

Math@Mac Online Mathematics Competition

Wednesday, November 27, 2013

Instructions:

There are **ten** multiple choice questions. Select **one** of A, B, C, or D for each question. Check your answers carefully before submitting them online. You will only be able to submit your answers once. Non-programmable, non-graphing calculators are permitted. You may not use any other resources including web-based ones.

Good luck!

1. The number of pairs (m, n) of positive integers such that $m^4 + n = 100000001$ is

- (A) 150
- (B) 125
- (C) 100
- (D) 75

2. If $x + y + z = 0$, then

$$\left(\frac{y-z}{x} + \frac{z-x}{y} + \frac{x-y}{z} \right) \left(\frac{x}{y-x} + \frac{y}{z-x} + \frac{z}{x-y} \right)$$

is equal to

- (A) 9
- (B) 0
- (C) 4
- (D) 12

3. Serena and Venus have been playing table tennis. To win the game, a player must have 11 or more points, AND must also have two more points than the other player. Right now, Serena has 13 points and Venus has 12. At any time, each player has a 50% chance of scoring the next point. What is the probability that Serena will win the game?

- (A) $1/2$
- (B) $2/3$
- (C) $3/4$
- (D) $4/5$

4. How many non-negative integers n are there such that $n + 2$ divides $(n + 18)^2$?

- (A) 1
- (B) 3
- (C) 8
- (D) more than 20

5. If 2^{2013} has m digits and 5^{2013} has n digits, then $m + n$ is

- (A) 2012
- (B) 2013
- (C) 2014
- (D) 2015

6. The pages of a book are 20cm tall and 10cm wide. If a page is folded appropriately, a corner of the page can stick out above the top of the book. What is the maximum amount that a page can protrude above the top of the book without tearing the page or separating it from the binding?

- (A) $5\sqrt{2}$
- (B) $\sqrt{10} - \sqrt{5}$
- (C) $\sqrt{20} - \sqrt{10}$
- (D) $10(\sqrt{5} - 2)$

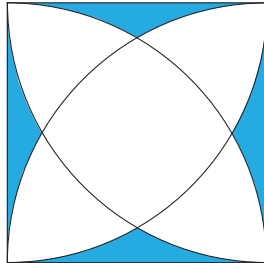
7. A student rolled a fair die until the sum of her rolls was a prime number. She rolled three times until this occurred. What is the probability that her last roll was a 6?

- (A) 0
- (B) $1/24$
- (C) $2/24$
- (D) $12/73$

8. The last three digits of the number $625^{376} + 376^{625}$ are

- (A) 001
- (B) 011
- (C) 021
- (D) 111

9. Take a unit square and draw a circle of radius 1 centred at each of its four vertices. What is the total area of the shaded regions?



- (A) $4 - 2\sqrt{3} - \frac{\pi}{3}$
(B) $2 - 2\sqrt{3} - \frac{2\pi}{3}$
(C) $8 - \sqrt{3} - \frac{\pi}{3}$
(D) $4 - \sqrt{3} - \frac{2\pi}{3}$

10. Assume that a polynomial $P(x)$ satisfies $(x + 1)P(x) = (x - 2)P(x + 1)$ and $P(3) = 12$. Then the coefficient of x^3 in $P(x)$ is

- (A) 0
(B) 1
(C) 2
(D) 3