From Sample Test 2

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(1st pdf link, but 2nd test)

2. Find the sum of the series

$$\sum_{n=2}^{\infty} n(n-1)x^{n-2}$$

(a) $\frac{6}{(1-x)^4}$ **(b)** $\frac{1}{(1-x)^2}$ **(c)** $\frac{-2}{(1-x)^3}$ **(d)** $\frac{2}{(1-x)^3}$ **(e)** $-\frac{6}{(1-x)^4}$

Sol'n: Recognizing this as a BINOMIAL SERIES.

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{n}{k} x^n.$$

$$(1+(-x))^k = \sum_{n=0}^{\infty} \binom{n}{k} (-x)^n$$

$$\binom{k}{n} = \frac{k(k-1)\cdots(k-n+1)}{n!}$$

Shifting index.

$$\sum_{n=2}^{\infty} n(n-1)x^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1)x^n$$

$$\binom{-3}{n} = \frac{(-3)(-4)\cdots(-3-n+1)}{n!}$$

cont'd on next page.

Factoring (-1) from each term in the numerator:

$$\binom{-3}{n} = (-1)^n \frac{\cancel{(3)}\cancel{(4)}\cdots(n+1)(n+2)}{(1)(2)\cancel{(3)}\cancel{(4)}\cdots\cancel{(n)}}$$

$$= (-1)^n \frac{(n+1)(n+2)}{2}.$$

$$(1+(-x))^{-3} = \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2} (-x)^n$$

$$= \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n$$

$$= \sum_{n=2}^{\infty} \frac{(n-1)(n-2)}{2} x^{n-2}$$

shifting index back.

$$\therefore \frac{2}{(1+x)^3} = \sum_{n=2}^{\infty} (n-1)(n-3) x^{n-2}$$