Math@Mac Online Mathematics Competition

Wednesday, November 28, 2012

Instructions:

There are **ten** multiple choice questions. Answer **one** of A, B, C, or D for each question. Check your answers carefully before submitting them online. You will only be able to submit your answers once. Non-programmable, non-graphing calculators are permitted. You may not use any other resources including web-based ones.

Good luck!

---

1. A cubical box contains 64 identical small cubes that exactly fill the box. How many of these small cubes touch a side, the bottom, or the top of the box?

(A) 48
(B) 52
(C) 56
(D) 60

2. A bag contains 11 candy bars: three cost 50 cents each, four cost 1 dollar each and four cost 2 dollars each. Three candy bars are randomly chosen from the bag, without replacement. What is the probability that the total cost of the three candy bars is 4 dollars or more?

(A) 1/4
(B) 14/33
(C) 4/11
(D) 7/11

3. Jill rides her bike around a course in the shape of an equilateral triangle. Her speed is 10 kilometers per hour on the first side of the course, 15 kilometers per hour on the second side of the course, and 20 kilometers per hour on the third and final side of the course. Then Jill's average speed during her ride

(A) is less than 13 kilometers per hour.
(B) is at least 13 but less than 14 kilometers per hour.
(C) is at least 14 but less than 15 kilometers per hour.
(D) is at least 15 kilometers per hour.

4. The average of 9 numbers $a_1, a_2, \ldots, a_9$ is equal to 200. If the number a_1 is replaced by $4a_1$, then the new average is 400. What is the value of a_1?

(A) 100
(B) 200
(C) 600
(D) 800

5. In Hamilton Spy high school each student is required to take a specialized science course. 280 students took electronics, 230 forensics and 230 robotics classes. 120 students took both electronics and forensics, 110 took electronics and robotics, 80 took forensics and robotics, and 50 took all three courses. How many students are there in the school?

(A) 460
(B) 480
(C) 520
(D) 536

6. What is the final digit of the product of all odd numbers *not divisible* by 5 from 1 to 999?

(A) 1
(B) 3
(C) 7
(D) 9

7. If the sides of a triangle satisfy $\dfrac{3}{a+b+c} = \dfrac{1}{a+b} + \dfrac{1}{a+c}$, what is the angle between the sides b and c?

(A) 30 degrees
(B) 45 degrees
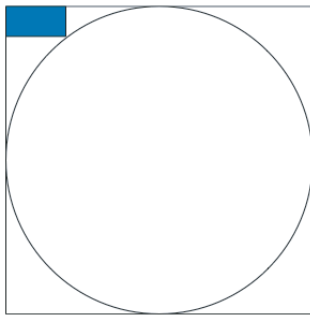(C) 60 degrees
(D) cannot be determined from the given information

8. The sum of the solutions of the equation

$$(x + 1)^2 + (2x + 1)^2 + (3x + 1)^2 + \cdots + (99x + 1)^2 = 99$$

is

(A) 6/189
(B) -6/199
(C) -4/199
(D) -6/189

9. The rectangle in the corner measures 8 cm by 16 cm. What is the radius of the circle in cm?

(A) 40
(B) 45
(C) 50
(D) 55

10. Two numbers a and b each have three digits. You are asked to calculate the product ab, but instead you put a to the left of b to form a six-digit number d. It turns out that d is three times the correct answer. Find a.

(A) 144
(B) 316
(C) 167
(D) 821

The End