Structured population models

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Introduction

Cycling populations

- Many populations seem to **cycle**
- Population densities increase and decrease more or less regularly
- Over multiple generations/years
- Name an example of a population that cycles
- Interesting (but maybe overreported)

Population regulation

- Population regulation is a **necessary** condition for cycling
- Things must get worse/*per capita* growth rates must decrease as population increases
- We will not count the special case of non-overlapping cohorts in structured populations

What we have so far

- Unregulated, unstructured models: exponential growth or decline
- Unregulated, structured models: exponential growth or decline in long-term (averaged across cohorts)
- Regulated, unstructured models in continuous time
  - $R < 1$: stable equilibrium at zero
  - $R > 1$, no Allee effects: stable positive equilibrium
  - $R > 1$, Allee effects: unstable and stable positive equilibria

Crossing

- If two populations are following the same **deterministic** rules
- e.g. $dN/dt = Nr(N)$
- And are in the same **state**
- Then they must go the same place next
- Trajectories can't cross
- Cycles are impossible

Why not cycles?

- If two populations are following the same **deterministic** rules
- e.g. $dN/dt = Nr(N)$
- And are in the same **state**
- Then they must go the same place next
- Trajectories can't cross
- Cycles are impossible

What can allow cycles?

- Discrete time
- Age structure
- Delayed effects (e.g. childhood crowding lowers adult fecundity)
- Seasonal variation
- Interactions with other populations (prey/depletable resources, predators)
- (Regulation is always necessary)
- Give an example of one of these effects in a real population

Conceptual model

- Discrete-time, deterministic, unstructured, regulated
- $\lambda(N) = p(N) + f(N)$
- For simplicity we'll assume $p(N) = 0$
- $f(N)$ must decline as N gets large

Mathematical model

- \$N_{T+1} = $\lambda(N\_T)N\_TEquilibriumwhen:$

- We will assume $\lambda(N) = f_0 \exp(-N/N_c)$ (*Ricker* model)

- $N_{T+1} = N_T \exp(-N/N_c)$
- What is $R(0)$ for this model?

Simple case

- $f_0 = 1.5$, $N_c = 1$
- $f_0 > 1$ so the population should grow initially

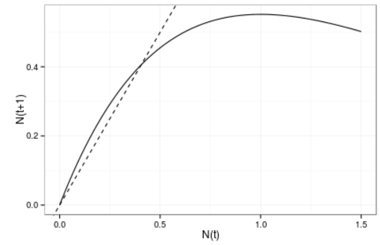


Figure 1: plot of chunk unnamed-chunk-1

Dynamics

- $f_0 = 1.5$, $N_c = 1$
- looks a lot like the continuous-time models we looked at previously

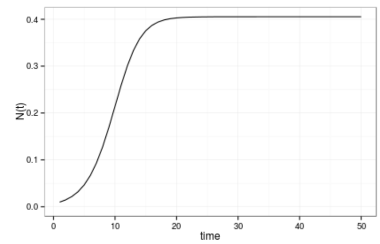


Figure 2: plot of chunk unnamed-chunk-2

Cobweb diagram

- Another way of visualizing the dynamics

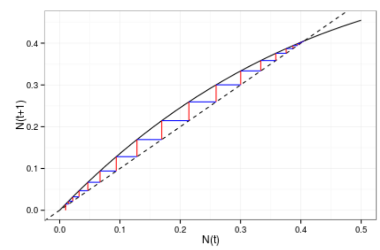


Figure 3: plot of chunk unnamed-chunk-3

Unhappy populations

- $f_0 < 1$

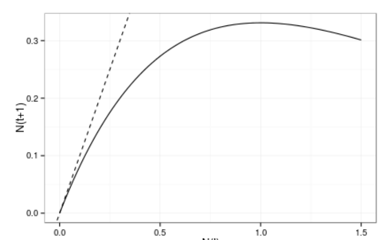


Figure 4: plot of chunk unnamed-chunk-4

Unhappy populations: dynamics

- Population declines to zero (stable equilibrium)

