Math@Mac Online Mathematics Competition

Wednesday, November 28, 2018

Instructions:
There are **ten** questions. For multiple choice, select **one** of the selected options for each question. Otherwise, fill in the blank space(s) as required.

Check your answers carefully before submitting them. You will only be able to submit your answers once. Non-programmable, non-graphing calculators are permitted. **You may not use any other resources including Internet-based ones.**

Good luck!

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1. Pick a random point (call it T) inside the rectangle with vertices $A(-1,0)$, $B(1,0)$, $C(1,1)$, and $D(-1,1)$. What is the probability that the point T is closer to the vertex A than it is to the point $P(2,1)$?

Express your answer as a fraction a/b.

2. Find the number of solutions (x,y) of the inequality

$$|x| + |y| < 50$$

where both x and y are integers.

3. Find all ordered triples (p,q,r) where p, q, and r are positive integers at least two of which are prime, such that

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$$

To answer this question, enter a single number: multiply the number of distinct solutions (i.e., distinct triples) by the largest value of p in all solutions.

4. Alice and Bob play a game where they take turns removing either 1, or 2, or 3, or 4, or 5 coins from a pile which originally contains 2018 coins. The player who takes the last coin(s) wins the game. (So, for instance, if there are 4 coins left then the player who goes next wins, as they can remove all 4 coins.)

If Alice goes first, there is a strategy that she can use to win. Using this strategy, the number of coins she should take on her first move is

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

5. The wheels of a car have a six-way radial symmetry (see diagram) and measure 0.75 m in diameter. A video camera which captures 12 frames per second creates a video of the car in motion. In the video, the wheels of the car appear to be not turning at all. If it is known that the car is traveling at a speed between 15 m/s and 20 m/s, which of the following is a possible speed for the car, measured in m/s?

(A) 4π

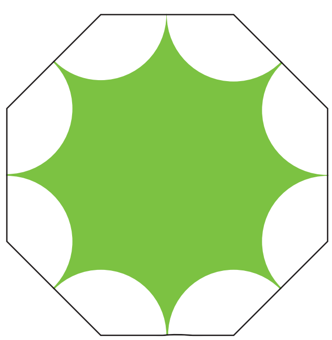
(B) 15

(C) 18

(D) 6π

(E) 24

6. Consider a regular octagon of side length 6. Draw arcs of radius 3 centred at each of the vertices of the octagon, thus creating circular sectors. The region inside the octagon but outside of the sectors is shaded. The area of the shaded region is:



(A) $9\left(12+8\sqrt{2}-2\pi\right)$

(B) $8\left(12+9\sqrt{2}-2\pi\right)$

(C) $9\left(8+9\sqrt{2}-2\pi\right)$

(D) $8\left(9+9\sqrt{2}-3\pi\right)$

(E) $9\left(8+8\sqrt{2}-3\pi\right)$

7. Given that $f(x) = 3x^2 - x + 4$, find all polynomials $g(x)$ so that $f(g(x)) = 3x^4 + 18x^3 + 50x^2 + 69x + 48$. The sum of all coefficients of all solutions $g(x)$ is

(A) 6

(B) 1/3

(C) 69

(D) 48

(E) None of the above

8. Four players are left in a game of paintball. Each player randomly selects one of the remaining three players. At the same time, they fire, and each player hits the player they targeted. What is the chance that no player remains "alive," i.e., that all four players are hit?

Write your answer as a fraction a/b.

9. You are afraid that you will forget your PIN number, but also do not want to write it down as you are afraid that someone will see it.

Your PIN number is a four-digit number $\overline{abcd}$, where a, b, c, and d are its digits.

Instead of writing down your PIN, you compute the sum $\overline{abcd} + \overline{abc} + \overline{ab} + a$, and write that number on a piece of paper. (Notation: if $a = 4$, $b = 3$, $c = 8$ and $d = 0$, then $\overline{abcd} = 4380$, $\overline{abc} = 438$, and so on.)

Few days later, you need your PIN, and indeed, you forgot what it is. However, you look at the piece of paper, and see the number 2018 written there. What is your PIN?

10. How many non-negative integers n are there such that $n + 2$ divides $(n + 18)^2$?

(A) 1

(B) 3

(C) 8

(D) More than 12