ratio" of 4 points

$$r = \frac{AC}{AD} \Big/ \frac{BC}{BD} \qquad \text{(ratio of ratios)}$$

is invariant under projective transformation.

Theorem

The Euler characteristic of a shape

$$\chi = face - edge + vertex$$

is invariant under topological deformation.

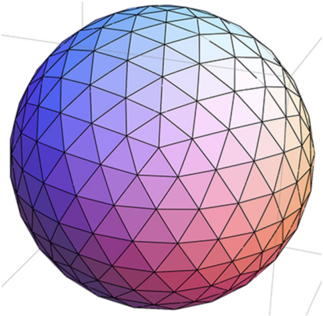


Figure: Triangulation of Sphere