Math@Mac Online Mathematics Competition

Wednesday, November 18, 2015

Instructions:
There are **ten** multiple choice questions. Select **one** of A, B, C, or D for each question. Check your answers carefully before submitting them online. You will only be able to submit your answers once. Non-programmable, non-graphing calculators are permitted. You may not use any other resources including web-based ones.

Good luck!

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1. Write $2015! = 6^k m$, where m is not divisible by 6. What is k?
(Recall that $2015! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot 2014 \cdot 2015$.)

(A) 1002

(B) 996

(C) 994

(D) 988

2. A list of consecutive positive numbers

$$1, 2, 3, 4, 5, \ldots, n-1, n$$

is written on a blackboard. One number is erased. The average (arithmetic mean) of the remaining numbers is $35\frac{7}{17}$. What number was erased?

(A) 14

(B) 13

(C) 7

(D) 11

3. A box contains 26 black and 13 white balls. There is a reserve supply of black balls. You pick randomly two balls at the same time from the box. If they are both white, you throw both balls away and put one black ball into the box. If they are both black, you throw away one ball and put the other ball back into the box. If one ball is white and the other black, then you throw away the black ball and put the white ball back into the box.

Every time you do this, there will be one ball less in the box. What is the probability that the last ball left in the box is black?

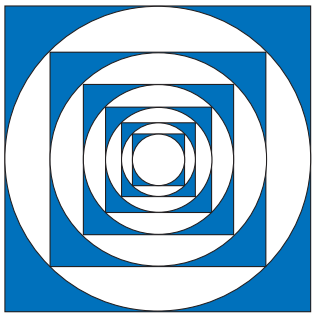
(A) 0

(B) 2/3

(C) 1/3

(D) 1/2

4. The side of the outer square has length 1. Into this square a circle is inscribed, and into this circle a square is inscribed, and into this square a circle is inscribed, and so on. Find the exact area of the shaded region.



(A) $2 - \pi/3$

(B) $4 - 2\pi/3$

(C) $3 - \pi$

(D) $2 - \pi/2$

5. Scientists observing how children socialize in the playground have found that girls congregate in groups of two or three. Each time a new girl arrives, she randomly choses a group. If a chosen group has two girls in it, the new girl joins the group, forming a group of three. If the chosen group has three girls in it, the new girl takes one of the girls away from the group of three girls and forms a separate group of two girls (so, in this case, a group of three girls is replaced by two groups of two girls).

Initially there are five girls in the playground. Assuming that new girls arrive one at a time, what is the probability that the fourth new girl joins a group of two girls?

(A) 4/9

(B) 3/7

(C) 4/7

(D) 5/9

6. A group of 500 high school students, 260 girls and 240 boys, is randomly divided into two rows of 250 students each. Each student in one row stands directly opposite from a student in the other row, and they shake hands. Which statement is true?

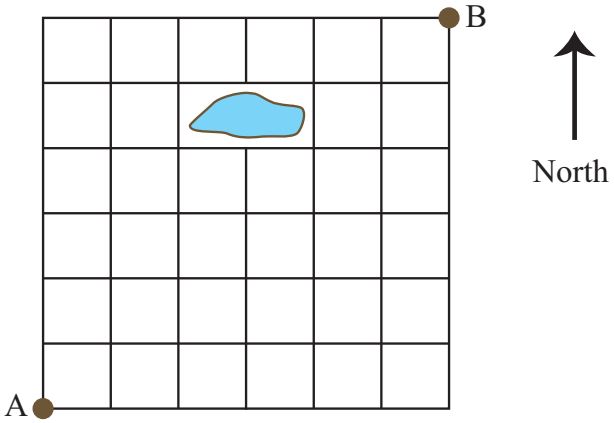
(A) Number of girl-girl handshakes is the same as the number of boy-boy handshakes

(B) Number of girl-girl handshakes is 5 more than the number of boy-boy handshakes

(C) Number of girl-girl handshakes is 10 more than the number of boy-boy handshakes

(D) Number of girl-girl handshakes is 20 more than the number of boy-boy handshakes

7. The streets in a town are laid out in a grid, as shown below. A small pond on the north side of town eliminates one segment of the grid. If you are allowed to move along the segments in the north and east directions only, in how many different ways can you walk from A to B?



(A) 924

(B) 826

(C) 844

(D) 784

8. Find the $12,345$ th term in the sequence

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \ldots$$

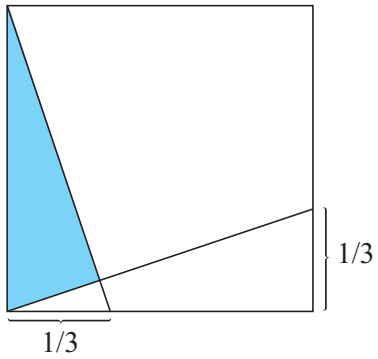
(A) 155

(B) 156

(C) 157

(D) 158

9. The length of the side of the square is 1. Determine the area of the shaded triangle.



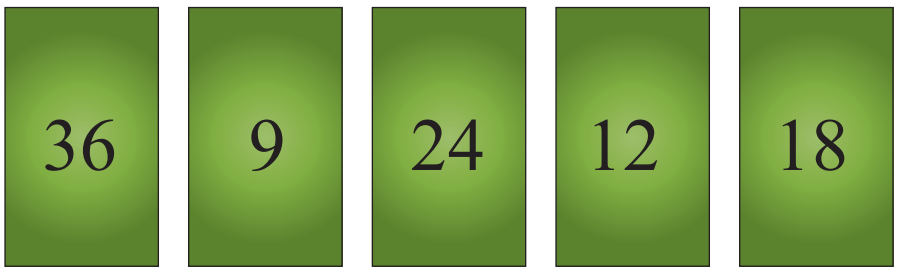
(A) 1/10

(B) 5/40

(C) 3/20

(D) 7/40

10. Each card is covering up a positive real number. The number written on each card is the product of all numbers covered by all of the other cards. Find the product of the three smallest numbers covered up by the cards.



(A) 3

(B) 4

(C) 6

(D) 12