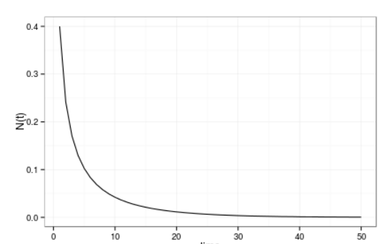


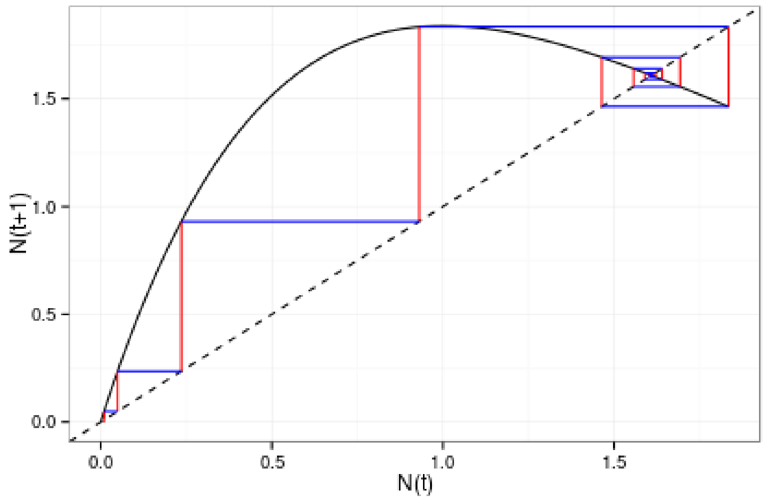
Figure 5: plot of chunk unnamed-chunk-5

Overshooting

- Interesting stuff starts to happen if we increase f_0 (steeper curve)
- $f_0 = 5$
- overshooting
- only possible in discrete time!

Overshooting: cobweb

- Equilibrium is now to the right of the peak



- Map has a negative slope at the equilibrium

Cycles

- $f_0 = 10$
- even stronger overshoot
- now we get a two-year cycle

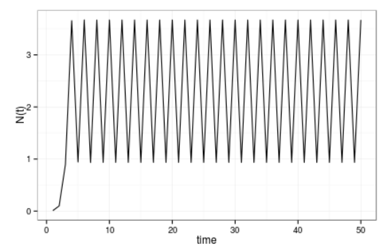


Figure 7: plot of chunk unnamed-chunk-8

Cycles: cobweb

- zoom into a cycle

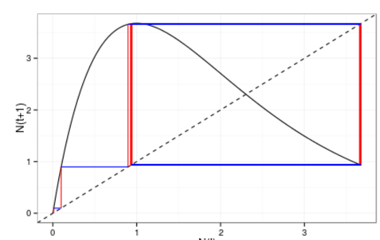


Figure 8: plot of chunk unnamed-chunk-9

Cycles: starting from within

- same f_0, N_c but different start
- zoom out to a cycle

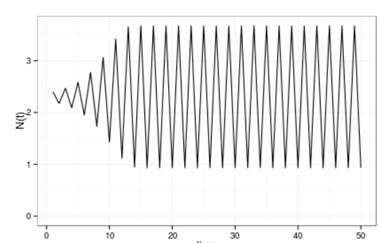
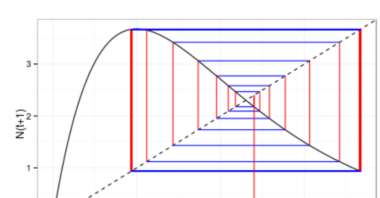


Figure 9: plot of chunk unnamed-chunk-10

Cobweb

- start near equilibrium
- unstable!



Even more extreme

- $f_0 = 15$
- even more overshooting
- 4-point cycle (maybe 8-point?)

