Math@Mac Online Mathematics Competition

Wednesday, November 30, 2016

Instructions:
There are **ten** multiple choice questions. Select **one** of A, B, C, or D for each question. Check your answers carefully before submitting them online. You will only be able to submit your answers once. Non-programmable, non-graphing calculators are permitted. You may not use any other resources including web-based ones.

Good luck!

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1. Suppose that a bag contains the nine letters of the word AOXOMOXOA. If you take one letter out of the bag at a time and line them up left to right, what is the probability that you will spell the word AOXOMOXOA?

   (A) between 0.01 and 0.1

   (B) between 0.001 and 0.01

   (C) between 0.0001 and 0.001

   (D) between 0.00001 and 0.0001

2. A motorist travels the first 10 kilometers of a trip at 30km/hour. How fast would he have to drive for the next 10 kilometers if the total trip has an average speed of 50 km/hour?

   (A) 70 km/h

   (B) 80 km/h

   (C) 110 km/h

   (D) 150 km/h

3. For how many integers $n \geq 1$ does the expression $3^{2n+1} - 4^{n+1} + 6^n$ yield a prime number?

(A) 1

(B) 2

(C) 4

(D) infinitely many

4. A set C of positive integers is called *cool* if any two numbers in C are relatively prime. Bob wants to build a cool set from numbers between 1 and 30 (inclusive), in such a way that his set contains as many numbers as possible. How many different cool sets can he build?

(A) 12

(B) 16

(C) 24

(D) 30

5. Carly plots a point A, and then starts drawing rays starting at A, so that all angles she gets (i.e., between any two rays) are integer multiples of 10^{o}. What is the largest number of rays she can draw so that all the angles at A between *any two* rays (not just adjacent rays) are distinct?

(A) 5

(B) 6

(C) 7

(D) 8

6. The diagonals of square $ABCD$ meet at the point O. The bisector of the angle OAB meets the segment BO at N, and meets the segment BC at P. The length of NO is x. What is the exact length of PC?

(A) $x\left(\sqrt{2}+\frac{1}{2}\right)$

(B) $2x$

(C) $x\sqrt{5}$

(D) $x\left(\frac{5}{4}+\frac{\sqrt{2}}{2}\right)$

7. Alice walks two-thirds across a railroad bridge from point A to point B when she sees a train approaching at 45 km/h. She does a very quick calculation and realizes that if she runs at a certain speed r, she can make it to *either* end of the bridge and avoid the train. What is the *smallest* value of r, i.e., what is the slowest speed at which she can do it?

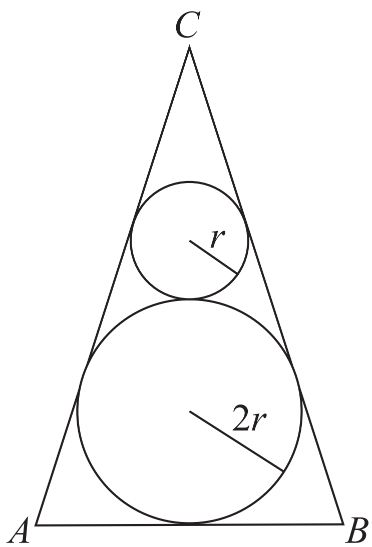
(A) 11 km/h

(B) 12 km/h

(C) 15 km/h

(D) 16 km/h

8. Triangle ABC is an isosceles triangle with two inscribed circles. The larger circle has radius $2r$, and the smaller circle with radius r is tangent to the larger circle and to the two equal sides of the triangle. The area of the triangle ABC is xr^2. What is x?



(A) $16\sqrt{2}$

(B) $8\sqrt{2}$

(C) $8\sqrt{2}+4$

(D) $8\left(\sqrt{2}+1\right)$

9. The sum

$$\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\frac{5}{3!+4!+5!}+\cdots+\frac{2016}{2014!+2015!+2016!}$$

is equal to

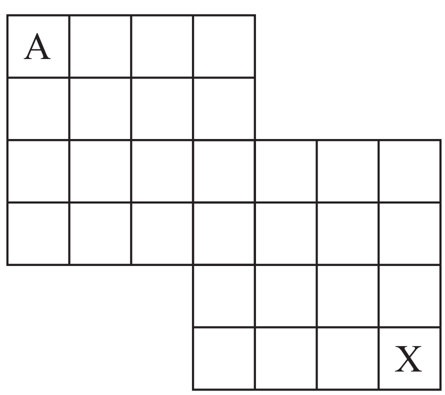
(A) $\dfrac{2016!+2}{2\cdot 2016!}$

(B) $\dfrac{2016!+1}{2\cdot 2016!}$

(C) $\dfrac{2016!-1}{2\cdot 2016!}$

(D) $\dfrac{2016!-2}{2\cdot 2016!}$

10. Consider a game board shown below. You are to move a piece from A to X by moving it to an adjacent square either to the right or down. In how many different ways can you do it?



(A) 245

(B) 280

(C) 300

(D) 320