Math@Mac Online Mathematics Competition
Wednesday, November 27, 2019

Instructions:

There are **ten** questions. For multiple choice, select **one** of the selected options for each question. Otherwise, fill in the blank space(s) as required.

Check your answers carefully before submitting them. You will only be able to submit your answers once. Non-programmable, non-graphing calculators are permitted. **You may not use any other resources including Internet-based ones.**

Good luck!

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1. What is the sum of all positive integers from 1 to 500 which are NOT divisible by 4 or 5?

(A) $56,750$

(B) $68,500$

(C) $66,200$

(D) $75,000$

(E) $72,250$

2. Two polynomials $f(x)$ and $g(x)$ satisfy

$$f(x) \cdot g(x) = 4x^4 + 4x^3 + 13x^2 + 6x + 9$$
$$f(x) + g(x) = 4x^2 + 2x + 6$$

What is the product of all coefficients of $f(x)$? (Note: *all coefficients* includes free terms as well; for example, the product of all coefficients of the polynomial $x^3 - 3x^2 + 2x + 4$ is $(1)(-3)(2)(4) = -24$.)

3. Consider the following list of numbers:

$$y^2 + 9, \ \sqrt{x}, \ 68, \ 111, \ 182, \ 296, \ 481$$

The numbers x and y also satisfy the equation

$$3y^2 - 9y + \frac{x}{4} = 484,$$

which makes the value of y unique. Find that value of y.

4. How many prime numbers are there whose digits add up to four and are such that none of their digits are equal to zero?

5. A square room with dimensions 4m by 4m has a trap door in the very centre of the floor. The trap door is a 1m by 1m square. A round barrel with straight vertical sides and a circular bottom of diameter 1m is placed upright randomly in the room (so the flat round bottom is sitting flat on the floor). If every allowable placement of the barrel in the room is equally likely, what is the probability that some part of the barrel will cover some part of the trap door?
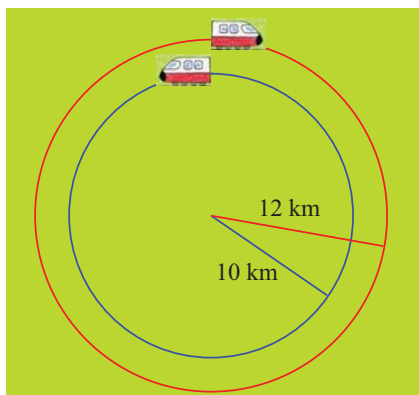
(A) $1/3 + \pi/36$

(B) $1/16$

(C) $\pi - 1/9$

(D) $1/4 + \pi/24$

(E) $4/9 + \pi/18$

6. Let $x_n = 1 + 2 + 3 + \cdots + n$. Find the largest value of n such that

$$x_n^3 - 127x_n^2 + 2331x_n - 2205 = 0$$

7. Two trains travel in circular paths nested inside one another (i.e., their centres are at the same point) Train 1 travels around a circle of radius 12 km at the speed of 150 km/h, and train 2 travels around a circle of radius 10 km at the speed of 120 km/h. If the trains begin at the same point on their respective circles and travel in opposite directions for 4 hours straight, how many times will they pass each other?

(A) 12 times or less

(B) 13

(C) 14

(D) 15

(E) 16 times or more

8. What are the last two digits of the number

$$2^{2^{2019}}$$

(i.e., 2 raised to the power 2^{2019})?

9. Kelly is creating strings from letters A, B, C, or D, with the following rules:

(a) A string must be 9 characters long

(b) Two adjacent characters must be different (e.g., ABCDABCDA is acceptable, but ABCDDACBA and ABBBACABA are not)
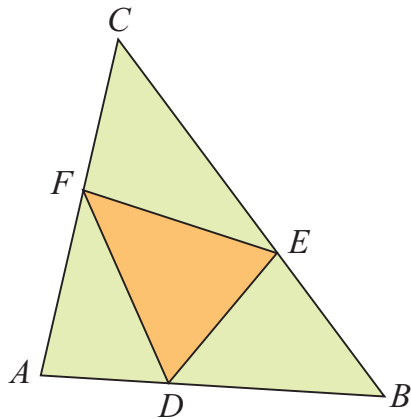
(c) No letter can appear more than 4 times (e.g., ABCACACAC and BABCBDBDA are acceptable, but ABACADADA is not).

By following these rules, how many different strings can Kelly make?

10. In the figure below ($\overline{XY}$ denotes the length of the line segment from X to Y)

$$\frac{\overline{AD}}{\overline{DB}} = \frac{\overline{BE}}{\overline{EC}} = \frac{\overline{CF}}{\overline{FA}} = \frac{2}{3}$$

The area of the triangle ABC is 1. What is the area of the triangle DEF?

(A) 1/4

(B) 6/25

(C) 7/25

(D) 3/16

(E) 5/16