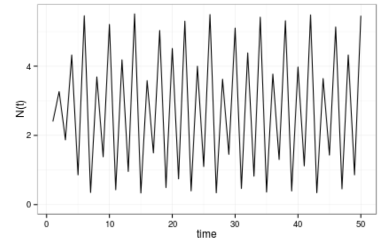


Figure 11: plot of chunk unnamed-chunk-12

Cobweb

- $f_0 = 15$
- even more overshooting
- 4-point cycle (maybe 8-point?)

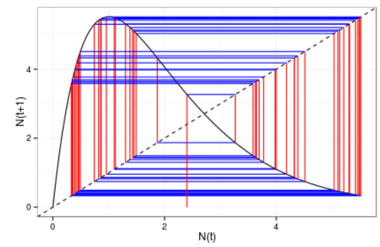


Figure 12: plot of chunk unnamed-chunk-13

Chaos!

- $f_0 = 25$
- doesn't seem to be settling down

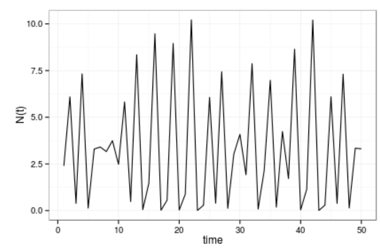


Figure 13: plot of chunk unnamed-chunk-14

Longer time scale

- run for 1000 steps
- never settles down

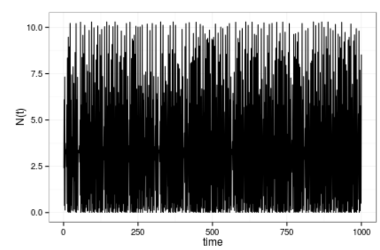


Figure 14: plot of chunk unnamed-chunk-15

Cobweb

- fills in entire space (maybe with gaps?)

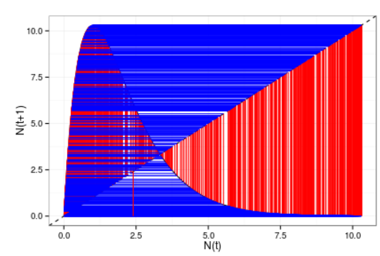


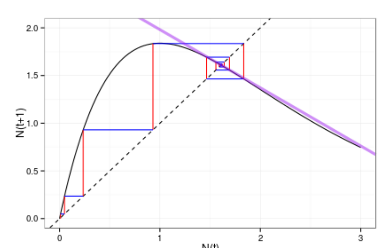
Figure 15: plot of chunk unnamed-chunk-16

Stability of equilibria

- Slope of the curve at equilibrium is called J
- Determines population dynamics around equilibrium

Stability of equilibria

- More steeply negative = bigger overshoot



Understanding J

- J says how the *deviation* from equilibrium grows in the next time step
- $0 < J < 1$: stable, direct approach
- $-1 < J < 0$: stable, damped oscillations
- $J < -1$: unstable (cycles/chaos)

Is there some biology here?

- The simple mechanism here produces a broad spectrum of dynamics
- But very limited: crashes must happen in one year
- Basic idea is reasonable (overshoot/overcompensation causes instability)
- Biologists are still arguing about what drives cycles in real populations
- Chaos or noise? (May 1976; Hastings et al. 1993)

References

Hastings, A, C L Hom, S Ellner, P Turchin, and H C J Godfray. 1993. "Chaos in Ecology: Is Mother Nature a Strange Attractor?*." *Annual Review of Ecology and Systematics* 24 (1): 1–33. doi:10.1146/annurev.es.24.110193.000245. http://www.annualreviews.org/doi/abs/10.1146/annurev.es.24.110193.000245.

May, Robert M. 1976. "Simple Mathematical Models with Very Complicated Dynamics." *Nature* 261 (5560): 459–467. http://isites.harvard.edu/fs/docs/icb.topic1313269.files/Lecture22/May_Simple%20mathematical%20models%20with%20very_76.pdf.