PEANO Existence Theorem

Consider the IVP: $y'(t) = f(t, y), \;\; y(t_0) = y_0,$
where $t \in \mathbb{R}, \;\; y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$ and
$f(t, y) = (f_1(t, y_1, y_2, \ldots, y_n), \ldots, f_n(t, y_1, y_2, \ldots, y_n)) \in \mathbb{R}^n$.

Assume f is **continuous** on some open set $\mathcal{D} \subseteq \mathbb{R} \times \mathbb{R}^n$ that contains the point (t_0, y_0).
Then there **exists** a solution to the IVP, defined for all t in **some** interval $[t_0 - h, t_0 + h]$, where h is a positive constant.

Fundamental Existence & Uniqueness Theorem

Consider the IVP: $y'(t) = f(t, y), \;\; y(t_0) = y_0,$
where $t \in \mathbb{R}, \;\; y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$ and
$f(t, y) = (f_1(t, y_1, y_2, \ldots, y_n), \ldots, f_n(t, y_1, y_2, \ldots, y_n)) \in \mathbb{R}^n$.

Assume f is **continuous** on some open set $\mathcal{D} \subseteq \mathbb{R} \times \mathbb{R}^n$ that contains the point (t_0, y_0), and assume that f is **locally Lipschitz** with respect to y on $\mathcal{D}$.

Then there **exists** a **unique** solution to the IVP, defined for all t in **some** interval $[t_0 - h, t_0 + h]$, where h is a positive constant.
The solution can either **be extended to the boundary of** $\mathcal{D}$ or is **unbound**.

Global Picard Theorem - Continuation

Consider the IVP: $y'(t) = f(t, y), \;\; y(t_0) = y_0,$
where $t \in \mathbb{R}, \;\; y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$ and
$f(t, y) = (f_1(t, y_1, y_2, \ldots, y_n), \ldots, f_n(t, y_1, y_2, \ldots, y_n)) \in \mathbb{R}^n$.

Assume f is **continuous** on some open set $\mathcal{D} \subseteq \mathbb{R} \times \mathbb{R}^n$ that contains the point (t_0, y_0), and assume that f is **uniformly Lipschitz** with respect to y on $\mathcal{D}$.

Then there **exist** a **unique** solution of the IVP that **can be extended to the boundary of** $\mathcal{D}$.